TYPE-1 FUZZY FUNCTIONS APPROACH BASED ON THE PARTICLE SWARM OPTIMIZATION FOR TIME SERIES FORECASTING

Ali Zafer Dalar

Abstract

Type-1 fuzzy functions (T1FFs) are a type of fuzzy logic that can be used to model uncertain systems and are particularly useful in situations where the information available is incomplete or uncertain. In this study, the T1FFs approach based on the particle swarm optimization (PSO) algorithm is proposed for time series forecasting, and the forecasting performance of the approach is compared with other time series forecasting methods on real-world time datasets such as ISE 100, FTSE 100, NASDAQ 100 and TAIEX. The proposed approach, used to develop robust and accurate time series forecasting models, involves using T1FFs to model the uncertain and incomplete information in the time series data and using the PSO algorithm, a metaheuristic optimization algorithm that is used to optimize the parameters of the T1FFs, to improve the forecasting accuracy. This approach provides better results than traditional time series forecasting in situations where the data is noisy or uncertain.

Key words: Time series forecasting, Particle swarm optimization, Type-1 fuzzy functions approach, Robust

JEL Code: C53, C61, E17

1 Introduction

Time series forecasting (Song and Chissom 1993a, 1993b; Chen 1996, 2002; Aladag et al. 2012; Egrioglu et al. 2014, 2015, 2016) is an essential tool in various domains, including finance, economics, weather prediction, and energy management. Accurate forecasting models enable decision-makers to make informed choices, optimize resource allocation, and mitigate risks. However, traditional time series forecasting methods often struggle to cope with noisy, uncertain, or incomplete data, which is commonplace in real-world scenarios.

Type-1 fuzzy functions (T1FFs) approach that proposed by Turksen (2008) offers a powerful framework for modeling uncertain systems, as they are capable of handling uncertainty and imprecision inherent in real-world data. Particle swarm optimization (PSO), proposed by (Kennedy and Eberhart 1995), on the other hand, is an effective metaheuristic optimization algorithm inspired by the social behavior of birds and fish. It has been successfully applied to a wide range of optimization problems.

In this study, a novel approach to time series forecasting that leverages the strengths of type-1 fuzzy functions and the particle swarm optimization algorithm is proposed. This approach, referred to as the T1FFs-PSO algorithm, aims to create robust and accurate forecasting models by utilizing T1FFs to model uncertain and incomplete information in time series data and employing the PSO algorithm to optimize the parameters of the T1FFs, ultimately improving the forecasting accuracy.

The performance of the proposed T1FFs-PSO approach is evaluated using real-world time series datasets such as ISE 100, FTSE 100, NASDAQ 100, and TAIEX. The results are compared with other time series forecasting methods to demonstrate the forecasting performance of the T1FFs-PSO algorithm.

This article is structured as follows: Section 2 provides background information on type-1 fuzzy functions, and the particle swarm optimization algorithm. Section 3 presents a detailed description of the proposed T1FFs-PSO method, including its formulation and integration of the two techniques. Section 4 outlines the experimental setup, including the datasets and performance metrics used for evaluation. Section 5 discusses the results and compares the performance of the proposed method with existing techniques. Finally, Section 6 concludes the article and suggests future research directions.

2 Background

This section provides a brief overview of the key concepts and techniques used in this study, namely T1FFs, and PSO.

2.1 Type-1 Fuzzy Functions Approach

Type-1 fuzzy functions approach is a type of fuzzy logic used to model uncertain systems. Unlike crisp logic, fuzzy logic is capable of handling imprecision and uncertainty by allowing variables to take on membership values between 0 and 1. T1FFs are particularly useful in situations where the available information is incomplete or uncertain, such as noisy or incomplete time series data. They provide a flexible and intuitive framework for representing and manipulating uncertain information.

2.2 Particle Swarm Optimization

Particle swarm optimization is a metaheuristic optimization algorithm inspired by the social behavior of birds and fish. The algorithm mimics the way these creatures search for food in a cooperative and decentralized manner. In PSO, a swarm of particles moves through a multi-dimensional search space to find the optimal solution. Each particle adjusts its position based on its own best-known position and the best-known position of the swarm. The algorithm has been successfully applied to various optimization problems, including parameter optimization for machine learning models and optimization of complex functions.

In this study, we propose a novel approach to time series forecasting that combines the strengths of T1FFs and the PSO algorithm. The proposed T1FFs-PSO method aims to develop robust and accurate forecasting models by using T1FFs to model uncertain and incomplete information in time series data and employing the PSO algorithm to optimize the parameters of the T1FFs, improving the forecasting accuracy. The proposed approach incorporates these techniques and guides the optimization process, ultimately providing better results than other time series forecasting methods in the literature.

3 Proposed Method

In this section, we describe the proposed method for time series forecasting using type-1 fuzzy functions and particle swarm optimization. The approach aims to create a robust and accurate forecasting model by combining the strengths of T1FFs and PSO.

3.1 T1FFs Approach for Time Series Forecasting

To model the uncertain and incomplete information in the time series data, we employ T1FFs. The input time series data, a set of fuzzy rules, and membership functions for each rule are taken. These rules are created based on the patterns and relationships found within the historical data. The membership functions assign a degree of membership to each data point in the time series, representing the uncertainty and imprecision of the data. The fuzzy rules and membership functions together form the basis of the T1FFs-based forecasting model (Aladag et al. 2016; Dalar et al. 2013, 2015).

3.2 PSO for Parameter Optimization

To improve the forecasting accuracy of the T1FFs-based model, we use the PSO algorithm to optimize the parameters of the membership functions. PSO algorithm iteratively searches for the optimal parameters in a multi-dimensional search space. Each particle in the swarm represents a potential solution, and the algorithm guides the particles to explore the search space and converge toward the best solution. The particles adjust their positions based on their own best-known positions and the best-known position of the swarm.

3.3 Integration of T1FFs and PSO

The proposed approach integrates the T1FFs and PSO techniques to create a comprehensive time series forecasting model. The approach takes the input time series data and other relevant parameters, such as the number of fuzzy rules, the type of membership functions, and the PSO parameters. Then applies the T1FFs to model the uncertain and incomplete information in the time series data and uses the PSO algorithm to optimize the parameters of the membership functions, thus improving the overall forecasting accuracy.

Here is the algorithm of the proposed approach step by step:

Step 1. Initialize parameters and variables:

Set initial values for the PSO algorithm, such as inertia weight (w), acceleration coefficients (c_1, c_2), swarm size (ps), and the maximum number of iterations (maxitr).

Step 2. Apply FCM clustering on the input data to obtain cluster centers (C) and membership matrix (U).

$$J(U, C) = \sum \left(\sum \left(u_{ij}^{m} \times \|x_i - c_j\|^2 \right) \right) \qquad (1)$$

where u_{ij} is the membership value of the i-th data point to the j-th cluster, x_i is the i-th data point, c_j is the center of the j-th cluster, and m is the fuzziness parameter.

Step 3. Initialize the PSO particles' positions (A_1, A_2) and velocities (V_1, V_2).

Step 4. Calculate the fitness of each particle for the training set. Fitness function:

$$RMSE = \sqrt{(\Sigma((y_i - \hat{y}_i)^2)/N)} \qquad (2)$$

where y_i is the actual value, $\hat{y}_i$ is the predicted value, and N is the number of data points.

Step 5. Set the initial personal best positions (pbest1, pbest2) and fitness (pbestf) for each particle.

Step 6. Determine the global best position (gbest1, gbest2) and fitness (gbestf) among all particles.

Step 7. For each iteration in the PSO algorithm:

Step 7.1. Update the inertia weight (w), and acceleration coefficients (c_1, c_2) based on the iteration number.

$$w = w_{max} - (w_{max} - w_{min}) * (iter/maxitr) \quad (3)$$

where w_{max} and w_{min} are the maximum and minimum values for the inertia weight, *iter* is the current iteration number, and *maxitr* is the maximum number of iterations.

Step 7.2. Update the velocities (V_1, V_2) and positions (A_1, A_2) of each particle using the PSO update equations.

$$V_i(t+1) = w * V_i(t) + c_1 * r_1 * \left(pbest_i - A_i(t)\right) + c_2 * r_2 * \left(gbest - A_i(t)\right) \quad (4)$$

where $V_i(t)$ is the velocity of the i-th particle at iteration *t*, *w* is the inertia weight, c_1 and c_2 are the acceleration coefficients, r_1 and r_2 are random numbers in the range [0, 1], $pbest_i$ is the personal best position of the i-th particle, $A_i(t)$ is the position of the i-th particle at iteration t, and *gbest* is the global best position.

$$A_i(t+1) = A_i(t) + V_i(t+1) \quad (5)$$

where $A_i(t+1)$ is the position of the i-th particle at iteration $t+1$.

Step 7.3. Calculate the fitness of each updated particle.

Step 7.4. Update the personal best positions (pbest1, pbest2) and fitness (pbestf) for each particle if the new fitness is better.

Step 7.5. Update the global best position (gbest1, gbest2) and fitness (gbestf) if a new best is found among all particles.

Step 8. After completing the PSO iterations, compute the RMSE and MAPE for both the training set ($RMSE_{train}, MAPE_{train}$) and the test set ($RMSE_{test}, MAPE_{test}$) based on the global best positions (gbest1, gbest2).

Step 9. Follow these steps:

Step 9.1. Calculate the membership values for the input data using the global best positions (gbest1, gbest2) and the cluster centers (C) and membership matrix (U) from FCM clustering.

Step 9.2. Apply the type-1 fuzzy functions using the binary and continuous parameters found by the PSO algorithm.

Step 9.3. Construct a regression matrix X using the membership values and the input data, and calculate the coefficients β using the Moore-Penrose pseudoinverse (Barata and Hussein 2012).

$$\beta = pinv(X' \times X) \times X' \times Y \quad (6)$$

where *pinv* denotes the Moore-Penrose pseudoinverse. The pseudoinverse of a matrix is a generalization of the inverse that can be used when the matrix is not invertible. The pseudoinverse can be used to obtain a solution that is “as close as possible” to the true solution, even when the matrix is ill-conditioned.

Step 9.4. Compute the forecasts as a weighted sum of the predictions from each fuzzy set, where the weights are the membership values.

Step 9.5. Calculate the RMSE and MAPE for the training set and the test set.

Step 10. Calculate the RMSE (given in equation 2) and MAPE for a given set of predicted and actual values.

$$MAPE = (100/N) * \Sigma(|(y_i - \hat{y}_i)/y_i|) \qquad (7)$$

where y_i is the actual value, $\hat{y}_i$ is the predicted value, and N is the number of data points.

Step 11. The final outputs include the RMSE and MAPE for the training and test sets, as well as the predicted values for both sets and the global best positions (gbest1, gbest2).

4 Experimental Setup

In this section, we describe the experimental setup and the dataset used to evaluate the performance of the proposed type-1 fuzzy functions approach based on particle swarm optimization for time series forecasting.

Four different stock exchange time series (ISE 100, FTSE 100, NASDAQ 100 and TAIEX) are used in order to show the forecasting performance of the proposed method. The datasets are divided into a training set and a testing set, with the testing set (*ntest*) containing the last 10 samples of the time series.

The T1FFs-PSO method’s performance is evaluated via the dataset for different values of the number of lag variable (*nl*) and the number of cluster (*nc*). The number of lagged variable is taken between 2 and 10, with increment 1. The number of cluster is taken between 3 and 10, with increment 1. The α-cut threshold for membership value is taken between 0 and 0.4, with increment 0.1. The maximum number of iterations for the PSO algorithm is set to 100. The performance of the forecasting models is measured using the RMSE and MAPE for both the training and testing datasets.

The T1FFs-PSO method’s performance will be compared with other time series forecasting methods such as ANFIS-G (Jang 1993), ANFIS-S (Jang 1993), MANFIS (Egrioglu et al. 2014), and T1FFs (Turksen 2008). This comparison will help to validate the effectiveness of the proposed T1FFs-PSO method and demonstrate its suitability for handling complex and uncertain time series data.

5 Results and Discussion

In this section, we present the results of the proposed T1FFs-PSO method applied to the ISE 100, FTSE 100, NASDAQ 100 and TAIEX time series data of the year 2014 and compare its performance with other forecasting methods, including ANFIS-G, ANFIS-S, MANFIS, and T1FFs.

The Table 1 shows the RMSE values of forecasted values for the test data for the 10 days for each method.

Tab. 1: RMSE values of all methods

Serie	*ntest*	Error Metric	ANFIS-G	ANFIS-S	MANFIS	T1FFs	Proposed Method
ISE 100	10	RMSE	1092.9095	1145.0725	**1023.9106***	1062.9051	1061.1744
		MAPE	0.0105	0.0110	0.0090	0.0101	0.0095
FTSE 100	10	RMSE	82.7541	68.6621	63.3645	62.4647	**62.2393***
		MAPE	0.0084	0.0085	0.0071	0.0074	0.0075
NASDAQ 100	10	RMSE	55.2201	47.2483	43.3360	42.6593	**42.4276***
		MAPE	0.0095	0.0084	0.0077	0.0075	0.0074
TAIEX	10	RMSE	65.3081	63.2321	58.8302	59.5211	**56.9990***
		MAPE	0.0059	0.0057	0.0053	0.0054	0.0056
	medRMSE		74.0311	65.9471	61.0973	60.9929	**59.6191***

In the experiment conducted with the four different stock exchange time series data, the three out of four best RMSE values were found by the proposed method. Median RMSE value of the all data was obtained by T1FFs-PSO. Parameters's value of the proposed method is given in Table 2.

Tab. 2: Parameters' value of the proposed method

Serie	*ntest*	*# of lag.*	*# of cluster*	*α-cut*	Membership transformations				
					Cubic root	Square root	Quadratic	Cubic	Exp.
ISE 100	10	4	4	0.1901	1	1	0	1	0
FTSE 100	10	3	4	0.0572	1	1	1	1	1
NASDAQ 100	10	3	8	0.0311	1	1	1	1	1
TAIEX	10	3	10	0.1908	1	1	1	1	0

Table 2 shows that, for example, the proposed method, for TAIEX, obtained the best RMSE value when X contains all transformations of membership except exponential, α-cut =

0.1908, number of lagged variable is taken 3, and number of cluster is taken 10. This means that the proposed T1FFs-PSO method achieved its highest forecasting accuracy with these parameter values. By including the transformations, the model can capture various relationships between input variables and the output variable.

α-cut = 0.1908 represents the threshold used for filtering out insignificant fuzzy sets during the model optimization process. By choosing this value, the model effectively eliminates fuzzy sets that do not contribute significantly to the forecasting performance, thus simplifying the model and reducing computational complexity.

The parameter # of lag. = 3 represents the order of the time series model, which is the number of lagged observations used as inputs in the forecasting model. In this experiment, it was found that using three lagged observations provided the best forecasting accuracy.

The # of cluster = 10 parameter represents the number of fuzzy sets in the T1FFs model. This value determines the granularity of the fuzzy partitions and the complexity of the model. With # of cluster = 10, the T1FFs model has ten fuzzy sets, which proved to be the most suitable level of complexity for modeling the Giresun Province daily temperature time series data.

The combination of these parameter values resulted in the lowest RMSE value of 56.9990, indicating that the proposed T1FFs-PSO method provides more accurate forecasts compared to other methods tested in the experiment. This demonstrates the potential of the T1FFs-PSO approach for time series forecasting. It can be inferred that the combination of T1FFs to model the uncertainty and the PSO algorithm to optimize the parameters of the T1FFs leads to better forecasting accuracy compared to other methods.

In summary, the experimental results demonstrate the effectiveness of the proposed T1FFs-PSO method for time series forecasting, and it shows potential for further research and application in various fields where accurate forecasting is crucial.

6 Conclusion

In this study, we proposed a novel time series forecasting method based on type-1 fuzzy functions and particle swarm optimization algorithm. The main motivation behind this approach was to address the challenges of modeling uncertain and incomplete information in time series data, which are common in real-world scenarios. The proposed method was applied to the four different stock exchange time series data and compared with several well-established forecasting techniques.

In conclusion, the T1FFs-PSO method offers a promising approach for time series forecasting. By leveraging the flexibility of T1FFs to model uncertain information and the optimization capabilities of the PSO algorithm, this method provides robust and accurate forecasts that can be beneficial in various applications, such as weather forecasting, finance, and resource management. Future research could explore the potential of incorporating other optimization techniques or hybrid models to further improve the forecasting performance of the T1FFs approach. Additionally, testing the method on different types of time series data and application domains could provide valuable insights into its generalizability and potential use cases.

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Contact

Ali Zafer Dalar
Giresun University, Department of Statistics
Gaziler Mh. Prof. Ahmet Taner Kışlalı Cd. No:1, Merkez/Giresun
ali.zafer.dalar@giresun.edu.tr