ROBUST ESTIMATORS FOR THE PARAMETERS OF THE JOINT LOCATION AND SCALE MODEL

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Abstract

Joint modeling location and scale have still been of interest to many authors for modeling variance heterogeneity in homogenous populations. In general, the joint location and scale model based on the normal distribution was considered because of the tractability of normal distribution. However, in the case of outlying observations besides heteroscedasticity, there need for some robust methods to deal with outlying observations. In this study, we consider the joint location and scale model based on the M-regression estimation method with the Huber function. We provide the estimation procedure for the parameters of the joint location and scale model. For the application, we conduct a simulation study and a real data example to compare the performances of the estimators in the case of with outliers and without outliers. We give both estimation results obtained from the joint location and scale model based on the normal distribution and M-regression estimation method in the numerical analysis.

Keywords: joint location and scale, M-regression estimation, normal distribution

JEL Code: C13, C15, C18

Introduction

Regression analysis is a very popular tool for examining the relationship between variables. In regression models, one of the main assumptions is variance homogeneity. However, it is impossible to ensure variance homogeneity for every data set. Therefore, joint location and scale models were offered to deal with heteroscedasticity, a commonly used model in recent years. Many authors studied the joint location and scale model (JLSM). For instance, Park (1966) proposed a log-linear model for the scale parameter. Harvey (1976) described a likelihood ratio test for heteroscedasticity. Aitkin (1987) offered to model the normal regression when the variances are not homogenous. Verbyla (1993) proposed to model normal regression for log-linearly dependent variances. Engel and Huele (1996) extended the Taguchi-type robust designs using the response surface approximation.

Furthermore, the JLSMs were considered in robust analyses such as Taylor and Verbyla (2004) proposed the JLSM of t distributions; Lin and Wang (2009) offered a robust approximation for the JLSM applied in longitudinal data; Lin and Wang (2011) proposed to use Bayesian analysis in the JLSM of the t distribution for longitudinal data; Li and Wu (2014) investigated the JLSM of the skew-normal distribution; Doğru and Arslan (2019) offered a joint location, scale, and skewness models of skew Laplace normal distribution; Doğru et al. (2019) used the mixtures of skew Laplace normal distributions in joint location, scale, and skewness models and Eskin and Doğru (2023) explored a JLSM based on the generalized normal distribution. In addition to that, robust methods were used for JLSM in some studies. For example, Güney et al. (2021) used the least favorable distribution for M-regression estimation in the variable selection of JLSM, and Hatipoğlu (2023) proposed some different robust methods in her master thesis.

In this paper, we consider the JLSM based on the M-regression estimation (JLSM-M) with the Huber function. We will summarize the estimation methodology in Section 1. In the next section, we provide a simulation study and a real data example to illustrate the performance of the JLSM-M over the JLSM based on the normal distribution (JLSM-N). Then, we finalize the paper with a conclusion section.

1 Joint location and scale model based on normal distribution

Let us consider the following JLSM-N:

$$\begin{cases} \quad y_i \sim N(\mu_i, \sigma_i^2), \;\; i = 1,2,\dots,n \\ \mu_i = \boldsymbol{x}_i^T \boldsymbol{\beta} \,, \\ \log \sigma_i^2 = \boldsymbol{z}_i^T \boldsymbol{\gamma} \,, \end{cases} \tag{1}$$

where y_i is the ith observed response, $\boldsymbol{x}_i = \left(x_{i1}, \dots, x_{ip}\right)^T$ and $\boldsymbol{z}_i = \left(z_{i1}, \dots, z_{iq}\right)^T$ are observed covariates corresponding to y_i, $\boldsymbol{\beta} = \left(\beta_1, \dots, \beta_p\right)^T$ is a $p \times 1$ vector of unknown parameters in the location model, and $\boldsymbol{\gamma} = \left(\gamma_1, \dots, \gamma_q\right)^T$ is a $q \times 1$ vector of unknown parameters in the scale model. It is not necessary that covariate vectors $\boldsymbol{x}_i$ and $\boldsymbol{z}_i$ are identical.

1.1 Joint location and scale model based on the M-regression estimation

Let $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\gamma})^T = \left(\beta_1, \dots, \beta_p; \gamma_1, \dots, \gamma_q\right)^T$ be the parameter vector for the JLSM-N defined in (1). To obtain the robust estimators for $\boldsymbol{\theta}$, we modify the log-likelihood function for the JLSM-N as follows:

$$\ell(\boldsymbol{\theta}) = \ell(\boldsymbol{\beta}, \boldsymbol{\gamma}) = -\frac{1}{2}\sum_{i=1}^{n} \boldsymbol{z}_i^T\boldsymbol{\gamma} - \sum_{i=1}^{n} \rho\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\frac{\boldsymbol{z}_i^T\boldsymbol{\gamma}}{2}}}\right).$$

where $\rho(\cdot)$ indicates Huber's ρ function defined by Huber (1964) with the following function:

$$\rho_k(x) = \begin{cases} x^2 & , \quad |x| \le k \\ 2k|x| - k^2 & , \quad |x| > k \end{cases}.$$

Further, the first and the second derivatives of ρ function has the following forms:

$$\rho_k'(x) = \psi_k(x) = \begin{cases} x & , \quad |x| \le k \\ sgn(x)k & , \quad |x| > k \end{cases},$$

$$\rho_k''(x) = \psi'_k(x) = \begin{cases} 1 & , \quad |x| \le k \\ 2\delta(x)k & , \quad |x| > k. \end{cases}$$

Here, $\delta(\cdot)$ is the delta-Dirac function and k is the tunning constant that controls the efficiency.

First, we differiantite the $\ell(\boldsymbol{\theta})$ concerning the parameters of interest and have the following score function:

$$U(\boldsymbol{\theta}) = \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \left(U_1{}^T(\boldsymbol{\beta}), U_2{}^T(\boldsymbol{\gamma})\right)^T,$$

where

$$U_1(\boldsymbol{\beta}) = \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} \frac{1}{e^{\boldsymbol{z}_i^T\boldsymbol{\gamma}/2}} \psi\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\boldsymbol{z}_i^T\boldsymbol{\gamma}/2}}\right),$$

$$U_2(\boldsymbol{\gamma}) = \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\gamma}} = -\frac{1}{2}\sum_{i=1}^{n} \boldsymbol{z}_i + \frac{1}{2}\sum_{i=1}^{n} \left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\boldsymbol{z}_i^T\boldsymbol{\gamma}/2}}\right) \psi\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\boldsymbol{z}_i^T\boldsymbol{\gamma}/2}}\right) \boldsymbol{z}_i^T.$$

Further, the second derivative of $\ell(\boldsymbol{\theta})$, information matrix, concerning $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ can be obtained as:

$$H(\boldsymbol{\theta}) = \frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T} = \begin{bmatrix} \frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T} & \frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}^T} \\ \frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\gamma} \partial \boldsymbol{\beta}^T} & \frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\gamma} \partial \boldsymbol{\gamma}^T} \end{bmatrix},$$

where

$$\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T} = -\sum_{i=1}^{n} \frac{1}{e^{\boldsymbol{z}_i^T\boldsymbol{\gamma}}} \rho''\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\frac{\boldsymbol{z}_i^T\boldsymbol{\gamma}}{2}}}\right) \boldsymbol{x}_i \boldsymbol{x}_i^T,$$

$$\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\gamma}^T} = -\frac{1}{2}\sum_{i=1}^{n} \frac{1}{e^{\frac{\boldsymbol{z}_i^T\boldsymbol{\gamma}}{2}}} \rho'\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\frac{\boldsymbol{z}_i^T\boldsymbol{\gamma}}{2}}}\right) \boldsymbol{x}_i \boldsymbol{z}_i^T + \frac{1}{2}\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\boldsymbol{z}_i^T\boldsymbol{\gamma}}}\right) \rho''\left(\frac{y_i - \boldsymbol{x}_i^T\boldsymbol{\beta}}{e^{\frac{\boldsymbol{z}_i^T\boldsymbol{\gamma}}{2}}}\right) \boldsymbol{x}_i \boldsymbol{z}_i,$$

$$\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\gamma} \partial \boldsymbol{\beta}^T} = -\frac{1}{2}\sum_{i=1}^{n} \frac{1}{e^{\frac{\mathbf{z}_i^T \boldsymbol{\gamma}}{2}}} \rho' \left(\frac{y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}}{e^{\frac{\mathbf{z}_i^T \boldsymbol{\gamma}}{2}}} \right) \boldsymbol{x}_i \boldsymbol{z}_i^T + \frac{1}{2}\left(\frac{y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}}{e^{\mathbf{z}_i^T \boldsymbol{\gamma}}} \right) \rho'' \left(\frac{y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}}{e^{\frac{\mathbf{z}_i^T \boldsymbol{\gamma}}{2}}} \right) \boldsymbol{x}_i \boldsymbol{z}_i,$$

$$\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\gamma} \partial \boldsymbol{\gamma}^T} = -\frac{1}{4}\sum_{i=1}^{n} \frac{y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}}{e^{\mathbf{z}_i^T \boldsymbol{\gamma}/2}} \left(\frac{y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}}{e^{\mathbf{z}_i^T \boldsymbol{\gamma}/2}} \right) \boldsymbol{z}_i \boldsymbol{z}_i^T - \frac{1}{4} \frac{(y_i - \boldsymbol{x}_i^T \boldsymbol{\beta})^2}{e^{\mathbf{z}_i^T \boldsymbol{\gamma}}} \rho'' \left(\frac{y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}}{e^{\mathbf{z}_i^T \boldsymbol{\gamma}/2}} \right) \boldsymbol{z}_i \boldsymbol{z}_i.$$

Finally, to get the M-estimators for JLSM we use the following estimation procedure algorithm.

Algorithm:

Step 1. Take $\boldsymbol{\theta}^{(0)} = \left((\boldsymbol{\beta}^{(0)})^T, \left(\boldsymbol{\gamma}^{(0)}\right)^T, s^{(0)}\right)^T$ as a starting point.

Step 2. Use $\boldsymbol{\beta}^{(k)}$, $\boldsymbol{\gamma}^{(k)}$, where $\boldsymbol{\theta}^{(k)} = \left(\left(\boldsymbol{\beta}^{(k)}\right)^T, \left(\boldsymbol{\gamma}^{(k)}\right)^T\right)^T$; and calculate the following $(k+1)$th estimates:

$$\widehat{\boldsymbol{\theta}}^{(k+1)} = \widehat{\boldsymbol{\theta}}^{(k)} + \left[-H(\widehat{\boldsymbol{\theta}}^{(k)})\right]^{-\mathbf{1}} U\left(\widehat{\boldsymbol{\theta}}^{(k)}\right).$$

Step 3. Repeat these 2 steps until convergence is satisfied.

2 Applications

This section consists of a simulation study and a real data example to illustrate the applicability of the JLSM-M over the JLSM-N.

2.1 Simulation study

This section provides a simulation study to compare the performance of the estimators obtained from the JLSM-M and the estimators obtained from the JLSM-N in the case of outliers and without outliers. We generate the random sample from JLSM-N given in (1), where $\boldsymbol{x}_i \sim Uniform(-1,1)$ and $\boldsymbol{z}_i \sim Uniform(-1,1)$. We consider the true parameter values as $\boldsymbol{\beta} = (1,0.7,0.5)^T$and $\boldsymbol{\gamma} = (0.5,0.3,0.2)^T$. For outlier case, the outliers are generated from the following mixture normal model:

$$(1-w)N(\mu_{1i}, \sigma_{1i}^2) + wN(\mu_{2i}, \sigma_{2i}^2),$$

where μ_{1i} and σ_{1i}^2 are set as the same parameter values as without outlier case and for μ_{2i} and σ_{2i}^2 we consider $\boldsymbol{\beta} = (5,5,0)^T$ and $\boldsymbol{\gamma} = (0.7,0.5,0)^T$. Additionally, the contamination proportion which shows the outliers rate is taken as 0.1. The sample sizes are set as $n = 50,100,150,200$ and the replication number is $N = 500$. For the comparison, we use the bias and the mean squared error (MSE) values with the following formulas:

$$\widehat{bias}(\hat{\theta}) = \bar{\theta} - \theta, \quad \widehat{MSE}(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_i - \theta\right)^2,$$

where θ is the real parameter value, $\hat{\theta}_j$ is the ith estimate of θ and $\bar{\theta} = \frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_j$.

Tab. 1: Bias and MSE values for JLSM-N and JLSM-M estimates in the case of without outliers

	$n = 50$				$n = 100$			
	JLSM-N		JLSM-M		JLSM-N		JLSM-M	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
β_1: 1	0.0062	0.0292	0.0027	0.0171	0.0014	0.0138	0.0010	0.0060
β_2: 0.7	-0.0014	0.0759	0.0171	0.1544	-0.0144	0.0355	-0.0091	0.0155
β_3: 0.5	-0.0025	0.0913	0.0021	0.0370	0.0101	0.0345	0.0088	0.0148
γ_1: 0.5	-0.1096	0.0409	-0.2021	0.0648	-0.0632	0.0188	-0.1702	0.0354
γ_2: 0.3	0.0090	0.1497	0.0150	0.0597	0.0093	0.0470	0.0081	0.0208
γ_3: 0.2	0.0189	0.1444	0.0138	0.0573	0.0167	0.0543	0.0109	0.0233
	$n = 150$				$n = 200$			
	JLSM-N		JLSM-M		JLSM-N		JLSM-M	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
β_1: 1	0.0019	0.0076	0.0005	0.0033	0.0047	0.0058	0.0023	0.0025
β_2: 0.7	-0.0020	0.0242	-0.0028	0.0105	0.0022	0.0163	0.0010	0.0073
β_3: 0.5	0.0001	0.0243	-0.0007	0.0113	0.0066	0.0174	0.0047	0.0076
γ_1: 0.5	-0.0427	0.0114	-0.1569	0.0290	-0.0271	0.0073	-0.1484	0.0249
γ_2: 0.3	0.0101	0.0304	0.0074	0.0141	0.0072	0.0193	0.0068	0.0083
γ_3: 0.2	0.0047	0.0289	0.0041	0.0128	0.0095	0.0225	0.0068	0.0101

Source: own computations in the MATLAB

Simulation results are summarized in Tables 1 and 2. We give the bias and MSE values of estimators in these tables. The first table gives the results for the case without outliers. We observe from this table that two models, JLSM-N and JLSM-M, show similar performance according to bias and MSE values. We further see that all MSE values for both models are getting smaller when the sample sizes are getting large and all estimators for both models are consistent.

In the next table, we give the simulation results for the case of outliers. We add randomly generated samples with 0.1 proportion from the outlier model to the original samples. We observe that the estimators obtained from JLSM-M have the smallest bias and MSE values contrary to estimators obtained from JLSM-N. It is because the estimators of JLSM-N have been badly affected by the outliers.

Tab. 2: Bias and MSE values for JLS-N and JLS-M estimates in the case of outliers

	$n = 50$				$n = 100$			
	JLSM-N		JLSM-M		JLSM-N		JLSM-M	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\beta_1: 1$	0.3416	0.1930	0.1014	0.0224	0.3936	0.2018	0.0882	0.0131
$\beta_2: 0.7$	0.3049	0.3752	0.0517	0.0439	0.4148	0.3512	0.0542	0.0199
$\beta_3: 0.5$	-0.0214	0.2865	-0.0022	0.0424	-0.0351	0.1595	-0.0032	0.0181
$\gamma_1: 0.5$	0.7003	0.6398	0.0350	0.0172	0.8399	0.7988	0.0840	0.0144
$\gamma_2: 0.3$	-0.0827	0.6311	-0.0165	0.0898	-0.1990	0.4310	-0.0365	0.0389
$\gamma_3: 0.2$	-0.0580	0.6495	-0.0052	0.0931	-0.1154	0.3933	-0.0184	0.0390
	$n = 150$				$n = 200$			
	JLSM-N		JLSM-M		JLSM-N		JLSM-M	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\beta_1: 1$	0.4332	0.2199	0.0910	0.0115	0.4473	0.2270	0.0895	0.0104
$\beta_2: 0.7$	0.4276	0.3005	0.0477	0.0124	0.4795	0.3258	0.0557	0.0106
$\beta_3: 0.5$	-0.0763	0.1138	-0.0117	0.0116	-0.0502	0.0898	-0.0066	0.0083
$\gamma_1: 0.5$	0.8511	0.7866	0.0957	0.0131	0.8507	0.7783	0.1020	0.0138
$\gamma_2: 0.3$	-0.2177	0.3245	-0.0320	0.0242	-0.2319	0.2547	-0.0372	0.0181
$\gamma_3: 0.2$	-0.1640	0.3022	-0.0216	0.0239	-0.1496	0.2185	-0.0178	0.0153

Source: own computations in the MATLAB

2.2 Real data example

This section provides a real data set called “Martin Marietta“ which includes the excess rate of incomes of the Martin Marietta company and the index for the excess proportion of income for the New York Exchange (CRSP). This data set was collected monthly for five years. This data set was used by many authors. Recently, Doğru and Arslan (2019) investigated this data set for the joint location, scale, and skewness model based on the skew Laplace normal distribution and Eskin and Doğru (2023) used this data set for the joint location and scale model based on the generalized normal distribution. In both studies, it was indicated that the data set consists of heteroscedasticity and the Breusch-Pagan test results also prove this result. In this study, we use this data set to perform the applicability of the JLSM-M over the JLSM-N. For this aim, we add some outliers on the y-direction generating from a uniform distribution to the original data set.

We summarize the estimation results in Table 3. According to this table, the best result is obtained from JLSM-M in terms of MSE value. Furthermore, we provide the scatter plot of the Martin Marietta data set along with the fitted regression lines obtained from JLSM-N and

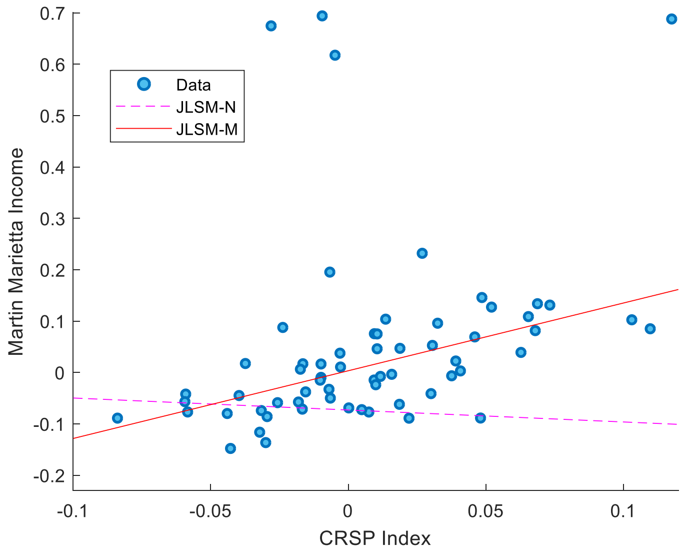
JLSM-M in Figure 1. This figure shows us that the JLSM-M can estimate the regression line perfectly although outliers. On the other hand, JLSM-N has been affected by the outliers in the y-direction and failed to estimate the regression line correctly.

Tab. 3: Estimation results for JLSM-N and JLSM-M

		JLSM-N	JLSM-M
		Estimate	Estimate
Location model	β_0	-0.0731	0.0033
	β_1	-0.2330	1.3175
Scale model	γ_0	-3.0807	-4.7492
	γ_1	0.6540	3.5642
Comparison criteria	MSE	0.0504	**0.0315**

Source: own computations in the MATLAB

Fig. 1: The scatter plot of the data set along with the fitted lines gained from JLSM-N and JLSM-M



Source: own computations in the MATLAB

Conclusion

This paper proposed robust estimators for the parameters of JLSM-M with the Huber function. We gave an estimation algorithm to obtain the ML estimators for the parameters of interest. We also provided a simulation study and a real data example to demonstrate the performance of the proposed estimators over the estimators obtained from JLSM-N. According to the simulation study results that the proposed algorithm is working well in estimating the parameters.

Furthermore, the results of the JLSM-M outperformed the results of JLSM-N when the simulated data set includes outliers. We had similar results from the real data example. The JLSM-M had the best fit compared with the JLSM-N in the case of outliers for the Martin Marietta data set. Consequently, we propose using the JLSM-M alternative to the JLSM-N when the data set has some potential outliers.

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