EVT BASED ANALYSIS OF PRICE CHANGES OF GME STOCK PRIOR TO YEAR 2021

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Abstract

The paper deals with the problem of evaluating price changes of GME stock (of GameStock corporation) on New York Stock Exchange from the point of view of extreme value analysis.

The stock price moves in the early January 2021 were considered as an extraordinary coordinated short squeeze resulting in massive losses of several hedge funds and limited availability of trading by several brokers, most notably Robinhood Markets, Inc.

The aim of the paper is to consider probability distribution of prior moves in GME stock price with emphasis on the tails of the distribution, to evaluate exceptionality of the price changes observed in January 2021. To this goal several diagnostic tools regarding heaviness of the tail, tail index estimate and estimates using extreme value theory are employed.

The results obtained indicates that right tail of GME stock price moves is heavy to fat-tailed with tail coefficient below 4 and possibly below 3. Even though the events in the January 2021 were extraordinary, especially their concentration, they were not outside of the realm of possible given the previous data.

Key words: Gamestock, Peaks over threshold, extreme value analysis

JEL Code: C13, G14

Introduction

During the late of 2020, a community of users of an online social network Reddit started targeting short sellers of stocks of Gamestop Corp. (GME) mostly by buying call options. As of the beginning of January 2021, the short sellers were selling over 130 % of all stocks available due to the possibility of repeated share loans and the most exposed short seller was Melvin Capital hedge fund, which later suffered estimated losses of 4.5 billion dollars. The buyers got the wide public attention which led to attract even more buyers and short sellers became forced to start shutting down short positions by buying them back due to the increasing unrealized losses, which in turn led to another increase in stock price and finally to a cascade usually called short squeeze (Chung, 2021).

Whole concept of the retail investors being able to buy financial securities immediately online is relatively new and it is likely one of the causes of the cascade of events observed in the early 2021. One of the most prominent companies in the sector was at the time Robinhood Markets, Inc. The company halted the possibility of buying GME stocks via its mobile application on January 28th, quickly followed by other similar companies, e.g. Trading212, e-Toro or Webull, which led to sharp fall of the GME price and accusations of organized action to cut the losses of large hedge funds. Official reason for this action was insufficient collateral required by the clearing houses, that ensures the brokers would be able to properly settle client´s orders on the stock exchange (Tenev, 2021a).

Vlad Tenev, cofounder and CEO of the Robinhood Markets, Inc., claimed on a social network Twitter on February 25th, that “This was a five sigma Black Swan event (1 in 3.5 million), so basically model breaking.” (Tenev, 2021b). The Black Swan clearly appeals to the Nassim Nicholas Taleb´s book (2009), where Black Swan is a rare event with extremely high impact that could not be foreseen from the past experiences. Five sigma combined with 1 in 3.5 million statements clearly appeals to normal distribution of the underlying random variable, since $1/P(X > 5\sigma) = 1/(1-\phi(5)) = 1/3{,}488{,}556$.

The motivation for this paper is thus to find out, whether using more appropriate tools not relying on normal distribution, would lead to more appropriate estimates of the probability of such extreme events, that would in turn bring them out of the Black Swan label to Grey Swan label, that is a rare event with extremely high impact, that could be foreseen from the past experiences.

The data for this paper were weekly and daily prices downloaded from Yahoo finance (2021). Only percentage increases of close prices, i.e. prices at the end of the trading day or week, are considered. The software used is R (R Core Team, 2020) with package POT (Ribatet and Dutang, 2019).

The paper is organized as follows – first graphical tools for the evaluation of heaviness of the tail of distribution are introduced, followed by simplified introduction of Pickands-Balkema-de Hann theorem and peaks over threshold method built upon it. Then these methods are employed to explore the tails in the data and to obtain estimates of the tail behaviour and of extreme quantiles. In the end the estimates of extreme quantile values are discussed and compared to the stock price moves at the beginning of the year 2021 to compare.

1 Methodology

If the random variable comes from heavy-tailed distribution, of which fat-tails are subset, several distinctive features follow and to diagnose heaviness of the right tail and its behaviour, several graphical tools can be employed. Among those are maximum to sum plot, log-log plot and mean exceedances plot. Then standard peaks over threshold method based on Pickands-Balkema-de Haan is introduced and used to obtain several estimates of the right tail behaviour (Pickands, 1975; Balkema and De Haan, 1974). The use of Pickands-Balkema-de Haan theorem in finance is not a new approach, which makes statements from Introduction based on normal distribution even more surprising (He et al., 2021)

Fat-tailed distribution in this paper is defined as a distribution of Fréchet maximum domain of attraction with regularly varying tail

$$S(x) = L(x)x^{-\alpha}, \qquad (1)$$

where $L(x)$ is slowly varying function that satisfies $\lim_{x\to\infty} L(rx)/L(x) \to 1$ for any positive value of r (Foss, Korshunov and Zachary, 2015; Taleb, 2020).

Putting $L(x)$ equal to 1, having thus power-law distribution, it follows:

$$log(S(x)) = -\alpha log(x) \qquad (2)$$

1.1 Maximum to sum plot

Among the fat-tailed distributions, it is common that some general moments are not finite. It follows from the strong law of large numbers, that if k-th general moment exists, then ratio of sample maximum to sample summation of k-th powers converges to 0 almost surely for increasing sample size n.

$$R_n^k = \frac{max(X^k)}{\sum_{i=1}^n X^k} \xrightarrow{a.s.} 0 \qquad (3)$$

This property can be simply tracked by maximum to sum (MS) plot – if the ratio converges, we can assume the specific moment is finite and vice versa. This tool is among first and rather crude graphical tools, used for example in (Cirillo and Taleb, 2016).

For distributions of regularly varying class, the k-th general moment does exists if and only if the tail coefficient $\alpha > k$.

1.2 Log-log plot

Fat-tailed distributions have power-law decaying survival function, which can be again graphically searched for in the data. One of the consequences is that logarithm of survival function plotted against logarithm of the value of a random variable is theoretically a straight line with slope equal to negative value of tail index – α (see equation (2)).

This property can again be visualized in the data using log-log plot. If the plotted points of logarithms of empirical survival function $\log(S_n(x))$ against of logarithms of values $\log(x)$ forms, usually after some initially declining slope, straight slope, that slope can be viewed as an intuitive rough estimate of tail coefficient α.

1.3 Conditional mean exceedances plot

Conditional mean exceedances (CME) defined as $E(X-\mu|X\geq\mu)$ with μ being threshold behaves accordingly to heaviness of the tails – if the CME is decreasing function of the μ, the tail is thin-tailed and belonging to the Weibull maximum domain of attraction; if the CME is constant function of μ, the tail is exponential belonging to Gumbel maximum domain of attraction; finally if CME is increasing function of μ, the tail is heavy belonging to Fréchet maximum domain of attraction.

The plot showing relation between values of threshold μ against sample conditional mean exceedances $(\bar{X}-\mu|X\geq\mu)$ thus belongs among graphical tools of heavy-tails detection. It is also used in peaks over threshold method for proper threshold determination.

1.4 Pickands-Balkema-de Haan theorem and peaks over threshold method

Dealing with distributions with possible heavy to fat tails, two main methods following two basic theorems of extreme value theory, are usually employed – block maxima method and peaks over threshold method. In this paper, peaks over threshold method is used as it possibly employ more information from the sample data.

Pickands-Balkema-de Haan theorem states that conditional excess distribution function converges in distribution to generalized Pareto distribution for large enough threshold μ

$$F_\mu(x) = P(X-\mu\leq x|X\geq\mu) = \frac{F(\mu+x)-F(\mu)}{1-F(\mu)} \xrightarrow{D} G_{\xi,\sigma}(x), \qquad (4)$$

where

$$G_{\xi,\sigma}(x) = \begin{cases} 1-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} \text{for } \xi\neq 0 \\ 1-\exp\left(-\frac{x-\mu}{\sigma}\right) \text{for } \xi = 0. \end{cases} \qquad (5)$$

For $\xi > 0$, the generalized Pareto distribution is a power-law distribution with $\alpha = \xi^{-1}$. For $\mu = \sigma/\xi$ and using parametrization $x_m = \sigma/\xi$ and it can be transformed to Pareto distribution.

Pickands-Balkema-de Haan theorem theoretically justifies use of generalized Pareto distribution to model tails of most of the distributions with focus on the fat-tailed distributions. Method employing this theorem is peaks over threshold method, in which firstly graphical tools are employed to find out proper value of threshold μ and then using only observations above the threshold parameters ξ and σ for the distribution of exceedances $(x - \mu)$ are estimated. Out of several possible estimation methods, method of maximum likelihood is used in this paper. Estimate of ξ can serve as an estimate of tail index α.

Graphical tools employed in this paper are CME, plot of estimated parameters ξ and σ against varying threshold μ, and a plot of sample third and fourth L-moments compared to theoretical values of the L-moments. Threshold μ is usually set at the points, after which CME shows stable trend (either decreasing, constant or increasing), and estimates of ξ and σ are stable (that is constant given estimate uncertainty). In a plot of L-moments it is usually set at the point at which the sample L-moments are close to the theoretical.

Setting threshold μ is also a form of bias-variance trade-off. The higher the threshold is set, the less biased the estimates will be due to the better convergence to generalized Pareto distribution. On the other hand, higher threshold leads to lower number of observations in the estimate and hence higher variance. Usual procedures advise to use as low threshold as possible given previous considerations, but also to try somewhat higher values (Tanaka and Takara, 2002).

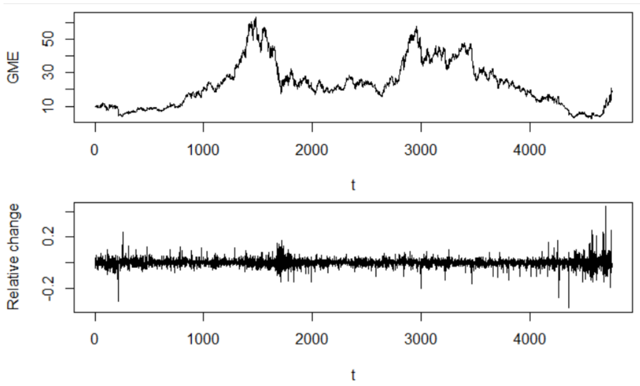
2 Analysis

The price development and price changes of the GME stock from introduction to New York Security Exchange on February 13th of 2002 up to the end of the year 2020 can be seen in the figure 1. As for the relative price changes, the maximum is daily +44.1 % increase occurring on October 8th of 2020 and weekly +55.5 % increase occurring in the week ending on September 14th.

2.1 Maximum to sum plots

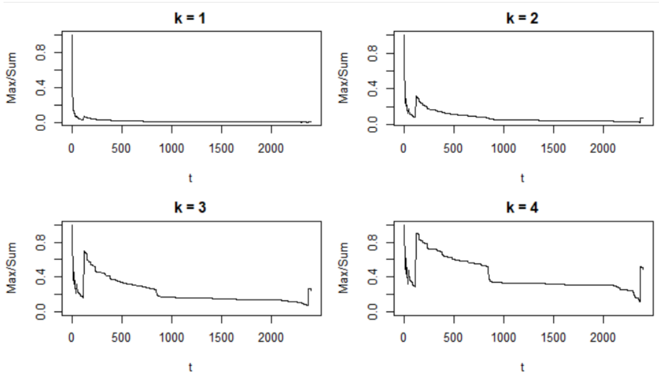
MS plots for the relative GME stock price increases are in the figures 2 and 3. The plot for daily increases shows, that fourth moment is likely non-existent and the third may not exists as well, as its convergence is very slow with large jump even after 2,365 previous observations. Figure 3 shows similar results for the weekly price increases event though there seems to be better convergence of the third moment.

Fig. 1: Daily price and relative price changes of GME up to the end of the year 2020.



Source: Yahoo finance, own work

Fig. 2: MS plots for first four moments of relative daily GME price increases.



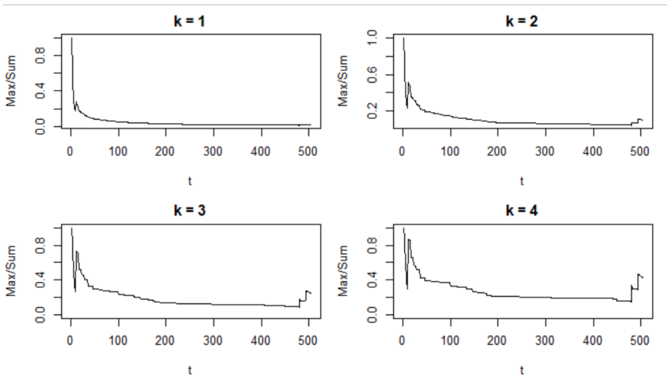
Source: Yahoo finance, own work

In general, the underlying distribution seems to have non-existent or very large third and fourth moments manifesting in very slow convergence. This would suggest tail coefficient α to be well below 4 and likely below 3 for daily increases and somewhere around 3 for weekly increases.

2.2 Log-log plots

Figure 4 displays log-log plots with charted lines representing tail indices 4, 3 and 2 subsequently going through the point $\log(x) = -3$ for daily and through $\log(x) = -2.5$ for weekly increases. Note that the x axis was limited to start at log(x) = – 5 even though data starts approximately at log(x) = -8.715.

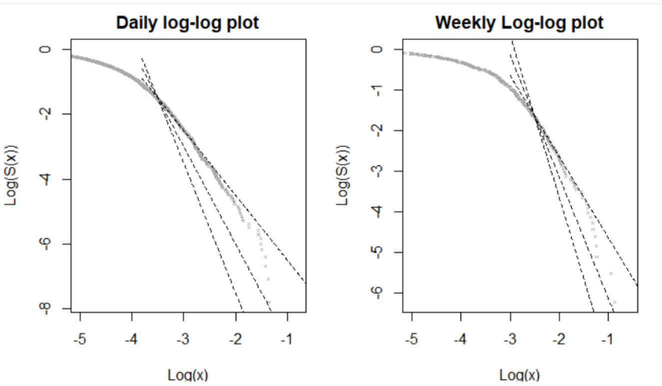
Fig. 3: MS plots for first four moments of relative weekly GME price increases.



Source: Yahoo finance, own work

The tail of daily increases seems to be following line with slope slightly below -2 except for the largest observations. This could mean that the underlying distribution do not have fat tail or, given the large number of observations more likely option, that the sample largest values underrepresents true tail distribution. Based on the plot the suggested value of α is below 3 consistent with non-existent third general moment in MS plot. The plot of weekly increases shows similar behaviour, again consistent with corresponding MS plot.

Fig. 4: Log-log plots of daily price increases.

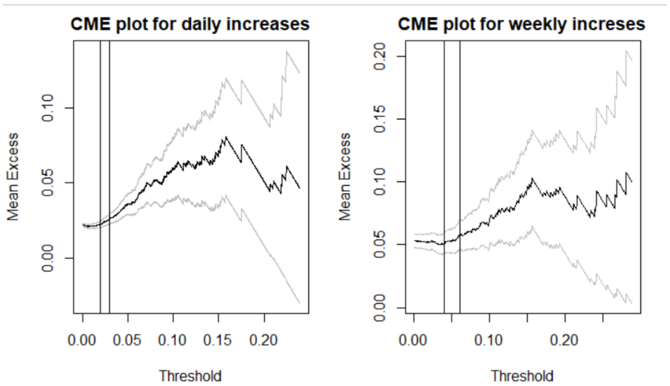


Source: Yahoo finance, own work

2.3 Conditional mean exceedances plots

CME plots in figure 5 for both daily and weekly increases show patterns in accordance with previous findings, that is conditional mean exceedances are, after some initial decrease, generally increasing suggesting heavy tails. The vertical lines show 0.02 and 0.03 thresholds for daily increases and 0.04 with 0.06 for weekly increases.

Fig. 5: Cumulative mean exceedances plots



Source: Yahoo finance, own work

2.4 Peaks over threshold analysis

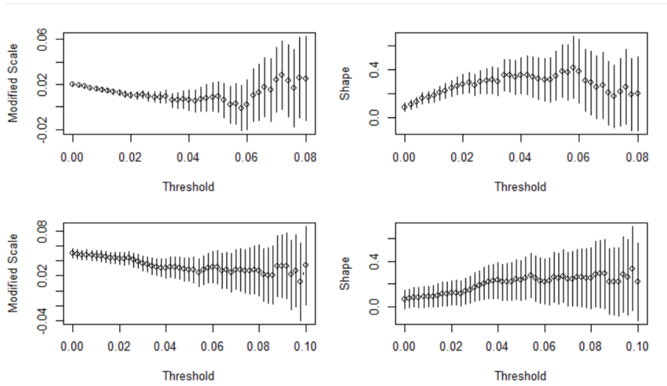
Cumulative mean exceedances suggest the threshold could be set at 0.02 at minimum and maybe up to at 0.03 to balance possible bias with variance for daily increases with corresponding values for weekly changes to be 0.04 to 0.06. Plot of various thresholds vs. parameter estimates in figure 6 suggests setting threshold at 0.02 up to 0.035 for daily increases and 0.04 to 0.06 for weekly increases.

Figure 7 shows compatibility of L-moments of exceedances distributions for various thresholds with generalized Pareto distribution. The line represents theoretical relation between these two given by equation $\tau_4 = \tau_3(1 + 5\tau_3)/(5 + \tau_3)$ and the dots represents sample L-moments for various thresholds. The points nearest to the theoretical lines are highlighted with suggested threshold 0.03 and 0.06 to be the best in terms of closeness of sample to theoretical L-moments for increases and absolute values of decreases, respectively, which is in line with previous findings. The proximity generally suggests that the tail distribution is truly generalized Pareto and peaks over threshold is appropriate method to use. (Ribatet and Dutang, 2019)

The following step is to estimate generalized Pareto distribution using selected threshold. For daily increases the two thresholds are used – 0.02 and 0.035 respectively; for weekly increases the thresholds chosen are 0.04 and 0.06. Results of parameter estimates are presented in table 1. For the given thresholds, estimated return levels (inverse of probability) for 1 in 10,000 trading days and 1 in 2,500 trading weeks are given in table 1 as well.

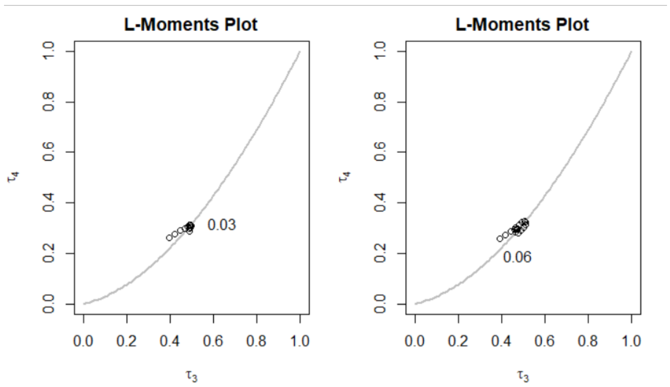
Assuming 200 trading days and 50 trading weeks per year, approximately half of them showing price increase, this would be 1 in 100 years return level.

Fig. 6: Thresholds vs. parameter estimates. Upper pair for price increases, bottom for absolute decreases.



Source: Yahoo finance, own work

Fig. 7: Sample vs. theoretical L-moments for various thresholds.



Source: Yahoo finance, own work

To accommodate the log-log plot suggesting $\alpha < 3$, the relation $\alpha = \xi^{-1}$, and most of the point estimates of $\xi < 1/3$, one more estimate for price increases is provided with thresholds 0.035 and 0.06 and $\xi = 0.4$ (hence only estimated parameter is σ).

The peaks-over threshold analysis shows, that 1 in 100 years event would be 125% increase in daily closing price and 100% increase in weekly closing price, but with the values close to 300% increase to be in a realm of possible. When the information from log-log plot suggesting heavier tail than the estimated is considered, it leads to estimated 174% increase in daily closing price and 223% increase in weekly closing price, with values over 330% and 440% being in line with confidence intervals.

Tab. 1: Estimated parameters of generalized Pareto distribution with 95% C.I.

μ	$\hat{\sigma}$	$\hat{\xi}$	1 in 100 years	Periodicity
0.02	0.016 (0.014; 0.017)	0.281 (0.201; 0.361)	0.718 (0.468; 1.178)	Daily
0.035	0.018 (0.015;0.021)	0.345 (0.214; 0.476)	1.251 (0.643; 3.092)	Daily
0.035	0.018 (0.015; 0.020)	0.4	1.739 (0.643; 3.389)	Daily
0.04	0.039 (0.031; 0.047)	0.233 (0.077; 0.388)	0.912 (0.545; 2.113)	Weekly
0.06	0.045 (0.045; 0.022)	0.221 (0.147; 0.432)	1.005 (0.545; 2.994)	Weekly
0.06	0.040 (0.031; 0.048)	0.4	2.233 (0.545; 4.464)	Weekly

Source: Yahoo finance, own work

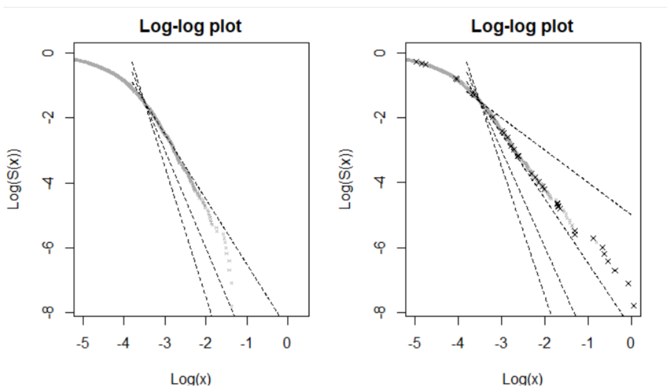
MS plots in figures 2 and 3 shows pattern consistent with the estimated parameters with $\hat{\xi}$ being around 1/3 for daily and 1/4 for weekly price increases suggesting the existence of 3rd and 4th moments are questionable, but not that of second.

3 Comparison with the price changes in the year 2021

Analysis provided in previous chapter shows what could be possible to expect based on the data up to the end of the year 2020. In this part the price increases recorded during the year 2021 are showed in respect to these estimates. The table 2 shows seven highest daily and 5 weekly increases of the year 2021 – those higher than previous highest daily price increase, with estimated return levels from three estimated generalized Pareto distribution.

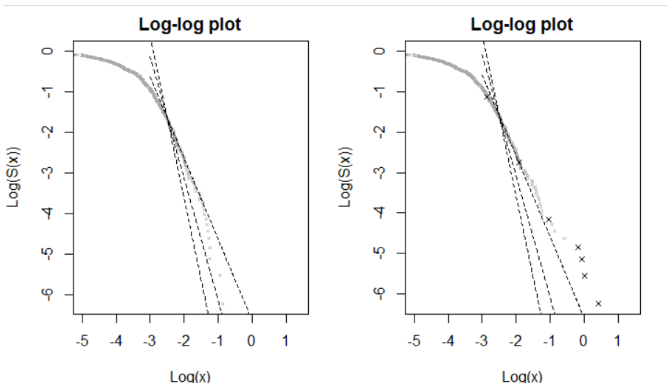
Figures 8 and 9 shows how including new data shifted the relation of logarithm of survival function to logarithm of data. For both daily and weekly price increases the slope is close to -2 with the highest value being above instead of below the slope indicating significant move of the sample right tail.

Fig. 8: Comparing log-log plot with and without daily increases of 2021.



Source: Yahoo finance, own work

Fig. 9: Comparing log-log plot with and without weekly increases of 2021.



Source: Yahoo finance, own work

Tab. 2: Estimated return levels of the highest daily price increases in the year 2021

Price increase	Return level 1 (Years)	Return level 2 (Years)	Return level 3 (Years)	Periodicity
+ 135 %	837	124	53	Daily
+ 104 %	337	59	28	Daily
+ 93 %	226	43	21	Daily
+ 68 %	77	18	10	Daily
+ 57 %	43	11	6	Daily
+ 53 %	32	9	5	Daily
+ 51 %	29	7	5	Daily
+ 399 %	562	409	36	Weekly
+ 151 %	205	143	19	Weekly
+ 107 %	133	91	15	Weekly
+ 92 %	42	28	7	Weekly
+ 83 %	23	15	5	Weekly

Source: Yahoo finance, own work

Conclusion

Analysis provided in chapters 2 and 3 showed, that respecting fat tailed properties of the price increases of both daily and weekly price changes leads to more realistic expectation about the events seen in the year 2021, leading it from the totally unexpected Black Swan territory to more realistic Grey Swan territory.

Although the return levels estimated in the chapter 3 for the price changes seen in the year 2021 are very high, they are by no means impossible, just improbable. Especially return levels deduced from generalized Pareto estimate using fixed value of inverse tail coefficient respecting behaviour visible in the log-log plots are seemingly close to reality observed in the year 2021.

This is not to underplay massive shift in the way small new investors use their money on stock market that likely started the spiral of 2021 events, only to remind, that analysis of random variables with possible fat tails needs to be done with respect to specific properties driven by them and not to use thin tailed statistics and probabilistic heuristics like ones presented by the Tenev´s quotation in the Introduction. Using inappropriate tools leads to underestimating impact of rare events with possible massive losses.

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