CALCULATION OF RECTIFYING ACCEPTANCE SAMPLING PLANS IN LTPDvar PACKAGE

Nikola Kaspříková

Abstract

R extension package LTPDvar is a free software for the calculation of rectifying acceptance sampling plans minimizing the mean inspection cost per lot of average process quality. The lot tolerance percent defective plans and the average outgoing quality limit plans for the acceptance sampling which minimize the mean inspection cost per lot of the process average quality when the remainder of the rejected lots is inspected are covered in the package. The rectifying sampling plans for the inspection by variables or the rectifying sampling plans for the inspection by variables when the remainder of the rejected lots is inspected by attributes can be used. The calculations can be made under the assumption that the standard deviation of the quality characteristic is known or under the assumption that the standard deviation of the quality characteristic is unknown. The paper discusses some of the characteristics of calculation of these plans, such as the time performance of calculation of average outgoing quality limit sampling plans covered in LTPDvar package.

Key words: acceptance sampling, rectifying sampling plans, LTPDvar package

JEL Code: C44, C80

Introduction

R extension package LTPDvar is a free software for the calculation of rectifying acceptance sampling plans minimizing the mean inspection cost per lot of average process quality. The lot tolerance proportion defective (LTPD) plans and the average outgoing quality limit (AOQL) plans for the acceptance sampling which minimize the mean inspection cost per lot of the process average quality when the remainder of the rejected lots is inspected are covered in the package. The rectifying sampling plans for the inspection by variables or the rectifying sampling plans for the inspection by variables when the remainder of the rejected lots is inspected by attributes can be used. The calculations can be made under the assumption that the standard deviation of the quality characteristic is known or under the assumption that the standard deviation of the quality characteristic is unknown. The paper discusses some of the characteristics of calculation of these plans, such as the time performance of calculation of some of the types of sampling plans covered in LTPDvar package.

The AOQL sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed by Dodge and Romig for the inspection by attributes. Plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of rejected lots is inspected by attributes) were then proposed and it has been shown that these plans are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. The AOQL plans for the inspection by variables and attributes have been introduced in (Klůfa, 1997), using an approximate operating characteristic function for calculation of the plans. Exact plans, using the non-central t distribution in calculation of the operating characteristic, have been reported in (Klůfa, 2008) and implemented in (Kaspříková, 2015). The operating characteristics used for these plans are discussed in (Jennett and Welch, 1939) and (Johnson and Welch, 1940). It has been shown that these plans are in many situations superior to the original attribute sampling plans as – see the analysis in (Klůfa, 2015) and in (Kaspříková and Klůfa, 2016). With the aim of obtaining further savings in the cost of inspection, the new AOQL plans for the inspection by variables and attributes, designed to use the EWMA statistics, have been implemented in (Kaspříková, 2015). Recent repetitive rectifying plans are analysed in (Yen et al., 2020).

This paper considers the average outgoing quality limit plans implemented in (Kaspříková, 2015) and shows the calculation of these plans in (Kaspříková, 2015). A numerical experiment is run to provide an empirical analysis of time complexity of the calculation of plans depending on the operating characteristic function used.

The structure of the paper is as follows: the AOQL plans for the inspection by attributes and the AOQL variable sampling plans minimizing the mean inspection cost per lot of the process average quality are recalled first and the variable sampling plan is calculated. The results of the numerical experiment are reported then to show the time needed to calculate the plans in LTPDvar package.

1 AOQL attributes inspection plans

For the case that each inspected item is classified as either good or defective (the acceptance sampling by attributes), Dodge and Romig (1998) consider rectifying sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N-(N-n)\cdot L(p_a;n;c) \qquad (1)$$

under the condition $$max(AOQ(p))=p_L, \text{ for } p \text{ in } (0, 1) \qquad (2)$$

where *L(p, n, c)* is the operating characteristic (the probability of accepting a submitted lot with proportion defective p when using plan *(n, c)* for acceptance sampling), *N* is the number of items in the lot (the given parameter), p_a is the process average proportion defective (the given parameter), p_L is the maximum allowed value for the average outgoing quality (the mean proportion of nonconforming items in lots after inspection), *n* is the number of items in the sample $(n<N)$, *c* is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than *c*).

The average outgoing quality is a function of the proportion nonconforming in the incoming lots and equals zero in the rejected lots (since the remainder in the rejected lots is inspected completely and the nonconforming items are replaced by good ones if the rectifying inspection plan is used). The average outgoing quality may be nonzero in the accepted lots, since just the nonconforming items in the sample are replaced by good ones, the items in the remainder are not inspected.

The condition (2) guarantees that for any proportion of nonconforming items in the lots submitted for the inspection, the average outgoing quality will not exceed the given limit p_L.

2 AOQL variables inspection plans

The AOQL plans for the inspection by variables and attributes (the items in the sample are inspected by variables, the remainder of the rejected lots in inspected by attributes) have been designed in (Klůfa, 2008) and implemented in (Kaspříková, 2015) under the following assumptions:

The measurements of a single quality characteristic *X* are independent, identically distributed normal random variables with unknown parameter μ and known parameter σ^2. For the quality characteristic *X* there is given either an upper specification limit *U* (the item is defective if its measurement exceeds *U*), or a lower specification limit *L* (the item is defective if its measurement is smaller than *L*).

This assumption is kind somehow limits the number of cases in which the plans may be applied (for example it excludes the situations when the quality characteristic of interest is just a binary variable) but on the other hand it allows to make use of the inspection by variables which allows significant savings in the mean inspection cost.

The mean inspection cost per lot of the process average quality p_a for such plans is

$$I_{ms} = n \cdot c_{m} + (N-n) \cdot (1-L(p_a;n;k)), \quad (3)$$

where c_m is the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes. The units of measurement for the I_{ms} function values is the cost of inspecting an item by attributes. The I_{ms} cost function is to be minimized when searching for the sampling plans in (Kaspříková, 2015).

Regarding the operating characteristic (OC) function, three options are available for calculations of the sampling plans in (Kaspříková, 2015). Either the plans without memory are used and then the user may choose the exact OC with noncentral t distribution or the approximate OC, for details see (Klůfa, 2008), or the plans with memory using the EWMA statistic can be used, see (Kaspříková, 2020).

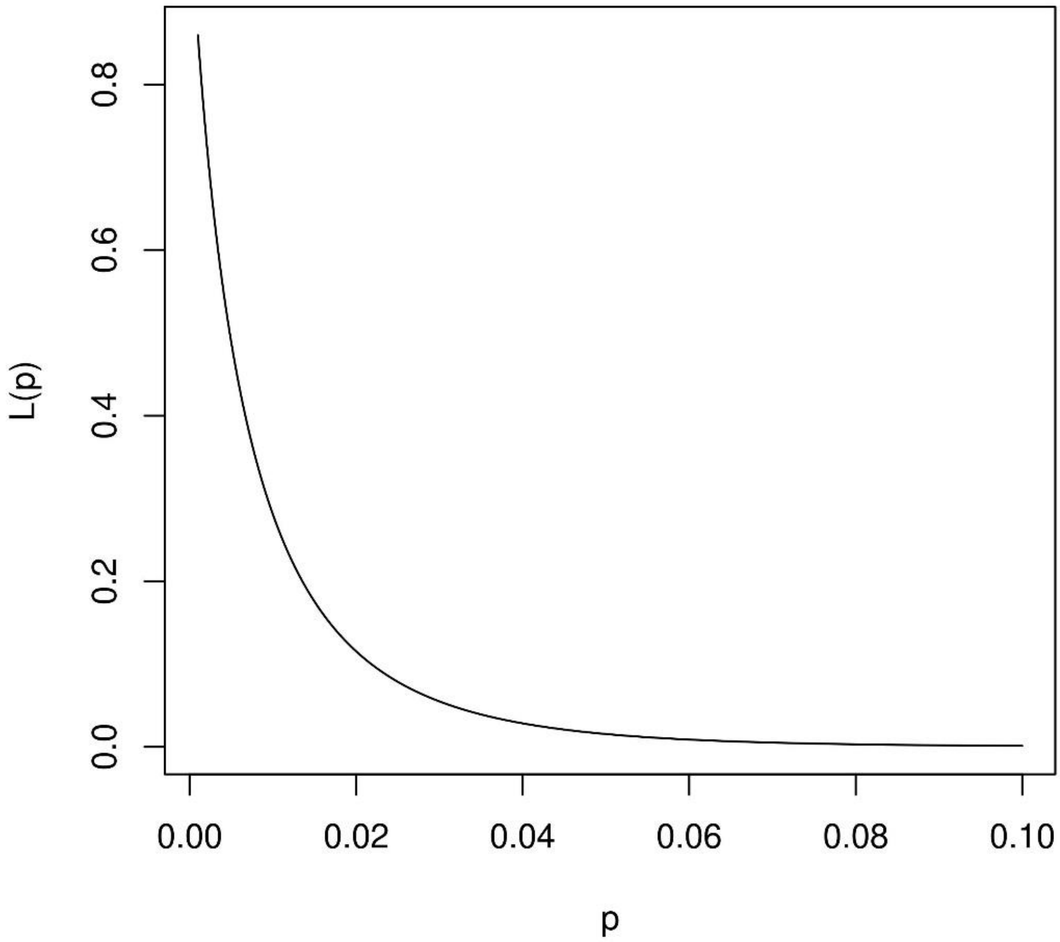
3 Example of AOQL plan calculation

Let's calculate the AOQL acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes and if the standard deviation of the quality characteristic is unknown in a case study below.

Example. We consider a lot of N = 200 in the acceptance procedure. The average outgoing quality limit is set to p_L = 0.0025. It is known that the average process quality is p_a = 0.001. The cost of inspecting an item by variables is known to be 2 times higher than the cost of inspecting an item by attributes, so c_m parameter equals 2. Find the AOQL acceptance sampling plan without memory for sampling inspection by variables when the remainder of rejected lots is inspected by attributes, using the exact operating characteristic, which works with the noncentral t distribution.

The plan can be calculated using the planAOQL function in the LTPDvar package (Kaspříková, 2015) for the R software (R Core Team, 2021). The solution is n = 22, k = 2.628. The operating characteristic plot is shown in Figure 1. The plot of the operating characteristic of the sampling plan can be easily produced using the generic function plot in the LTPDvar package.

Fig. 1: The operating characteristic of the plan (22, 2.628)



Source: the figure has been produced by the author in R software

4 Performance of functions for plans calculation

The analysis of complexity is an important part of the algorithm development and its efficient implementation. The complexity of the computation may be evaluated analytically or empirically using numerical experiments. The theoretical, analytical evaluation may be more rigorous and independent on the platform later used for the actual calculations, but empirical analysis may be useful for practical purposes, since the users are mostly interested in concrete time needed to perform the calculation in practice using concrete hardware and software environment.

AOQL plans calculation using LTPDvar package is analysed empirically using numerical experiments in this paper. More detailed analysis of the time needed to calculate the plan can be obtained using the profiling tools (Rprof function) in the R computing environment. The outcome of such analysis in case of the function for calculating the AOQL plan suggests that the functions which are demanding the largest time in the call include the predefined R functions for calculating the t distribution function. Similarly, the operating characteristic evaluation for the case of plans with memory is consuming a lot of time. So, the number of OC function evaluations should be minimized for the efficient calculation of the plans.

The calculation of the AOQL plan is much more complicated than the calculation of the LTPD plan, but still the calculation of the AOQL plan implemented in LTPDvar package may be considered fast enough. Table 1 shows the time needed to calculate 100 sampling plans for lot size from 151 to 250, with other input parameters p_L =0.0025, p_a =0,001, c_m =2. The time to calculate one sampling plan with memory or sampling plan without memory using exact operating characteristic function takes slightly more than one second on average with hardware that roughly corresponds to current performance standard. The calculation of plan without memory using the approximate OC is much faster, roughly by factor 10.

Tab. 1: Time to calculate 100 sampling plans (in seconds)

Plans with memory	Plans without memory, approximate OC	Plans without memory, exact OC
128	9	132

Source: calculation in LTPDvar package

The performance of the functions for calculation of the sampling plans in LTPDvar package is sufficient for practical purposes. The calculation could be made even faster with more efficient code, to be implemented in future release of the package. New release of the package is planned which would contain updated user documentation, cleaner, easier to maintain code and which will also add several functions and packages to the namespace of the package.

Conclusion

The calculation of the AOQL plan is more complicated in comparison with the calculation of the LTPD plan, but the performance of the functions for calculation of the AOQL sampling plans in LTPDvar package is sufficient for practical purposes, even when the plans with memory or the plans without memory using the exact operating characteristic are calculated. New release of the LTPDvar software is planned which would contain cleaner, easy to maintain code, and which would better correspond to the recent development in R software (R Core Team, 2021).

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Contact

Nikola Kaspříková

Prague University of Economics and Business, Department of Mathematics

nám. W. Churchilla 4, 130 67 Praha

nikola.kasprikova@vse.cz