BANKRUPTCY FUZZY PREDICTION INDEX FOR BUSINESSES REFLECTING THE COVID-19 ECONOMIC CONSEQUENCES

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Abstract

Many companies currently face an economic crisis due to the outbreak of the Covid-19 pandemic and related measures applied. This implies that the future existence of these firms is uncertain. The aim of the paper is to construct a fuzzy approach-based system for prediction a tendency to go bankrupt in the short-term period 2021-2022. The tool is a three-stage fuzzy model of multicriteria evaluation. The input parameters are objective and subjective measured data between 2011-2020 given in the form of intervals (financial ratios used for bankruptcy evaluation by Taffler´s model, dividend yield and the economic sentiment indicator). The model output is the interval of subjectively expected values of non-Bankruptcy fuzzy index (non-BFI) prediction. The general procedure is demonstrated on the real data of Škoda Auto factory. The own fuzzy computation was demonstrated at the third stage of the model – block *K*. The innovation of the paper lies in the fuzzy procedure for bankruptcy prediction by the three-stage model and identification of a measure of business safety for risk-averse bankruptcy evaluators.

Key words: Index of bankruptcy, fuzzy approach, three-stage fuzzy model

JEL Code: C44, C58, C63

Introduction

The measurement of risk of corporate bankruptcy is an important issue of financial practitioners. The last global financial crisis 2008 and the current economic downturn due to the Covid-19 pandemic accelerated economic changes and revealed new sources of increased financial risk that corporations face (Li, 2012, Usheva & Vagner, 2021). Thus, the focus on the reliability of the forecasting procedures should be placed that are strong enough to point out to the bankruptcy tendency within the reasonable horizon prior to the bankruptcy itself.

The bankruptcy consequences are serious for many subjects in economy (e. g. shareholders, investors, employees, financial creditors).

The question is not whether to use bankruptcy predictive models but what models to use to increase the forecasting reliability. Many approaches have been used; these are summarised in Korol (2019). Let us mention multivariate discriminant analysis by Altman, which was a pioneer model, a wide variety of statistical methods and logistic regression models (Barboza et al, 2017). The statistical techniques widespread in practice became the most often used in bankruptcy forecasting, despite the fact they are burdened by many statistical preconditions (linearity, normality, and independence among variables) – more in Jayasekera (2018). The current approach is targeted to bankruptcy predictions based on neural networks, genetic algorithms, vector support machines (Shen et al., 2020) and fuzzy logic (Succurro et al., 2019).

The main concern in the expert literature has been aimed to assess which procedure was the most reliable in making predictions. As shown, most of the models have a significant predictive ability within a short horizon of the forecasting; the prediction accuracy has been declining in the period beyond three years (Štefko et al, 2020).

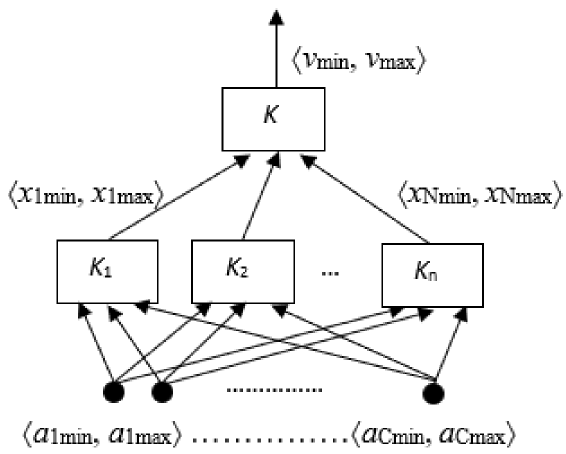
To answer the question of reliability of bankruptcy prediction, the main aim has been set to construct a bankruptcy model based on the three-stage model using the fuzzy sets. The key point is to develop an approach that will consider financial data and the judgments and attitudes of experts and other opinions relevant to the problem. The paper is divided into the methodological and practical part. Methodology provides the insight to the structure of the three-stage fuzzy multicriteria ex-ante evaluation approach, which is followed by the description of the computation fuzzy algorithm. The technique is applied to the solution of a multicriteria ex-ante evaluation of bankruptcy prediction of Škoda Auto factory.

1 A three-stage fuzzy multicriteria model of bankruptcy

The ex-ante fuzzy evaluation graphically presented in Fig. 1 is performed by a three-stage fuzzy system of multicriteria evaluation. In the ex-ante evaluation regime, the fuzzy system appears in conditions of *interval* input data that might include both the numerical and linguistic values. The model consists of three evaluation stages. The first stage is "uploaded" with the data expressed by means of intervals of values of their measured/possible occurrences (in Fig. 1 see the intervals $\langle a_{\min}, a_{\max} \rangle$ to $\langle a_{C\min}, a_{C\max} \rangle$, which are in the application part 3 presented by basic data of Annual Reports of Auto Škoda necessary for Z-score calculation, where $a_{\min}$ / $a_{\max}$ stands for minimum / maximum value calculated in the period 2011-2020, i.e., R_1 – R_4, D, CI) or mixes of uncertain data and numerical data. At the second stage these external inputs are transferred by the criterion functions to the internal data of the model which will undergo a fuzzy process in block *K* (see intervals $\langle x_{1\min}, x_{1\max}\rangle$ to $\langle x_{N\min}, x_{N\max}\rangle$ in Fig. 1, that in the application part 3 are presented by Z-score, D and CI intervals, in which minimum / maximum stands for measured/calculated value in the period 2011-2020). In the third stage (block *K*) the outputs of the second stage are converted to the fuzzy output (the interval $\langle v_{\min}, v_{\max}\rangle$ - see Fig. 1).

Thus, N intervals are created at the second stage of the model, which are presented by at most two-element sets of their limit values. Cartesian product of the N interval results in up to 2^N N-dimensional numerical vectors entering block *K*. The gradual processing of each of them by the computational fuzzy algorithm leads to a creation of a set of partial calculation results (numbers *v*) – the fuzzy processing is described by the block diagram in Fig. 2. The arithmetic average $v_x = (v_{\min} + v_{\max}) / 2$ of the minimum and maximum of this set is a subjectively expected numerical value of the output linguistic variable non-Bankruptcy fuzzy index (BFI) prediction of an evaluated firm.

Fig. 1: Structure of a general three-stage fuzzy system of ex-ante multicriteria evaluation



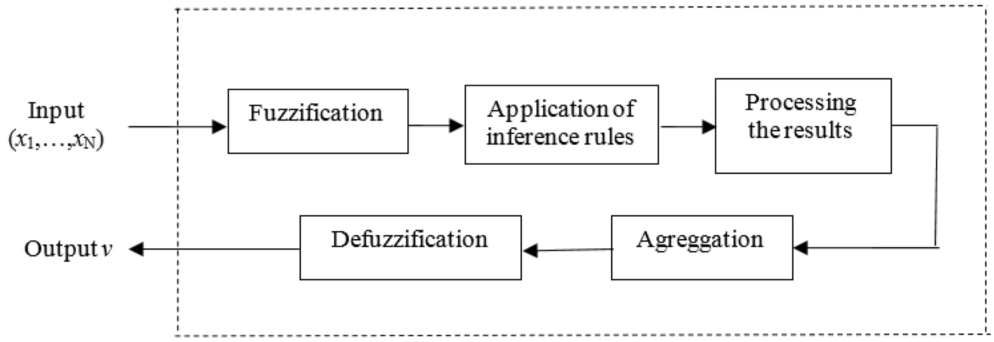
Source: own processing

2 Computational fuzzy algorithm – block *K*

At the third stage of the fuzzy system (block *K*) processing-related to the numerical inputs take place that are analogous to the processes by which the human mind evaluates visual, auditory, tactile, and other stimuli based on which it generates the corresponding reactions.

At this stage the N-dimensional vector $(x_1,\ldots,x_N)$ entering the block K is transfigured to a vector $(u_1,\ldots,u_N)$ by converting the coordinates x_i into a scale in the range 0 to 100. The coordinates u_i, i = 1,…,N are "integrated" in the adequate input fuzzy sets. The adequate inference rules for the manipulation with them are selected, their "strength" is defined and the membership function μ_{agg} is generated on the domain $V = \langle 0, 100\rangle$ of the output linguistic variable non-BFI. The horizontal coordinate of the centre of gravity of the surface below its course $\mu_{agg}(v)$ represents the result.

Fig. 2: Phase diagram of the computational algorithm of the fuzzy approach at the third stage of the model



Source: own processing

The phase diagram of the computational algorithm in Fig. 2 consists of five steps. In the fuzzification phase it is necessary to take into account the expected influence of the x_i coordinate of the vector $(x_1,\ldots,x_N)$ on the output v. The superiority of positive effects will result in low expectations of bankruptcy – the non-Bankruptcy fuzzy index will be high; on the contrary, if negative effects prevail, the high value of bankruptcy can be expected – the non-Bankruptcy fuzzy index will be low.

To every i-*th* input, i = 1 to N, a fuzzification table is compiled (see Tab. 1) that generates a set $h(u_i) = \{\underline{T}_i: \underline{T}_i \in \{\underline{L}_i, \underline{M}_i, \underline{H}_i\}, \mu(\underline{T}_i, u_i) > 0$. The Cartesian product $H = h(u_1) \times\ldots\times h(u_N) = \{(\underline{T}_1,\ldots,\underline{T}_N): \underline{T}_1 \in h(u_1),\ldots,\underline{T}_N \in h(u_N)\}$ with 2^α elements is formed from them, where α, $0 \le \alpha \le N$ is the number of two-element sets $h(u_i)$ in the Cartesian product H.

Tab. 1: Fuzzification table with *a, b, c, d* ∈ ⟨0, 100⟩, 0 ≤ a ≤ b ≤ c ≤ d ≤ 100

Interval	$u < a$	$a \le u < b$	$b \le u < c$	$c \le u < d$	$u \ge d$
$\underline{L}$	1	$(b-u)/(b-a)$	0	0	0
$\underline{M}$	0	$(u-a)/(b-a)$	1	$(d-u)/(d-c)$	0

$\underline{H}$	0	0	0	$(u-c)/(d-c)$	1

Source: own processing

The inference rule is described as the element of projection p: $\{\underline{L}_1, \underline{M}_1, \underline{H}_1\} \times \ldots \times \{\underline{L}_N, \underline{M}_N, \underline{H}_N\} \rightarrow \{\underline{L}, \underline{M}, \underline{H}\}$, where $\underline{L}$, $\underline{M}$ and $\underline{H}$ are the terms of the output linguistic variable with a domain of definition of numerical values $V = \langle 0, 100 \rangle$. Thus, the set of inference rules consists of a total 3^N pairs of the type $((\underline{T}_1, \ldots, \underline{T}_N), T)$; these rules are given by experts in the subject area in question. In the inference rules application phase, three classes of decomposition of the set H according to the terms of the output linguistic variable are constructed, $H(\underline{T}) = \{(\underline{T}_1, \ldots, \underline{T}_N): (\underline{T}_1, \ldots, \underline{T}_N) \in H \cap p^{-1}(\underline{T})\}$, $\underline{T} = \underline{L}, \underline{M}, \underline{H}$, where the relation p^{-1} is the inversion of projection p. Each resulting class $H(\underline{T})$ is then assigned by its characteristic number $\mathrm{M}_{\underline{T}} \in \langle 0, 1 \rangle$ as follows:

- if the class is empty ($H(\underline{T}) = \emptyset$), then $\mathrm{M}_{\underline{T}} = 0$;
- if $H(\underline{T}) \neq \emptyset$, then $\mathrm{M}_{\underline{T}} = \max\{\{\min\{\mu(\underline{T}_1, u_1), \ldots, \mu(\underline{T}_N, u_N)\}: (\underline{T}_1, \ldots, \underline{T}_N) \in H(\underline{T})\}\}$.

In the phase of the result processing, the fuzzy algorithm by means of the numbers $\mathrm{M}_{\underline{L}}$, $\mathrm{M}_{\underline{M}}$ and $\mathrm{M}_{\underline{H}}$ restricts (cuts off) the course of the functions $\mu_{\underline{L}}(v)$, $\mu_{\underline{M}}(v)$ and $\mu_{\underline{H}}(v)$ of the terms $\underline{L}$, $\underline{M}$ and $\underline{H}$ of the output linguistic variable on the domain of its numerical values $V = \langle 0, 100 \rangle$.

In the aggregation phase, the fuzzy algorithm fuzzy logically adds their torso, thus aggregating them into the resulting function $\mu_{\mathrm{agg}}(v)$, which creates their wrapper.

The final step is defuzzification, in which the output value y_x of the multicriteria model is obtained as the mean value of the elements the aggregation phase weighted by the values $\mu_{agg}(v)$ of their significance – see (2).

$$v_x = \int v \cdot \mu_{agg}(v)dv / \int \mu_{agg}(v)dv \qquad (1)$$

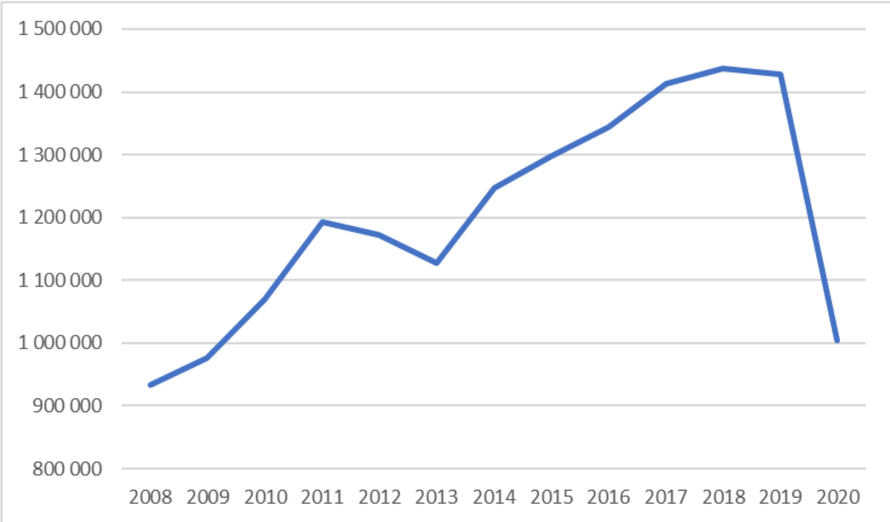
where $\int$ is the symbol of a definite integral over the universe V.

3 Application – introduction to a study

Economists and politicians have often discussed the strong dependence of the Czech economy on the automotive industry and strong export focus on Germany. This dependence has been manifested most drastically during economic crises – see Figure 3 that captures the development of passenger cars production in the Czech Republic (CZ) between 2008 and 2020 (OICA, 2021).

The coronavirus outspread brought a significant decline in the production and car sales, automotive components, and technologies. For example, Škoda Auto factory that is understood as the heart of the Czech industry stopped all lines. This factory creates on average for about five percent of GDP and accounts approximately 9 % of exports.

Fig. 3: Passenger cars production development in the Czech Republic



Source: OICA (2021) – own processing

3.1 The task database and data processing

The following procedures are based on the data of Annual Reports of Auto Škoda a.s. (Škoda, 2021) and Czech Statistical Office (ČSÚ, 2021).

The short-time fuzzy bankruptcy prediction of Škoda Auto factory leans on I. a series of financial ratios, which are essential components of a modified version of the Taffler´s model based on Z-score evaluation – see Horak et al. (2020), which are R_1 = Income before tax / Current liabilities, R_2 = Current assets / Debt, R_3 = Current liabilities / Total assets, R_4 = Total sales / Total assets, II. Dividend yield expressed in % (D = Forward full year dividend / Current share price) and III. Basic index of the confidence indicator balance expressed in points (CI – the economic sentiment indicator, expressed as a basic index).

The Z score and Dividend yield are economic criteria which computation is based on objective sources; the CI expresses a sensitivity indicator of future expectations in economic development based on subjective judgements. From the viewpoint of external and internal interest groups these criteria perform data for which the highest possible value is desirable.

All the results measured in period 2011 – 2020 are reported within the intervals (the worst, respectively the best results are the extremes determining the interval limits): $R_1 \in \langle 0.17, 0.52\rangle$, $R_2 \in \langle 3.15, 8.6\rangle$, $R_3 \in \langle 0.31, 0{,}47\rangle$, $R_4 \in \langle 1.52, 1.9\rangle$, $D \in \langle 0.1, 3.8\rangle$, $CI \in \langle 82, 106\rangle$; most of the "worst" extremes are recorded in 2020, i.e., R_1, R_2, R_3 and CI.

These measured/calculated criteria at the first stage of the three-stage model are subjected to its second stage where Taffler´s bankruptcy score is calculated from R_1 – R_4 resulting to Z-score $\in \langle 0.88, 1.63\rangle$. As the left limit Z (0,88) > 0,3, Taffler´s model excludes an imminent threat of bankruptcy. D and CI criteria are input parameters that enter the bankruptcy evaluation to the second stage of the model directly from the first stage.

The rough assessment of a bankruptcy threat made from the interval values Z, D and CI indicates that the firm is not in an imminent danger of bankruptcy. With what degree of veracity can be argued that the company is out of immediate danger of bankruptcy? To answer the question, we need to solve the problem at the third stage, i.e., in block *K* (see Fig. 1 and part 2).

Fuzzy approach in block *K* requires the transformation of the interval values Z score, D and CI to a uniform rating scale. Within the period 2011-2020 the company experienced a decline and expansion. The best results of the criteria in this period are the maximal values that can be achieved in the near future 2021-2022 under condition of favourable development in the automotive industry. Analogically, the worst results are considered the possible minimal values. The common scale is a percentage scale on the interval $\langle 0, 100\rangle$, where the 100 % corresponds to the value of the right interval extreme and signals zero tendency to go bankrupt; the left extreme indicates a definite bankrupt.

The block *K* includes numerical vectors in % (Z, D, CI), $Z \in \langle 54, 100\rangle$, $D \in \langle 3, 100\rangle$, $CI \in \langle 77, 100\rangle$ of the triple of linguistic variables Z, D and CI with terms $\underline{L}_i$, $\underline{M}_i$ and $\underline{H}_i$, i = z, d, ci. For simplicity, let us suppose that their fuzzification tables are symmetric and identical except for subscripts (i.e., for all i = z, d, ci it applies: $a_i = 20$, $b_i = 40$, $c_i = 60$, $d_i = 80$). The fuzzy procedure of block *K* enables us to calculate a number $v \in \langle 0, 100\rangle$ of the output linguistic variable of a non-Bankruptcy fuzzy index (BFI) taking into account the terms $\underline{L}$, $\underline{M}$ and $\underline{H}$ ($\underline{L}$ – low, $\underline{M}$ – middle, $\underline{H}$ – high). The set of inference rules of the type $((\underline{T}_z, \underline{T}_d, \underline{T}_{ci}), \underline{T})$, $\underline{T}_i \in \{\underline{L}_i, \underline{M}_i, \underline{H}_i\}$, $\underline{T} \in \{\underline{L}, \underline{M}, \underline{H}\}$ has 27 elements formed by the strategies of the predominant element. It assigns to the given left side the very term $\underline{T}$, which predominates on the left side. If there is no such term, the term $\underline{M}$ is assigned.

For the recalculated vector (u_1, u_2, u_3) the following applies: $u_1 = z$, $u_2 = d$ and $u_3 = ci$. To obtain the outputs $v_{\min}$ and $v_{\max}$, we use a combination ($u_{1\min}$, $u_{2\min}$, $u_{3\min}$) for $v_{\min}$ and ($u_{1\max}$, $u_{2\max}$, $u_{3\max}$) for $v_{\max}$. Then, it applies:

- $h(u_{1\min}) = h(54) = \{\underline{M}_z\}$, $h(u_{2\min}) = h(3) = \{\underline{L}_d\}$, $h(u_{3\min}) = h(77) = \{\underline{M}_{ci}, \underline{H}_{ci}\}$, $H = \{(\underline{M}_z, \underline{L}_d, \underline{M}_{ci}), (\underline{M}_z, \underline{L}_d, \underline{H}_{ci})\}$, $M_{\underline{L}} = M_{\underline{H}} = 0$, $M_{\underline{M}} = \max\{\{\min\{1, 1, 0.15\}, \{\min\{1, 1, 0.85\}\}\} = \max\{0.15, 0.85\} = 0.85$. For the value $\boldsymbol{v_{\min}}$ it holds: $\boldsymbol{v_{\min}} = \int v \cdot \mu_{\underline{agg}}(v)\mathrm{d}v \,/\, \int \mu_{\underline{agg}}(v)\mathrm{d}v = 1827.5 / 36.55 =$ **50 %**.
- $h(u_{1\max}) = h(100) = \{\underline{H}_z\}$, $h(u_{2\max}) = h(100) = \{\underline{H}_d\}$, $h(u_{3\max}) = h(100) = \{\underline{H}_{ci}\}$, $H = \{(\underline{H}_z, \underline{H}_d, \underline{H}_{ci})\}$, $M_{\underline{L}} = M_{\underline{M}} = 0$, $M_{\underline{H}} = \max\{\{\min\{1, 1, 1\}\}\} = 1$. For the value $\boldsymbol{v_{\max}}$ it holds: $\boldsymbol{v_{\max}} = \int v \cdot \mu_{\underline{agg}}(v)\mathrm{d}v \,/\, \int \mu_{\underline{agg}}(v)\mathrm{d}v = 2533.33 / 30 =$ **84 %**.
- The average interval fuzzy value $v_x = (50 + 84) / 2 =$ **67 %**.

3.2 Discussion

The procedure shows a multicriteria fuzzy approach to the short-time bankruptcy prediction using the three-stage fuzzy model. The own process of fuzzy analysis takes place at the third stage in the block *K* (see part 2). Its input numerical vectors (*Z*, *D*, *CI*) are intervals with right extremes of the value 100 %, which corresponds to the assumption that the best measured criterial inputs between 2011 – 2020 will not exceed this value in the next two years.

The fuzzy interval analysis last step (see the computation algorithm in Fig. 2) provides us with the output in the form of the fuzzy interval $v = \langle 50, 67, 84 \rangle$ %, where the value 67 % is the arithmetic average of extreme values $v_{\min}$ and $v_{\max}$. This value indicates the tendency of the company to go bankrupt in the short term. Since non-BFI = v_x = 67 %, the average susceptibility to bankruptcy of Škoda Auto is below average.

Given the fact that the left extreme $v_{\min}$ is the bottom limit of possible (expected) values, which will not be likely lower (with the 68 % probability) in the next two years, the value $v_{\min}$ can be interpreted as a measure of business safety from the viewpoint of the bankruptcy evaluators.

4 Summary and conclusion

The topic of the paper is the issue of multicriteria ex-ante evaluation in terms of interval input data. A three-stage fuzzy system is presented, which sophisticatedly projects the results of partial criteria into a range of values of the non-Bankruptcy fuzzy index (non-BFI).

At its first stage, the partial criteria enter the system in the form of the extreme points of intervals of its values. For them, at the second stage, the sets of their corresponding results expressed in the language of interval algebra are generated. The elements of the Cartesian product of these sets are then gradually processed at the third stage by the computational fuzzy algorithm. The processing of each element goes through five steps. The arithmetic mean of the maximum and minimum of the algorithm results above all elements of the Cartesian product is a subjectively expected value of the non-Bankruptcy fuzzy index.

The subjectively expected value is derived from the basic parameters of the fuzzy system (the shape of μ functions of relevant terms and the set of inference rules), which are generally given by experts based on their knowledge, skills, and experience, thus influencing the function of the fuzzy system by means of "human factor".

In the application part the course of calculations is performed based on the real data of Škoda Auto factory to assess a short-term prediction to bankruptcy. The own fuzzy computation was demonstrated at the third stage of the model – block *K*. The fuzzy interval output $v = \langle 50, 67, 84 \rangle$ % of non-BFI does not indicate the imminent danger of bankruptcy in 2021-2022. The left extreme v_{min} is the bottom limit of possible values, which will not be likely lower in the next two years. The value v_{min} can be thus interpreted as a measure of business safety from the viewpoint of bankruptcy evaluators.

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