EWMA-BASED LTPD RECTIFYING ACCEPTANCE SAMPLING PLANS

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Abstract

The lot tolerance percent defective plans for the acceptance sampling which minimize the mean inspection cost per lot of the process average quality when the remainder of the rejected lots is inspected are addressed in this paper. The sampling plans for the inspection by variables when the statistic based on the exponentially weighted moving average is used and the remainder of the rejected lots is inspected by attributes are discussed. The calculations are made under the assumption that the standard deviation of the quality characteristic is known.

The optimal acceptance sampling plan is calculated in a short case study and the effects of some of the parameters such as the process average proportion defective and the lot size on the mean inspection cost per lot of the process average quality is shown. The R extension package LTPDvar is used for the calculations of optimal plan and for the economic evaluation of the plan.

Key words: inspection cost, acceptance sampling, LTPD

JEL Code: C44, C80

Introduction

In acceptance sampling, the decision whether the submitted lot should be accepted or rejected is based on the result of the inspection of the sample of items from the lot. If the item is inspected by attributes it is just classified as either good or defective (nonconforming). The other option is to do the inspection by variables. Sampling plans for inspection by variables in many cases allow obtaining the same level of the protection as the corresponding sampling plans for the inspection by attributes while using a lower sample size. The basic notions of the variables sampling plans are addressed in (Jennett and Welch, 1939).

The LTPD sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed by Dodge and Romig for the inspection by attributes. Plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of rejected lots is inspected by attributes) were then proposed and it has been shown that these plans are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. The LTPD plans for the inspection by variables and attributes have been introduced in (Klůfa, 1994), using a sort of an approximate calculation of the plans. Exact plans, using the non-central t distribution in calculation of the operating characteristic, have been reported in (Klůfa, 2010) and implemented in (Kasp říková, 2015). The operating characteristics used for these plans are discussed in (Jennett and Welch, 1939) and (Johnson and Welch, 1940). It has been shown that these plans are in many situations superior to the original attribute sampling plans and similar results have been obtained for the AOQL plans – see the analysis is discussed in (Klůfa, 2015) and (Kaspříková and Klůfa, 2016). With the aim of obtaining further savings in the cost of inspection, the new LTPD plans for the inspection by variables and attributes, designed to use the EWMA statistics, have been proposed in (Kaspříková, 2017).

This paper considers the plans proposed in (Kaspříková, 2017) and shows the economic characteristics of these plans measured by the mean cost of inspection per lot of the process average quality. The optimal acceptance sampling plan for the known standard deviation case is calculated in a short case study and the effects of some of the parameters such as the process average proportion defective and the lot size on the mean inspection cost per lot of the process average quality is shown.

The structure of the paper is as follows: the LTPD plans for the inspection by attributes and the LTPD variable sampling plans minimizing the mean inspection cost per lot of the process average quality are recalled first and then the variable sampling plan is calculated and evaluated with respect to the mean inspection cost per lot of the process average quality. The values of the mean inspection cost per lot of the process average quality are then calculated for plans obtained for different values of the lot size and process average quality.

1 LTPD attributes inspection plans

For the case that each inspected item is classified as either good or defective (the acceptance sampling by attributes), Dodge and Romig (1998) consider sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N\text{-}(N\text{-}n) \cdot L(p_a;n;c) \qquad (1)$$

under the condition $$L(p_t;n;c) \leq \beta \qquad (2)$$

where $L(p, n, c)$ is the operating characteristic (the probability of accepting a submitted lot with proportion defective p when using plan (n, c) for acceptance sampling), N is the number of items in the lot (the given parameter), p_a is the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, P_t=100 p_t is the lot tolerance per cent defective, denoted LTPD), n is the number of items in the sample (n<N), c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c).

Condition (2) provides a guarantee for the consumer that lots of unsatisfactory quality level, with proportion defective p_t are going to be accepted only with specified probability β (consumer's risk). The value β =0.1 is used for the consumer's risk in Dodge and Romig (1998).

2 LTPD variables inspection plans

The LTPD plans for the inspection by variables and attributes (the items in the sample are inspected by variables, the remainder of the rejected lots in inspected by attributes) have been designed in (Kaspříková, 2017) under the following assumptions:

The measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameter μ and known parameter σ^2. For the quality characteristic X there is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L).

This assumption is kind of restrictive but on the other hand it allows to make use of the inspection by variables which allows significant savings in the mean inspection cost (Kaspříková, 2017).

The mean inspection cost per lot of the process average quality p_a for such plans is

$$I_{ms} = n \cdot c_{m} + (N\text{-}n) \cdot (1\text{-}L(p_a;n;k)), \qquad (3)$$

where c_m is the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes. The units of measurement for the I_{ms} function values is the cost of inspecting an item by attributes. The I_{ms} cost function is to be minimized when searching for the sampling plans in (Kaspříková, 2017).

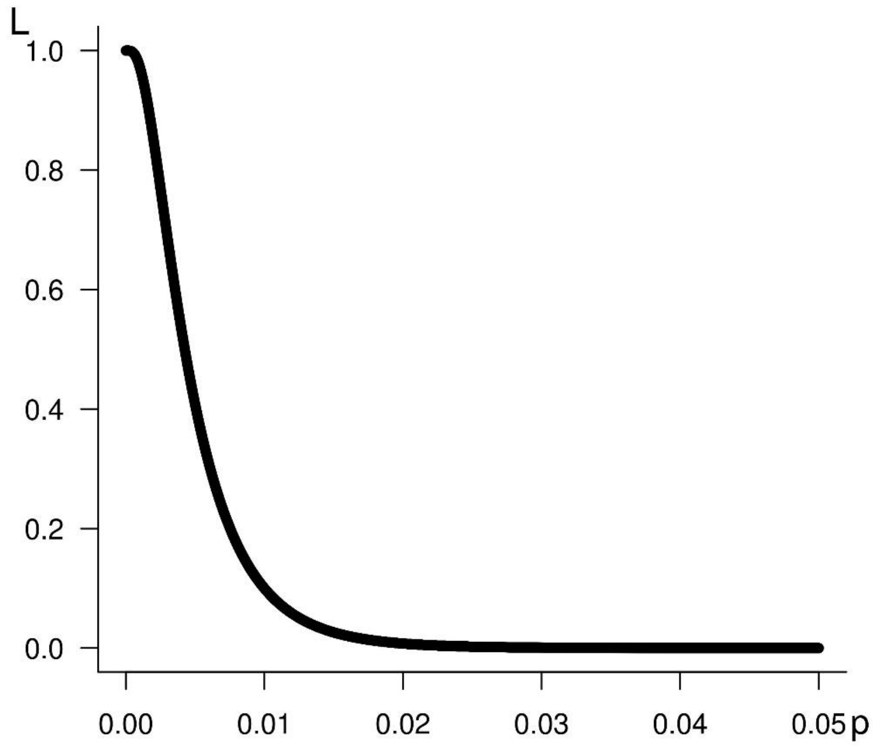
3 Calculation and cost of LTPD variables inspection plan

Let's calculate the LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes and if the standard deviation of the quality characteristic is known in a case study below.

Example. We consider a lot of N = 1000 in the acceptance procedure. Lot tolerance proportion defective is given to be p_t =0.01 and the consumer's risk β =0.1. It is known that average process quality is p_a = 0.001. A cost of inspecting an item by variables is known to be 5.5 times higher than the cost of inspecting an item by attributes, so c_m parameter equals 5.5. Find LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes, using the EWMA statistic with smoothing constant 0.95. The plan can be calculated using the LTPDvar package (Kaspříková, 2015) for the R software (R Core Team, 2020).

The solution is n = 16, k = 2.631. The operating characteristic plot is shown in Figure 1. The mean inspection cost per lot of the process average quality for this plan is 114.33

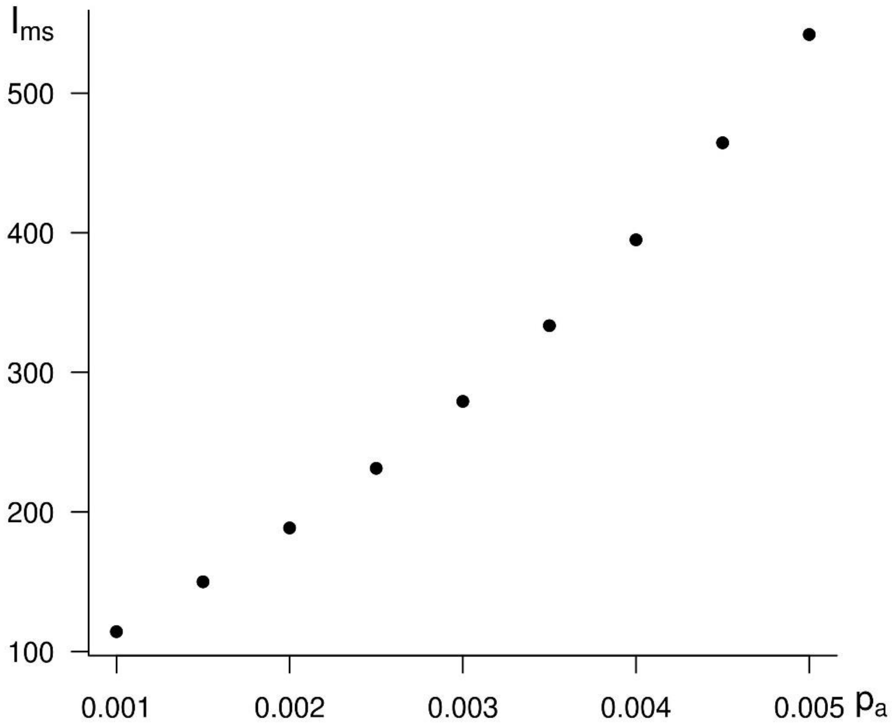
Fig. 1: The operating characteristic of the plan (16, 2.631)



Source: the figure has been produced by the author in R software

The values of the input parameters influence the resulting sampling plan and its economic characteristics. The Figure 2 shows the mean inspection cost per lot of the process average quality for plans calculated for various values of the process average quality, keeping the other parameters unchanged. The values of the mean inspection cost (see also Table 1) are increasing in the process average quality, i. e. when the process average quality becomes closer to the lot tolerance proportion defective, the mean inspection cost increases.

Fig. 2: The inspection cost by process average quality (*N*=1000, p_t =0.01)



Source: the figure has been produced by the author in R software

Tab. 1: The cost function I_{ms} by process average quality (*N*=1000, p_t =0.01)

Process average quality	I_{ms}
0.0010	114.33
0.0015	150.02
0.0020	188.60
0.0025	231.30
0.0030	279.25
0.0035	333.49
0.0040	394.96
0.0045	464.44
0.0050	541.99

Source: the table has been produced by the author in R software

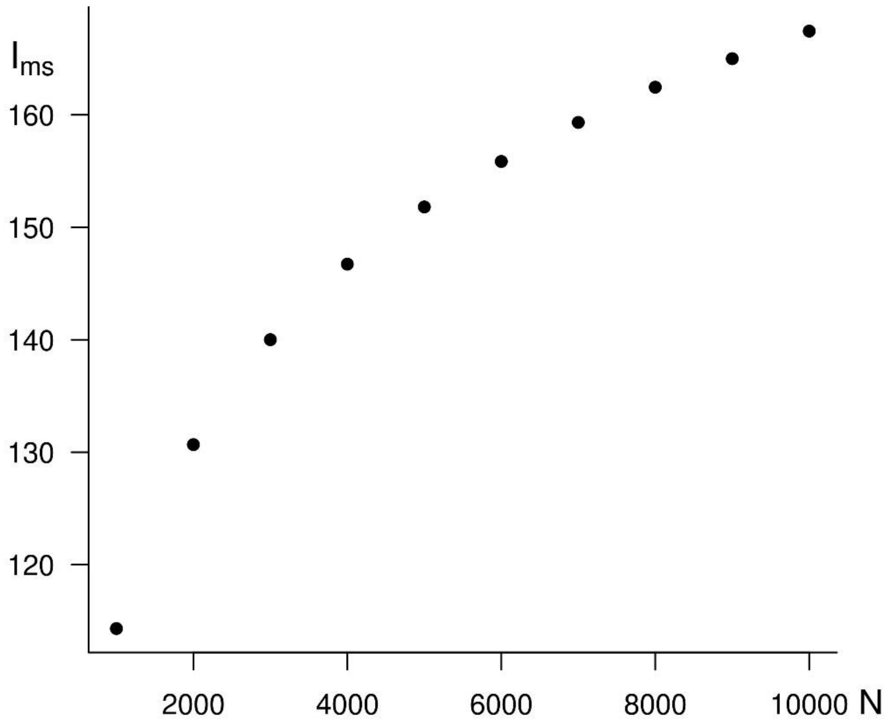
Similarly when the lot size increases, the mean inspection cost of the corresponding sampling plans becomes larger, see Table 2 and Figure 3.

Tab. 2: The cost function I_{ms} by process average quality (p_a =0.001, p_t =0.01)

Lot size	I_{ms}
1000	114.33
2000	130.68
3000	140.01
4000	146.73
5000	151.81
6000	155.86
7000	159.33
8000	162.46
9000	165.00
10000	167.45

Source: the table has been produced by the author in R software

Fig. 3: The inspection cost by lot size (p_a =0.001, p_t =0.01)



Source: the figure has been produced by the author in R software

Conclusion

The LTPD sampling plans for the inspection by variables when the statistic based on the exponentially weighted moving average is used and the remainder of the rejected lots is inspected by attributes were addressed under the assumption that the standard deviation of the quality characteristic is known. The mean inspection cost per lot of the process average quality has been used as the economic characteristic of the plans. The effects of the process average proportion defective and the lot size on the mean inspection cost per lot of the process average quality were shown and it has been observed that the mean inspection cost is increasing in both.

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