ŠIMERKA'S CZECH TEXTBOOK OF CALCULUS

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Abstract

The paper is devoted to the beginning of calculus in the Czech secondary school. School mathematics had delay behind mathematics-science; it took two centuries for mathematical analysis to penetrate into it as a newly created mathematical discipline. Václav Šimerka for nine years he taught mathematics and physics at the grammar school. He was not allowed to continue in teaching because he fell into disgrace at the education authorities. From 1862 he worked as a pastor. In 1863, Šimerka published the grammar school textbook "*Algebra or General Calculating for Higher Grammar Schools*", whose supplement on calculus is the first Czech textbook of calculus for higher schools. Since at the time the calculus were not part of the curriculum of high school mathematics. The text "*Supplement to Algebra for Higher Grammar Schools*" was published separately in 1864. The main difference from today's textbooks, however, is that Šimerka viewed the basic concepts of differential and integral calculus, such as derivative and differential, intuitively, whereas derivation is not the basic term of differential calculus. Šimerka's textbook supports the belief that even deeper concepts of mathematical analysis can be presented in school mathematics in intuitive and accessible way, in line with historical developments.

Key words: calculus, differential of a function, integral, textbook, Šimerka

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Introduction

The article follows up on the contribution by Coufal, Dotlačilová, Tobíšek (2019), in which he is

a) a brief biography of Václav Šimerka, who according to Petržilka (1926) and Hykšová (2006) he was born in 1819 in Vysoké Veselí and ended his career in 1887 in the village of Praskačka near Hradec Králové,
b) interpreted his main mathematical work Power of Conviction (Šimerka, 1882, 1883), which becomes the forerunner of theories of subjective probability and Šimerka is the first Czech mathematician who dealt with applications of mathematics in psychology.

The article is devoted to one of the other activities of Václav Šimerka. School mathematics had delay behind mathematics-science it took two centuries for mathematical analysis to penetrate into it as a newly created mathematical discipline. Differential and integral calculus first became part of university mathematics courses; at that time, the textbook "Elementa calculi differentialis et integralis" by Professor Stanislav Vydra (1741 – 1804), which was published in 1783 in Prague and Vienna, was well known. In the year 1853, by the decree of the Ministry of Culture and Teaching of 29th October 1853, Šimerka was named as a substitute teacher at the Piarist Grammar School in České Budějovice. In the first year of his pedagogic activity he taught physics and Czech language, and in the following years he taught Czech language and mathematics. At that time, Šimerka's scientific treatises (Šimerka, 1858, 1858a, 1859, see Petržílka, 1926, Garcia, Miller, 2019, p. 552) were published in the Sitzungsberichte of the Vienna Academy of Sciences. However, even after nine years, Šimerka did not wait the tenure. Therefore, at the end of the school year of 1861/62, he returned to spiritual administration at his own request. On 20th July 1862, he was appointed pastor.

The process of penetrating mathematical analysis into secondary school mathematics in European countries became more intense after the Meeting of German Naturalists and Physicians in Merano in 1905, where Felix Klein (1849-1925) declared functional thinking as the axis in teaching of mathematics and called for the foundations of differential and integral calculus to high school mathematics. This programme has later become known asthe Meraner Programm (see Krüger, 2001). These tendencies were reflected very early in Austria-Hungary. In his lecture "*On the issue of infinitesimal calculus at the Austrian high school*" in 1906, Karel Zahradníček emphasized the importance of introducing the basics of mathematical analysis into school mathematics by saying: "*It is very desirable for secondary school mathematics to include concept of functtion, differential and integral calculus ; it is necessary in the modern concept of didactics of mathematics, if it is to correspond to the current scientific concept, and it is also necessary for the use of mathematics in physics, which by its nature falls into the field of infinitesimal analysis, whose methods can be easily used here*. "(Potůček, 1993, p. 9).

Nevertheless, already in the 1860s, the need for Czech high school textbooks and the effort to create them appeared. Šimerka submitted a textbook of algebra (Šimerka, 1863) to school authorities for approval, which also included an introduction to differential and integral calculus. He stated that in reviews of the manuscript of his book, he met with support, but also with criticism of his "pioneering attempt" to open up the issue of differential and integral

calculus. Šimerka was so convinced of the interest and need of the mentioned calculus that he originally intended to include it as a part of the textbook itself.

Šimerka eventually decided to publish the part devoted to calculus only as an addition to the textbook of algebra (Šimerka, 1864). This supplement was intended for more inquisitive students who would like to become acquainted with the issue. Although at the time of publishing this book, the ministry that decided on the school plan was not in favor of the idea of introducing the issue to high school, this act is clear evidence that this topic seemed interesting to some high school professors and in a suitable way accessible to high school students (Kopáčková , 2007).

First we will briefly discuss Šimerka's algebra (Šimerka, 1863), then we will go through the Addition to Algebra more thoroughly with the subtitle The Beginnings of the Differential and Integral Calculus (Šimerka, 1864).

1 Šimerka's Algebra

In one of his important writings, the textbook *Algebra or General Calculating for Higher Grammar Schools* (Šimerka, 1863), Šimerka worked in České Budějovice for several years and submitted a manuscript (including an introduction to differential and integral calculus) in May 1861 to the local government for approval. as a high school textbook. After a year, the manuscript was returned to him with four reviews.

One of them from prof. Jandečka from Hradec Králové praised the work, the other two were written decently with scientific justification, but the fourth from the German Mack from the Lesser Town Gymnasium in Prague was full of hateful commentary, no doubt because it was written by the Czechs.

The c. and k. ministry agreed in august 1862 that the book, after omitting the differential and integral calculus (so that students would not be overloaded), be a high school textbook of mathematics. Šimerka reworked the file once more and decided to publish the infinitesimal calculus separately as an addition to algebra.

The book printer Augusta from Litomyšl offered himself to him. Šimerka went to him in the autumn of 1862 and discussed the whole matter with him. In a short time, however, he received the manuscript without the first three sheets back with the message that they did not have a fractional font in the local printer. Soon after, Augusta went bankrupt and fled to America. So far, Šimerka has found another publisher, Dr. Ed. Grégr in Prague, who in 1863 published a textbook of 169 pages.

In Czech newspapers, this work was praised as a new work based on the basic concept of the equation and characterized by the amount of content, but also the great clarity of interpretation. Šimerka writes in the textbook (Šimerka, 1863, p. 1 of the preface): "*Algebra is not a doctrine of equations*", but "*equations are the thread connecting all parts of algebra*".

Šimerka also encountered philological difficulties, because he writes in the preface (Šimerka, 1863, p. 1 of the preface): "*The grammatical side caused me considerable difficulty. The dictionary of scientific nomenclature gives the meanings used in the review; however, many of them come from exaggerated purism, which does more harm than good to science.* " In addition to the usual names today, Šimerka uses names that are unusual today.

The book is divided into an introduction (introductory initial concepts) and four parts containing a total of 18 chapters. From the point of view of mathematical analysis, the most important is probably XVII. a chapter called Series, in which the author deals with sequences, series and the concept of function. It is clear that Šimerka understood the function as a general analytical formula. Such a concept is in Euler's spirit. The name is misleading today.

The term series Šimerka sometimes refers to series, sometimes sequence (in today's concept). Because the functions could not be loaded as displays at that time, it was not possible to define sequences in this way. In the "definition" of this term, the author writes: "*There are several consecutive numbers at all, which are related by a certain rule*." But he also uses the word sequence when talking about arithmetic or geometric sequences.

The terminology associated with series and sequences corresponds approximately to today's. In other parts of this chapter, he deals with arithmetic (calculating) and geometric (measuring) sequences, here Šimerka chooses a description similar to today's. The sequence is described by the relation for the n-th member. We will also find a section devoted to higher-order arithmetic series.

The author first acquaints students with the concepts of constant and variable quantities, these designations then appear in the "definition" of the concept of function. The following is a definition of the term arithmetic function, which corresponds to today's term polynomial. It should be noted that from today's point of view, this is not a definition of a term.

In the part devoted to geometric sequence there are interesting word problems: Achilles and the tortoise (as a solution to Zenon's paradox, which the author states as Zenon's deceptive judgment), reward for the author of chess (both Šimerka, 1863, p. 150), determining the sum of all divisors of a given number (Šimerka, 1863, p. 151).

2 Supplement to Algebra for Higher Grammar Schools

This part *Supplement to Algebra for Higher Gymnasium* (Šimerka, 1864) in the range of 58 pages (including appendices) devoted to differential and integral calculus is divided into six parts:

I. Differentials of Operations ("Differencialy daných úkonů" in Czech).

II. Transformation of Operations into Series ("Proměňování úkonů v řady" in Czech).

III. Trigonometrically Operations ("Úkony trigonometrické" in Czech).

IV. Taylors Theorem and its Corollaries ("Taylorova poučka a její následky" in Czech).

V. Foundations of Integral Calculus ("Základy počtu integrálního" in Czech).

VI. Use of Infinitesimal Calculus in Geometry ("Upotřebení počtu nekonečného v geometrii" in Czech).

Šimerka's conception of differential and integral calculus differs considerably from today's, both in its scope and content. In the part devoted to differential calculus, we will not find concepts or continuity or limits, without which differential and integral calculus cannot be done today. The whole issue is discussed only intuitively with a focus on practical use. Individual terms are not exactly defined. For simplicity, the author uses only continuous functions (discontinuity is considered only in connection with indefinite expressions). If a connection of function is assumed (which is not stated in the publication), the statements made by Šimerka are true. The author focuses on explaining the basic knowledge and algorithms that can be applied in solving mathematical problems. Its aim is to teach students to use mathematics in practical tasks. The theoretical background states only to the extent necessary for an idea of the concepts that are necessary for practical tasks. It is appropriate to show some interesting things.

The author immediately introduces the concept of differential function, but it is not a precise definition, but rather the creation of a kind of intuitive idea. The way of interpreting this term and its use is similar to the approach of mathematicians at the end of the 17th and the whole of the 18th century work with an infinitesimal quantity.

Šimerka's differential (1864, p. 1) means: "*Extremely or infinitely small part, by which we let the continuous variable quantity* (x, y, z, etc.) *grow, it is called the differencial and it means the letter* δ *before the quantity built* (δx, δy, δz, etc.)."

Furthermore, Šimerka (1864, p. 2)*:* "*Differentials are therefore quantities found between zero and the smallest fractions that ever come in practical calculus.*"

The author further builds the theory of differentials on such an intuitively understood concept. First, it presents some "rules for the use of differentials". Here the author states that if the differentials are added to the finite quantities (or subtracted from them), they disappear. Furthermore, the differentials of higher powers also disappear if they are added to the differentials of lower powers (or subtracted from them). On the example of the function $f(x) = a + bx$ Šimerka concludes that "constant quantities" do not have a differential and that during differentiation it is possible to place the constants before the differential. The following is the derivation of rules for differentiating the sum, the product of two, resp. more functions and the share (denoted by a fraction) of functions. The procedure is always the same, because it is based on the expression of the differential: $\delta f(x) = f(x + \delta x) - f(x)$.

Here is a first notice about the proportions $\frac{\delta y}{\delta x}, \frac{\delta^2 y}{\delta x^2}, \frac{\delta^3 y}{\delta x^3}, \ldots, \frac{\delta^k y}{\delta x^k}$, which Šimerka calls the first, second, third,…, k-th "derived act" (or derivative). "Thus, where there is speech about derived functions, they are always notified by the insurrection of differentiations." (Šimerka, 1864, p. 7)

Many of Šimerka's procedures are very cumbersome. The author tries to prove some statements on the basis of Taylor's theorem, but he encounters the problem of the absence of a definition of a limit.

Conclusion

Šimerka's textbook of differential and integral calculus is different from the textbooks used in teaching the basics of differential calculus in high school today. Some topics commonly included in current textbooks are completely missing in it (eg investigation of the course of a function). The textbook contains almost no pictures, there are no graphs of functions. The main difference compared to today's textbooks, however, is that derivation in Šimerka is not a basic concept of differential calculus, it is an "act derived" using a differential, which is based on intuitive infinitesimal calculations without using the term limit. However, Šimerka's textbook supports our belief that even relatively deep concepts of mathematical analysis can be presented in mathematics in accordance with historical development in a more intuitive and accessible way. One can only agree with the important Czech mathematician and philosopher Petr Vopěnka (2010) that the rejection of Newton's and Leibniz's conception of the infinitesimal calculus of mathematics of the 19th and 20th centuries – caused either by their reluctance or inability to guess and complete the basic concepts on which the original

concept of this calculus was based – was one of the greatest mistakes not only in mathematics but in European science in general.

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