ANALYSIS OF PREDICTION OF CURRENT PROFIT AND INTERVAL FUZZY PROFIT IN THE CASE OF SUBSIDIZED PROJECTS

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Abstract

The analysis aims at the prediction of the present value of profit from investment in subsidized renewable energy production operating under condition of uncertainty stemming from a priory ignorance on the input side. The state of such uncertainty does not provide the analyst with any objective evidence to construct a rational estimate. The prediction procedures should, therefore, be based on intervals of possible input values. This basic principle is derived from the fact that in terms of such uncertainty there is a better chance to correctly define the intervals in which priori expected inputs are found than to correctly predict a single value. This argument is developed in the methodology, in which the formula for quantifying the profit present value is translated from the language of the arithmetic into the corresponding formula of the interval language. In the application part, the biogas station is a specific representative of the system operating under conditions of mentioned uncertainty on the input side. Significant interval profit points compared to the spot value of current profit provide information about the degree of safety of the investment in terms of its resistance to possible loss; this conclusion is the main original superstructure of the paper.

Key words: current value of profit, interval fuzzy profit, subsidized project, safety of investment

JEL Code: C44, C58, C61

Introduction

The subsidies of environmentally clean resources of energy have contributed to a sharp increase in demand for production systems of renewable energy (OZE). An exemplar of rapid demand growth in the Czech Republic (CR) between 2009 - 2013 is solar photovoltaic plants. Under the current subsidy conditions and the level of capital expenditure the investments in biogas systems belong to one of the most profitable within OZE (Scarlat et al., 2018) both in the CR and the Central European countries, in which hundreds of biogas stations have been launched during the last two decades (Maroušek et al., 2018).

The current Czech legislation enables to subsidize biogas systems in the form of green bonus or feed-in tariff as an operating support for heat produced from biomass or liquids. Changes in feed-in tariffs during the production system lifetime result from significant decline in operating costs. The amount of green bonus is adjusted annually mainly according to the behaviour of the power electricity price on the market. A reduction in green bonuses is directly related to rising electricity prices, as it is expected that electricity producers will be able to sell the produced energy at a higher price (see Energetický regulační úřad, 2018) and thus, they are compensated for a subsidy reduction. In this respect, the forecast of the profit from the investment in OZE is burdened by uncertainty especially in the case of green bonus. It is hardly possible to confidently predict whether the producers will be able to negotiate a higher purchase price to substitute for a decrease in green bonus.

1 Vagueness, ambiguity and randomness in managerial problems

The type of the uncertain situation described above can be handled by the fuzzy approach. Not only can it bring the calculated values nearer to empirically detectable values, it can also prevent the emergence of a paradox of ambiguity in the expected value. This may occur in conventional approaches due to the application of *indifference principle* when inputs are of uncertain nature (Gelman & Hennig, 2017). The reason for the existence of the paradox of ambiguity is the unsatisfactory way of taking into account uncertainty and from it resulting risk within managerial calculations of IRR, NPV, E[NPV], etc.

Managers often face uncertainty of a triple nature. It may be uncertainty in terms of statistically measurable randomness, uncertainty arising from fuzziness or uncertainty stemming from a priori ignorance. Uncertainty in the sense of *randomness* can only be considered in conjunction with elements of the universe, whose objectively determined basic statistical characteristics are known; then it can be described, for example, by a probability distribution (Tartakovsky & Gremaud, 2017). If there is nothing available, it is fuzziness or uncertainty stemming from a priori ignorance (Bellman & Zadeh, 1970). The reason why it is necessary to distinguish between randomness, fuzziness and uncertainty stemming from a priori ignorance is that they cannot be treated equally (Hašková & Fiala, 2019).

The probability theory and mathematical statistics deals with uncertainty in the sense of randomness. In the case of fuzziness, it is the uncertainty that results from the inaccuracy, ambiguity or vagueness about the substance of the matter, content and the significance of intuitive terms or doubts about problem-solving procedures (see, for example, arbitrage and flexibility of cooking recipes). Mathematics models this type of uncertainty by fuzzy sets, i.e., classes in which no sharp transition between membership and non-membership exist (which occurs when the characteristic function of a class is more than two-valued). Uncertainty, which is brought to the problem by a priori ignorance of the values of future inputs into predictive models, is an essential component of the overall uncertainty associated with the expected results of managerial decisions (Atkinson et al., 2006). The conventional managerial approach often refers to the general principle of indifference, within which the assumption is accepted that uncertainty can be considered randomness with an even probability distribution on the universe of possible values. This is, according to Zadeh (1976), a doubtful assumption.

The paper shows how fuzzy approach deals with the uncertainty stemming from a priori ignorance of input data in managerial calculations, i.e., in situations, in which it is possible to reliably determine the limits within which the numeric values of input variables can be determined, but no relevant information exists that would help to justify prioritizing this or that particular value within the given limits.

2 Methodological approach: present value of profit versus fuzzy interval value of profit

Let one of the conventional criteria for assessing the profitability of the investment projects of n-year lifetime be the net present value (NPV) criterion, which is in our case defined as

$$\mathrm{NPV} = \mathrm{CF}_0 + \sum_{1}^{n} \mathrm{CF} / (1+\mathrm{r})^{i} \tag{1}$$

where $\sum_1^n$ is the summation symbol from i = 1 to n, CF_0 is the initial capital expenditure (negative payment), CF is the payment generated by the project in each year of its lifetime, and r is the discount rate p. a. for each year of the project run.

From this relationship, additional evaluation criteria can be derived to assess the project's profitability, such as, e.g., Return on Investment over the project lifetime (ROI) reflecting the project's profitability relative to the amount of investment (I)

$$\mathrm{ROI} = \mathrm{NPV} / \mathrm{I} \tag{2}$$

Suppose that each of the three inputs to the NPV formula are uncertain, i.e., only the intervals of possible values $\langle CF_{0\min}, CF_{0\max}\rangle$, $\langle CF_{\min}, CF_{\max}\rangle$, $\langle r_{\min}, r_{\max}\rangle$ can be estimated and no other relevant information is known. The question is: What input values should be inserted into the NPV formula?

The conventional approach often answers this question with the help of *principle of indifference*, which says that if there are multiple alternative values for which we do not have any relevant reasons to prioritize one over the other, then we can assign the same probability of occurrence for each. Within the interval $U = \langle x_{\min}, x_{\max}\rangle \subset \mathbb{R}$ the values can be regarded as values of continuous random variable α assigned to the interval U by constant probability density $f_\alpha(x) = 1 / (x_{\max} - x_{\min})$ with a statistically expected value $\mathrm{E}[\alpha] = \int_U (x / (x_{\max} - x_{\min})) \cdot \mathrm{d}x = (x_{\max} + x_{\min}) / 2$. Conventional approach inserts the statistically expected values $\mathrm{E}[\mathrm{CF}_0]$, $\mathrm{E}[\mathrm{CF}]$, $\mathrm{E}[\mathrm{r}]$ of uncertain inputs to the NPV formula, thereby achieving result

$$\mathrm{NPV} = \mathrm{E}[\mathrm{NPV}] = \mathrm{E}[\mathrm{CF}_0] + \sum_1^{\mathrm{n}} \mathrm{E}[\mathrm{CF}] / (1 + \mathrm{E}[\mathrm{r}])^{\mathrm{i}} =$$
$$= (CF_{0\min} + CF_{0\max}) / 2 + \sum_1^{\mathrm{n}} (CF_{\min} + CF_{\max}) / (2 \cdot (1 + (r_{\min} + r_{\max}) / 2)^{\mathrm{i}}) \qquad (3)$$

In contrast, the fuzzy approach interprets the interval $U = \langle x_{\min}, x_{\max}\rangle \subset \mathbb{R}$ as a support of non-fuzzy subset $\underline{\mathrm{A}} = \{(x, \mu_{\underline{\mathrm{A}}}(x)) : x \in \mathbb{R}\}$, $\mu_{\underline{\mathrm{A}}}(x) = 1$ for $x \in U$, $\mu_{\underline{\mathrm{A}}}(x) = 0$ otherwise. This support is a fuzzy number and the NPV formula is understood as projection NPV: $\langle CF_{0\min}, CF_{0\max}\rangle \times \langle CF_{\min}, CF_{\max}\rangle \times \langle r_{\min}, r_{\max}\rangle \rightarrow \langle NPV_{\min}, NPV_{\max}\rangle$, in the form $NPV(x,y,z) = w$, where $x \in \langle CF_{0\min}, CF_{0\max}\rangle$, $y \in \langle CF_{\min}, CF_{\max}\rangle$, $z \in \langle r_{\min}, r_{\max}\rangle$ and $w \in \langle NPV_{\min}, NPV_{\max}\rangle$.

By the conventional approach calculated *statistically* expected value $\mathrm{E}[\mathrm{NPV}] \in \langle NPV_{\min}, NPV_{\max}\rangle$, which is in this projection the reflection of centres of the input intervals, may not be the center of the $\langle NPV_{\min}, NPV_{\max}\rangle$ interval. Conversely, the *subjectively* expected value y_{NPV} will always be the central value. This is because of the projection of a Cartesian product (pattern) into an output interval (image) mediated by the *NPV* criterion function, which is purely a technical matter that does not provide any reason for giving preference to one value over another. Consequently, the output interval $W = \langle NPV_{\min}, NPV_{\max}\rangle$ is also the support of fuzzy number $\underline{\mathrm{NPV}} = \{(w, \mu_{\underline{\mathrm{NPV}}}(w)) : w \in \mathbb{R}\}$, $\mu_{\underline{\mathrm{NPV}}}(w) = 1$ for $w \in W$, $\mu_{\underline{\mathrm{NPV}}}(w) = 0$ otherwise, therefore

$$y_{\mathrm{NPV}} = \int_W w \cdot \mu_{\underline{\mathrm{NPV}}}(x) \cdot \mathrm{d}w \Big/ \int_W \mu_{\underline{\mathrm{NPV}}}(w) \cdot \mathrm{d}w =$$
$$= (NPV_{\max}^2 - NPV_{\min}^2) / (2 \cdot (NPV_{\max} - NPV_{\min})) = (NPV_{\max} + NPV_{\min}) / 2 \qquad (4)$$

The $NPV_{\min}$ and $NPV_{\max}$ values can be obtained in the following three steps:

- the formulation of triplets $\mathbf{CF_0} = (CF_{0\min}, \mathrm{CF}_0, CF_{0\max})$, $\mathbf{CF} = (CF_{\min}, \mathrm{CF}, CF_{\max})$, $\mathbf{r} = (r_{\min}, \mathrm{r}, r_{\max})$ of significant points of the input intervals in which the middle members are centers of intervals, and the triple **NPV** = ($NPV_{\min}$, E[NPV], $NPV_{\max}$) in which the middle member is the statistically expected value;

- substitution of variables CF_0, CF, r, NPV in the criterion function with triplets $\mathbf{CF_0}$, **CF**, **r**, **NPV** and operations of the arithmetic of real numbers +, –, · and / with operations of algebra intervals **(+)**, **(–)**, **(×)** and **(/)**; for example, for n = 3 the relation (1) transforms into the form of the formula of language of algebra of intervals: **NPV** = $\mathbf{CF_0}$ **(+) CF(×)**[1**(/)**(1 **(+) r**) **(+)** 1**(/)**((1 **(+) r**)**(×)**(1 **(+) r**)) **(+)** 1**(/)**((1 **(+) r**)**(×)**(1 **(+) r**) **(×)**(1 **(+) r**))];
- by means of the application of operations of algebra of intervals defined in the article Hašková (2017), in which the relations for significant **NPV** points are derived for a more general variant of NPV formula. In our case the system of relations is described as (5):

$$NPV_{\min} = CF_{0\min} + \sum_{1}^{n}[\max\{CF_{\min},0\}/(1+r_{\max})^{i} + \min\{CF_{\min},0\}/(1+r_{\min})^{i}],$$

$$\mathrm{E[NPV]=CF_0} + \sum_{1}^{n}\mathrm{CF/(1+r)^{i}}, \tag{5}$$

$$NPV_{\max} = CF_{0\max} + \sum_{1}^{n}[\max\{CF_{\max},0\}/(1+r_{\min})^{i} + \min\{CF_{\max},0\}/(1+r_{\max})^{i}].$$

3 Application of the conventional and fuzzy approach to analysis of the present value and the fuzzy value of profit

The proportional decline in feed-in tariffs driven by the decline in operating costs does not lead to a negative impact on the budgeted profit, and hence the profitability of the project. Therefore, we will consider an impact of future growth in market prices of electricity and with this related possible decrease of the green bonus on the value of estimated profit of the biogas plant project put into operation by the end of 2019 (between 2016 and 2019 prices of electricity in CR increased by CZK 0.337/kWh, see Kurzy.cz, 2019).

3.1 Input data: the biofuel plant project´s (BP) budgeted cash flow

To analyze the impact of a decrease in green bonus on the budgeted net profit of BP we draw from data of Tab. 1, in which the symbols a, b, c, d represent the values of budgeted revenues. Symbol α expresses the uncertainty in revenues for energy produced that includes green bonus and purchase price, for which it applies α = P / P_E. Symbols P / P_E represent the actual purchase price for the energy produced / the expected (budgeted) price. If $P = P_E$, then $\alpha = 1$ and the budgeted revenues correspond to current sales as follows: a = 1800, b = 1900, c = 2500, d = 3800 (the values are stated in thousands of CZK, further marked as kCZK). In the case of $P < P_E$, $\alpha \in \langle 0.8, 1)$; the lower limit is given by a pessimistic scenario expressing the maximum drop of the actual price compared to the price budgeted. Parameter β stands for uncertainty in investment expenditure given in the range $\beta \in \langle 1, 1.3 \rangle$, in which the lower limit corresponds to budgeted expenditures; the upper limit performs 30 % growth in spending.

Tab. 1: Yearly cash flows (CF) in kCZK generated by the BP under uncertainty

	Period (years)	0	1	2	3	4	5	6-21	22-31
	Year	2017	2018	2019	2020	2021	2022	2023-38	2039-48
1	Cap. subsidy	1000							
2	Cap. investment	3000	500·β						
3	Revenues			a·α	b·α	c·α	d·α	d·α	d·α
4	Operating costs			1300	1500	1900	2500	2400	2400
5	Depreciation in total			80	150	170	180	180	30
6	EBT (3–4–5)			a·α −1380	b·α −1650	c·α −2070	d·α −2680	d·α −2580	d·α −2430
7	Tax 24 % of EBT			0.24·a·α −330	0.24·c·α −390	0.24·c·α −500	0.24·d·α −640	0,24·d·α −620	0,24·d·α −580
8	EAT (6 – 7)			0.76·a·α −1050	0.76·b·α −1260	0.76·c·α −1570	0.76·d·α −2040	0,76·d·α −1960	0,76·d·α −1850
9	Operating CF (8+5)			0.76·a·α −970	0.76·b·α −1110	0.76·c·α −1400	0,76·d·α −2220	0,76·d·α −1780	0,76·d·α −1820
10	CF of cap. bud. (1-2+9)	−2000	−500·β	0.76·a·α −970	0.76·b·α −1110	0.76·c·α −1400	0,76·d·α −2220	0,76·d·α −1780	0,76·d·α −1820

Source: own processing

The data of Tab. 1 correspond to the BP cash flows of the installed electrical power 1000 kW, which is put into operation in 2019. The BP is financed from the firm resources and government subsidy. The budgeted revenues and operating costs result from the expert assessment, which is based on similar projects with regard to the unique characteristics of the particular project. The purchase tariffs and operating costs are not adjusted to inflation. Therefore, Tab. 1 corresponds to the situation with zero inflation or both the cash revenues and operating costs change in the same proportion as general price level. Since the relevant discount rate r represents alternative costs of capital in the same class of subsidized projects, we assume that $r_{\min} = r_{\max} = r$ (see the set of relations (5)).

The initial cash flows involving 32 payments captured in 10^{th} row of Tab. 1 can be replaced by the equivalent flows with seven payments as shown in Tab. 2. In it, the CF payments of the years 1 – 5 are the forecasted cash flows. The present values PV_5 and PV_{21} are "shadow" payments, which are equivalent to the effect of annuities that replace them. The set of relations (6) in Tab. 2 describes the reduced set of payments as follows:

Tab. 2: The set of relations (6) corresponding to the budgeted CF in Tab. 1 in kCZK, net internal yield r = 9 % reflects the yield of renewable resources production in the CR

CF_0	-2000	(6)
CF_1	$-500 \cdot \beta$	
CF_2	$0.76 \cdot a \cdot \alpha - 970 = 0.76 \cdot 1800 \cdot \alpha - 970 = 1360 \cdot \alpha - 970$	
CF_3	$0.76 \cdot b \cdot \alpha - 1110 = 0.76 \cdot 2100 \cdot \alpha - 1110 = 1600 \cdot \alpha - 1110$	
CF_4	$0.76 \cdot c \cdot \alpha - 1400 = 0.76 \cdot 2500 \cdot \alpha - 1400 = 1900 \cdot \alpha - 1400$	
CF_5	$0,76 \cdot d \cdot \alpha - 2220 = 0.76 \cdot 3800 \cdot \alpha - 2220 = 2900 \cdot \alpha - 2220$	
PV_5	For the annuity payments of the 16-year annuity it applies: $0.76 \cdot d \cdot \alpha - 1780 = 0.76 \cdot 3800 \cdot \alpha - 1780 = 2900 \cdot \alpha - 1780.$ Considering the internal yield of 9 % and the annuity factor of 8.3126 it corresponds to the present value PV_5 $(2900 \cdot \alpha - 1780) \cdot 8.3 = 24070 \cdot \alpha - 14774$	
PV_{21}	For the annuity payments of the 10-year annuity it applies: $0.76 \cdot d \cdot \alpha - 1820 = 0.76 \cdot 3800 \cdot \alpha - 1820 = 2900 \cdot \alpha - 1820$ Considering the internal yield of 9 % and the annuity factor of 6.4177 it corresponds to the present value PV_{21} $(2900 \cdot \alpha - 1820) \cdot 6.4 = 18560 \cdot \alpha - 11648$	

Source: own processing

3.2 Task solution

From the above data and calculations, the interval $\mathbf{CF_0}$ = (–2000, –2000, –2000) can be immediately compiled and the significant points of the intervals $\mathbf{CF_1}$ = (–650, –575, –500), $\mathbf{CF_{i=2\text{-}32}}$ = ($CF_{\min}$, CF, $CF_{\max}$) calculated valid for 100 % efficiency of BP system. For example, for $CF_{\min=2}$ = 118 = 1360 · 0.8 – 970 and $CF_{\max=2}$ = 390 = 1360 – 970. Since no relevant reason exist to assume that the values expected in $\mathbf{CF_1}$ or $\mathbf{CF_{i=2\text{-}32}}$ should be closer to one or the other extreme point, they were placed in the center of intervals (CF_1 = (–650 + (–500)) / 2 = –575 and e.g., CF_2 = (118 + 390) / 2 = 254). After substituting into the system of equations (5) significant points of interval **NPV** = (1221, 3719, 6217) in kCZK were calculated as follows:

- $NPV_{\min} = -2000 - 650 / 1{,}09 + 118 / 1{,}09^2 + 170 / 1{,}09^3 + 120 / 1{,}09^4 + 100 / 1{,}09^5 + 4482 / 1{,}09^5 + 3200 / 1{,}09^{21} = 1221$,
- $NPV = -2000 - 575 / 1{,}09 + 254 / 1{,}09^2 + 330 / 1{,}09^3 + 310 / 1{,}09^4 + 390 / 1{,}09^5 + 6889 / 1{,}09^5 + 5056 / 1{,}09^{21} = 3719$,
- $NPV_{\max} = -2000 - 500 / 1{,}09 + 390 / 1{,}09^2 + 490 / 1{,}09^3 + 500 / 1{,}09^4 + 680 / 1{,}09^5 + 9296 / 1{,}09^5 + 6912 / 1{,}09^{21} = 6217$.

4 Discussion

This example shows that the transformation of uncertain input data into criterial function-mediated output data does not result in factual favoring one data at the expense of others. The validity of the indifference principle over the interval of possible values is not disturbed by that, therefore, E[NPV] and y_{NPV} are identical, which can be justified by the linear nature of the criterial function. The case of lower E[NPV] compared to y_{NPV} would be attributed to the distortion caused by nonlinearities of the criterial function resulting from uncertainty of discount rates, which was not our case.

Unlike the fuzzy approach, the conventional approach says nothing about the limits of project profitability or whether and to what extent the project investment could be loss-making. This information, inter alia, is provided by the fuzzy approach in the form of the trio **NPV** = (1221, 3719, 6217): primarily, it shows that the project is profitable even in the case of pessimistic development. The possible profitability values expressed for the investment lifetime in (%) **ROI** = (47, 147, 253) – see relation (2). It can be stated that under condition of uncertain inputs the fuzzy approach offers more complex and useful information compared to the conventional approach based on one-value criterion.

Conclusion

The analysis was focused on the prediction of the present value of profit from the investment in subsidized renewable energy production system operating under conditions of uncertain data on the input side. This uncertainty is understood as a state of vagueness or fuzziness, which does not provide the decision-maker with adequate evidence based on which reliable estimate can be constructed. Therefore, one-value profit estimate in the form of net present value was supplemented by the calculation of interval of possible net present values calculated from the intervals of the possible input values. Basic principle of the procedure derives from the fact that in terms of such uncertainty there is a greater chance to correctly define the intervals in which a priori expected inputs are found than to correctly predict a single value. This was developed methodologically by means of the relevant formula translation from the language of the arithmetic into the corresponding formula of the language of intervals. We compared conventional and fuzzy approach in terms of processing uncertain inputs in a criterial function. The conventional approach is based on the principle of indifference that excludes the preference of any value, which means that the possible values are "rated" by the uniform probability; the fuzzy approach interprets the interval of possible values as the subset support, which is a fuzzy number, and a criterion function as a projection.

A conventional and fuzzy profitability of biogas station project was examined to demonstrate a typical system operating under conditions of uncertainty on the input side. Significant NPV points of interval were calculated and compared to the single value of current profit of the conventional approach. The results were identical. This consistency was a consequence of linearity of the criterial function, in which the uncertainty at discount rates was not considered. Generally, it can be concluded that the resulting interval of possible NPV values provides information about the degree of investment safety in terms of its resistance towards possible loss, which is regarded as the main original superstructure of the paper.

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