COMPARISON OF THE PROBABILITY APPROACH AND THE FUZZY APPROACH TO THE ASSESSMENT OF THE ECONOMIC EFFICIENCY OF INVESTMENT PROJECTS

Simona Hašková

Abstract

The paper aims to reduce the shortcomings of traditional methods based on discounted cash flows used to assess the economic efficiency of investment projects. These methods´ main problem is how to solve the non-standard inaccuracies evoked by uncertainty of the input data that are brought into a problem. A rational approach to limit this uncertainty is provided by probabilistic models in the case that these data can be assigned by means of their distribution function. These cases are restricted by a historically short observation period on which the accuracy of the distribution of input values in a newly considered investment can build. In situations, where this distribution cannot be estimated reliably, a fuzzy approach is more appropriate tool that approximates the unknown distribution of input values by an even distribution over the interval of their possible values. Within the mutual comparison the both approaches are presented. The conditions of their application are discussed in view of the economic efficiency of long-term projects estimation and their benefits and shortcomings are presented using a specific example.

Key words: fuzzy approach, probability approach, uncertainty, investment project

JEL Code: C15, C 44, G31

Introduction

Every company is limited by financial, material and human resources and also by the time that needs to be spent on new investment projects. Firms' competition for investment projects is very intense. The decisive parameter of success in this competition is, in particular, forecast of future investment flows (Flyvbjerg, 2008), on which traditional methods of economic evaluation is based. These methods (e.g. net present value, payback period, internal rate of return, profitability index) are in practice the most commonly used one-criterion methods. They reflect the time value of money and therefore, according to economists, they are consistent with the behaviour of a rational decision-maker. However, the inclusion of time and risk factors in the computational model does not reduce the inaccuracy of the calculation result (Maroušek et al., 2014). The greater part of the difference between the result of the calculation and the real result is due to the uncertainty associated with the ignorance of the exact future cash flow points, the discount rate, or inflation development.

Statistical approaches are limited by the historically short observation period, on which the accuracy of the distribution of the input values can build in the newly considered investment (Tufekci and Young, 1987). If a probability distribution of possible CF_i results cannot be assembled, the use of traditional methods is inappropriate.

This paper aims to reduce the shortcomings of traditional deterministic methods based on discounted cash flows, whose problem is how to solve the non-standard inaccuracies that the uncertainty of input data brings to a problem. For this purpose, the reasons for the incorrect investment decisions in terms of traditional and behavioural economics are summarized.

1 A brief outline of reasons leading to erroneous investment decisions

The Flyvbjerg (2008) research shows that most of the large investment projects are behind the schedule, significantly outperforming the budget, and many of them will never meet the expectations of investors.

Standard economic theory explains the failures in evaluating investment projects as a consequence of firms undergoing rational risks under uncertain conditions. Entrepreneurs evaluate the likelihood of success and failure, and accept such an investment if the reward resulting from success is sufficiently tempting. In the long run, profits from several successful investments should outweigh the losses from many failures.

Behavioral economists (Kahneman and Lovallo, 1993) do not believe that a large number of such failures are the result of the best rational choice whose outcome has failed, they rather tend to believe that this is the result of a mistaken decision based on ignoring the competitor's capabilities, overvaluation of own capabilities and the so-called deception of anchoring. As a result, investments that are unlikely to match the planned budget, implementation time, or expected benefits are preferred.

In the real world, the investment decision-making takes place in an environment where the consequences of the decisions cannot be predicted with certainty. Both the internal and external factors that the decision-maker is able to influence only partially or not at all, are blamed. In practice, however, it is often quietly assumed that uncertainty, regardless of its nature, can be considered as randomness, which is a false premise. Randomness can only be connected with the elements of the universe whose basic statistical characteristics are known. If nothing of this is known, it is uncertain (Lai and Xiao, 2018).

To cope with the existence of uncertainty the technique of fuzzy approach helps, while with randomness the concepts and techniques of the probability theory.

2 Methodological approach: probability theory versus philosophy of fuzzy approach

Statistical processes of the *probability theory* are, within investment decision-making, based on the expected value E[PV], which is the present value of the current expected annual cash flows $E[CF_i]$, for i = 1, 2 to n, given as:

$$E[PV] = \sum_{i=1}^{n} (E[CF_i]) / \prod\nolimits_{j=1}^{i} (1 + r_j) \qquad (1)$$

where CF_i, $i > 0$ are the net cash flows (positive or negative) perceived as statistically independent random variables generated by the project in the i-th year of its run, r_j is the positive discount rate per annum valid in the j-year of the project.

The *fuzzy set theory* represents an alternative approach in the case of uncertain variables in the structure of CF_i values or uncertain discount rates r_j. It is based on different fuzzy logic versions, which were created by adapting the binary numerical characteristics of the propositional operators to the interval $\langle 0,1 \rangle$ - in detail in Negoita (1988).

Briefly, the fuzzy set is a class of arranged pairs in which the first member is an element of the given universe of consideration, the second is the number from the interval $\langle 0,1 \rangle$, which assigns a degree of membership to a subset of the universe (the fuzzy set support). This membership degree corresponds to the extent to which the element is „compatible“ with the fuzzy set support. More specifically:

Let the set U be a field of consideration or discussion (universe), let $\mu_{\underline{A}}: U \to \langle 0,1 \rangle$ be a function of affiliation and let $\underline{A} = \{(y, \mu_{\underline{A}}(y)): y \in U\}$ be the set of all pairs $(y, \mu_{\underline{A}}(y))$ in which the numbers $0 \leq \mu_{\underline{A}}(y) \leq 1$ assign to the given $y \in U$ the degree of membership of the pair $(y, \mu_{\underline{A}}(y))$ to the set $\underline{A}$. Then $\underline{A}$ is a fuzzy subset on the universe U. An important characteristic of the fuzzy subset $\underline{A}$ is its support $U_{\underline{A}} = \{y: 0 < \mu_{\underline{A}}(y) \leq 1, y \in U\} \subset U$. In terms of fuzzy logic $\mu_{\underline{A}}(y) = |y \in U_{\underline{A}}|$, where $|y \in U_{\underline{A}}|$ denotes the degree of veracity of the statement that y is an element of the support of the fuzzy set $\underline{A}$. At values greater than 0.5, the element y rather belongs to $U_{\underline{A}}$, in the case of smaller ones, it rather does not (Herrera-Viedma, 2015).

We call fuzzy subset $\underline{A}$, whose support $U_{\underline{A}} \subset U \subset R$, where R is the set of real numbers, and its function $\mu_{\underline{A}}$ being gifted by the property of normality and convexity, i.e. at least in the case of one element $x \in U_{\underline{A}}$ it applies $\mu_{\underline{A}}(x) = 1$, and $\mu_{\underline{A}}(x') \geq \min\{\mu_{\underline{A}}(x_1), \mu_{\underline{A}}(x_2)\}$ for all $x' \in \langle x_1, x_2 \rangle \subset U_{\underline{A}}$, the *fuzzy number*. Theoretically, there may be different shapes of membership functions $\mu_{\underline{A}}$ of fuzzy numbers: triangular, trapezoidal, bell, sinusoidal, cosinusoidal (Yuan et al., 2009) etc. Such defined fuzzy numbers can be used for formal representation of uncertain variables.

Though basically two different things, there is a significant analogy between the function $f(x)$, which is the probability density of the random variable x and the function $\mu_{\underline{A}}(x)$, which is the degree of membership of the element x to the support of the uncertain variable (fuzzy number $\underline{A}$). For example, a similar role, which mean or expected value E[x] plays in the case of a random variable x, which is the horizontal coordinate of the centre of gravity of the area under the function $f(x)$ in its definition field, in the case of uncertain variable this role is played by the horizontal coordinate of the centre of gravity under the course of function $\mu_{\underline{A}}(x)$ above the interval defined by support of the fuzzy number $\underline{A}$ (Hašková, 2016).

This analogy can be used to solve a number of problems with variables that defy any statistical description. A trusted point estimate, which can hardly be obtained (if at all), can be replaced by the corresponding coordinate of the position of the centre of gravity of an adequate fuzzy number with a support matched to the set of all possible results. Then, for instance, the intervals **A**, **B** of the fuzzy numbers $\underline{A}$, $\underline{B}$ can be expressed by the triplets of the numbers **A** = (A_L, A, A_R), **B** = (B_L, B, B_R), where the indices L and R denote the left and right edges of the supports, numbers without index are psychologically expectable values for which it is assumed $\mu_{\underline{A}}(A) = \mu_{\underline{B}}(B) = 1$ (psychologically expected values usually lie at the centre of the fuzzy number supports, and in the case of symmetric probability densities they coincide with the statistically expected values).

By means of algebraic operations **(+)**, **(–)**, **(⋅)** and **(/)** of interval calculus (Yuan et al., 2009), which are e. g. **A (+) B** = ($A_L + B_L$, A + B, $A_R + B_R$), **A (–) B** = ($A_L - B_R$, A – B, $A_R - B_L$), k **(⋅) A** = ($k \cdot A_L$, $k \cdot A$, $k \cdot A_R$) and **A (/) B** = (A_L / B_R, A / B, A_R / B_L), the interval **PV** = (PV_L, PV, PV_R) of the support of fuzzy number $\underline{PV}$ in the case of uncertain CF_i and uncertain r_j can be formulated as the triple of relations (2):

$$PV_L = \sum_{i=1}^{n} [\max\{CF_{iL}, 0\} / \prod_{j=1}^{i} (1 + r_{jR}) + \min\{CF_{iL}, 0\} / \prod_{j=1}^{i} (1 + r_{jL})],$$

$$PV = \sum_{i=1}^{n} CF_i / \prod_{j=1}^{i} (1 + r_j), \tag{2}$$

$$PV_R = \sum_{i=1}^{n} [\max\{CF_{iR}, 0\} / \prod_{j=1}^{i} (1 + r_{jL}) + \min\{CF_{iR}, 0\} / \prod_{j=1}^{i} (1 + r_{jR})].$$

3 Application part: the statistically expected current value of oil extraction project versus the range of possible results

The above-mentioned approaches are applied to the problem of profitability assessment of investment in the oil extraction project specified and standardly solved in Hašková (2013) by means of the traditional method of expected net present value (E[NPV]) based on the data concerning:

- the probability occurrences of oil at a depth of 1, 2 and 3 thousand feet,
- costs of exploration well NV_i, where i = 1, 2 , 3 is the mining depth in thousand feet,
- investment costs I_i in mining equipment in millions USD for maximum mining depths up to i = 1, 2, 3 thousand feet,
- the income from the sale of the mining equipment after 10 years of use in millions USD,
- costs of capital of the company,
- net annual income values (NAI(i,j)) in millions USD depending on the depth of mining, richness of resource and demand,
- the payment terms regarding mining rights (TP) for a ten-year mining, initial investment I_i and the costs of exploration well NV_i.

3.1 Probabilistic evaluation based on knowledge of data distribution and expected cash flows

The performed study deals with an evaluation of the economic efficiency of the oil deposit of the mining project, whose oil occurence is determined by the exploration well method. The probabilistic evaluation of possible scenarios of project development is performed in Hašková and Kolář (2011).

The corresponding predictions of net annual income values NAI(i,j), calculated from the given data, are compiled in Tab. 1 (in which, for example, the value of NAI(2,3) = 4,25 indicates the NAI at the mining depth of 2,000 feet in the case of a lot of oil and low demand expressed in millions USD).

Tab. 1: Overview of potential NAI values from oil extraction and sale (in millions USD)

Mining depth (in feet)	**„Poor“ source of oil**		**„Rich“ source of oil**	
	Low demand	High demand	Low demand	High demand
1,000	2,5	3,75	5	7,5
2,000	2,125	3,19	4,25	6,375
3,000	1,5	2,25	3	4,5

From values of Tab. 1 and the given values of costs of the exploration well NVi, provided that the cost of exploitation rights TP = 0, the unbiased E[NPV] forecasts are then calculated according to the following formulas and recorded in Tab. 2:

$E[NPV](i,1) = 0.5 \cdot (NPV(i,1) + NPV(i,2))$,

$E[NPV](i,2) = 0.5 \cdot (NPV(i,3) + NPV(i,4))$ and

$E[NPV](i,3) = NV_i / 1.2$, for i = 1 to 3, where

$NPV(i,j) = (4{,}192 \cdot NAI\ (i,j) – (I_i + NV_i) + 0.2 \cdot I_i / 1.2^{10}) / 1.2$ for j = 1 or 2, i = 1 to 3,

$NPV(i,j) = (4{,}192 \cdot NAI\ (i,j) – (I_i + NV_i)) / 1.2$ for j = 3 or 4, i = 1 to 3.

Tab. 2: Overview of unbiased forecasts E[NPV] of the project (in millions USD)

Mining depth (in feet)	**„Poor“ source of oil**	**„Rich“ source of oil**	**No mining**
1,000	1,185	11,83	– 1,67
2,000	– 2,48	6,48	– 2,08
3,000	– 8,05	– 1,9	– 2,5

Data from Tab. 2 represent sheets of the pruned and evaluated model of decision tree shown in Fig. 1 in its upper part, in which the conditional probabilities attributed to tree branches are derived from the input data; the circles in it show the situation nodes, and the squares represent the option to terminate the project. It represents a model of a statistical solution to the problem based on unbiased forecasts of point estimates of random variables. Uncertain demand also considered as a random variable is described with an even distribution of probability density.

3.2 Evaluation of the project within fuzzy approach

The basic method for dealing with uncertain variables in the fuzzy approach is the change of point values for adequate interval values. In this case, we replace the point inputs of the decision model as recorded in Tab. 2. As in the previous case, we calculate for each of the twelve values of NAI(i,j) recorded in Tab. 1 the corresponding NPV(i,j) values, which are then arranged in the pairs recorded in Tab. 3.

Tab. 3: Symbolic record of intervals of the type ($NPV(i,j)_L$, E[NPV](i,j), $NPV(i,j)_R$)

Mining depth (in feet)	„Poor“ source of oil	„Rich“ source of oil
1,000	(NPV(1,1), NPV(1,2))	(NPV(1,3), NPV(1,4))
2,000	(NPV(2,1), NPV(2,2))	(NPV(2,3), NPV(2,4))
3,000	(NPV(3,1), NPV(3,2))	(NPV(3,3), NPV(3,4))

In general, this is a record of the interval limits of type ($NPV(i,j)_L$, E[NPV](i,j), in which the value E[NPV](i,j) is tabulated in Tab. 2 and hence known. Unlike the previous case, in which the formulas for calculating NPV(i,j) values are derived from the middle formula of relation (2), this time they are derived from the remaining formulas, and therefore:

$NPV(i,1) = (\alpha(r_R,10) \cdot NAI(i,1) – (I_i + NV_i) + 0.2 \cdot I_i / (1 + r_R)^{10}) / (1 + r_L)$, i = 1 to 3,

$NPV(i,2) = (\alpha(r_L,10) \cdot NAI(i,2) – (I_i + NV_i) + 0.2 \cdot I_i / (1 + r_L)^{10}) / (1 + r_L)$, i = 1, 2,

$NPV(3,2) = (\alpha(r_L,10) \cdot NAI(3,2) – (I_3 + NV_3) + 0.2 \cdot I_3 / (1 + r_L)^{10}) / (1 + r_R)$,

$NPV(i,3) = (\alpha(r_R,10) \cdot NAI(i,3) – (I_i + NV_i)) / (1 + r_R)$, i = 1, 2,

$NPV(3,3) = (\alpha(r_R,10) \cdot NAI(3,3) – (I_i + NV_i)) / (1 + r_L)$,

$NPV(i,4) = (\alpha(r_L,10) \cdot NAI(i,4) – (I_i + NV_i)) / (1 + r_L)$, i = 1 to 3.

The model from which the formulas are derived assumes that the ten-year geometric average of the discount rate is an uncertain variable with the interval values $\mathbf{r} = (r_L, r, r_R)$. If this is not the case, then $r_L = r_R = r$. In our example we work with the interval $\mathbf{r}$ = (0.15, 0.2, 0.25). α(r,10) is the value of the 10-year annuity factor at the average annual discount rate r.

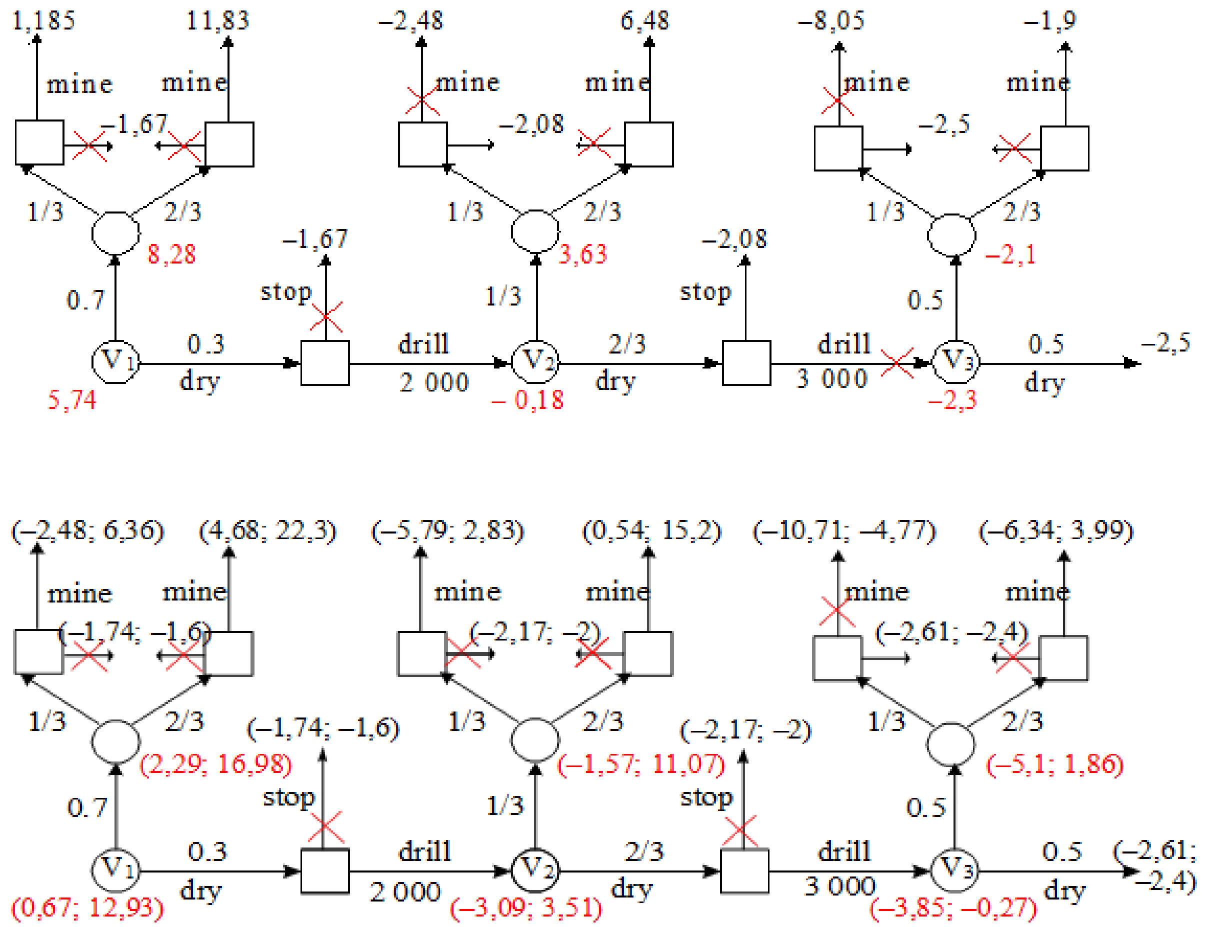
Extending Tab. 3 for a column with limit intervals of type $((–NV_i / (1 + r_L), –NV_i / (1 + r), –NV_i / (1 + r_R))$, i = 1 to 3 we get Tab. 4, which is an analogy to Tab. 2, transforming its point values into adequate intervals. Interval limits from Tab. 4 are relevant sheets of the bottom decision tree in Fig. 1.

Tab. 4: Analogy of Tab. 2 for input interval values in the fuzzy model in millions USD

Mining depth (in feet)	„Poor“ source of oil	„Rich“ source of oil	No mining

1,000	(–2,48; 6,36)	(4,68; 22,3)	(–1,74; –1,6)
2,000	(–5,79; 1,83)	(0,54; 15,2)	(–2,17; –2)
3,000	(–10,71; –4,77)	(–6,34; 3,99)	(–2,61; –0,27)

Fig. 1: Model of the decision tree: comparison of two approaches to solving the same problem - the standard approach (top) and fuzzy approach (bottom)



Source: own elaboration

From Fig. 1 it is clear that the fuzzy approach represented by the bottom tree, in which the new intervals are calculated by using the appropriate interval calculus tools mentioned at the end of section 2 and the basis for the option choice is the result of a comparison of the mean values of the intervals, extends the standard result (upper tree) by a number of useful information.

In particular, the statistical expected values with which the upper tree operates do not significantly differ from the psychological expected values (i.e. from the middle of the intervals) in the lower tree. The bottom tree node evaluation is formulated within the limit values that are taken into account by investors with a negative relationship to risk. For instance, it is not clear from the upper tree that mining at the depth of 1,000 feet can be unprofitable and vice versa (with a double probability) it is possible to gain a good profit, etc.

Conclusion

The paper deals with the issue of managerial decision-making when uncertain variables come into play; uncertainty is being understood as the absence of any objective statistical or probabilistic description. Statistics often tends to approximate uncertain variables with a random variable by means of a uniformly or symmetrically distributed probability density and then works with point estimates. The fuzzy approach derives from the idea of an adequate fuzzy number of the result and leans on the more easily identifiable interval limits of its support.

The statistical expected result value is considered the horizontal coordinate of the position of the centre of gravity area under the probability density curve above the range of possible results. The psychological expected result value is meant the horizontal coordinate of the position of the centre of gravity of the area under the curve of the membership function of the resulting fuzzy number over its support.

The functional analogy between the statistical and psychological expected value enables us to judge the problem from a number of perspectives, which therefore offers better (more qualified) decision. Possible benefits of fuzzy approach in this respect are illustrated by means of results of the managerial assessment of the adequacy of price of mining rights for oil production.

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Contact

Simona Hašková
Vysoká škola technická a ekonomická v Českých Budějovicích
Okružní 517/10, 370 01 České Budějovice
haskovas@post.cz