

# OPTIMIZATION OF QUALITY COSTS FOR INDUSTRIAL ENTERPRISE: MATHEMATICAL AND STATISTICAL APPROACH

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## Abstract

One of the tools of the industrial enterprise performance management in the sphere of quality management is analyses of the quality costs data. A striving to increase the efficiency of the operating activity by reducing losses from nonconforming products leads to an increase in costs for prevention of nonconformities. Thus, the important management problem is the determination of the balance between benefit by reducing losses from nonconforming products and the losses due to the increase in costs of the product conformity. The article presents the results of the analysis of the dynamics of costs of product quality management of the industrial enterprise. The authors used the correlation-regression analysis to build mathematical models of share of general quality costs in manufacturing costs and also mathematical models of shares of costs of conformity and nonconformity in general quality costs. The model parameters were estimated on the basis of the quarterly dynamics of quality costs for the years 2011-2016. The results of the research allow to estimate the enterprise budget for quality and share of costs of conformity and nonconformity and also to calculate the optimum level of the percentage of defects in product costs when the minimum of quality costs is reached.

**Keywords:** quality management, quality costs, mathematical modeling

**JEL Code:** D24, C55

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## Introduction

The main trend in the enterprise development is raising operating performance including reduction of operating costs affecting the product costs. Nonconformities and defects are one of the main types of the losses affecting the product costs. This type of losses is of no value to the client and affects directly the key competitive position on the contemporary market, such as speed and reliability of the fulfillment of orders. Due to the intensified competition, including price competition, the reduction in the amount of nonconformities and defects affects directly product costs, product quality, speed and reliability of the fulfillment of orders, which are the

important factors in the development of the enterprise economics and preservation the position of the enterprise on the market.

Management of most Russian enterprises focuses on the sales growth, when introducing and certifying the quality management systems. The problems of the economic resources costs stay in the background that leads to a gradual reduction of the effectiveness of the quality management system and to an increase of risk of a loss of the competitive capacity of the enterprise. International standards of ISO 9004:2000 involve the use of three approaches: the process cost analysis, the cost analysis of the product life cycle, the quality cost analysis. It is possible to integrate the approaches to ensure greater enterprise efficiency. At the same time the cost analysis of the product life cycle comprises almost all types of quality costs of specific products, which makes it more attractive, especially when analyzing the percentage ratio of costs and total costs for quality. (Dedusheva, 2006; Dzul, 2008).

Increasing the efficiency of the operating activity by reducing losses from nonconforming products leads to an increase in costs of conformities, i.e. costs of preventive measures and control. In this case, one should take into account that the efficiency of costs of prevention of nonconformities decreases as they increase. (Sansalvador, 2013). Thus, the most important management problem is the determination of the balance between the benefit by reducing losses from nonconforming products and the losses due to the increase in costs of the product conformity. The aim of the analysis is to build a mathematical model for estimation of an optimal enterprise budget for quality assurance and the cost structure for conformity and nonconformity.

The authors solved the following problems to achieve this aim:

- The system of indicators was developed and the database was created;
- The model was built for estimating the share of quality costs in product costs for determining the enterprise budget for quality assurance;
- The model was built for estimation of the cost structure for conformity and nonconformity;
- The authors estimated the optimal enterprise budget for quality assurance and the cost structure for conformity and nonconformity.

The subject of research is the costs for quality control of PJSC "Montazhkonstrukciya".

## **1. The study of the current situation**

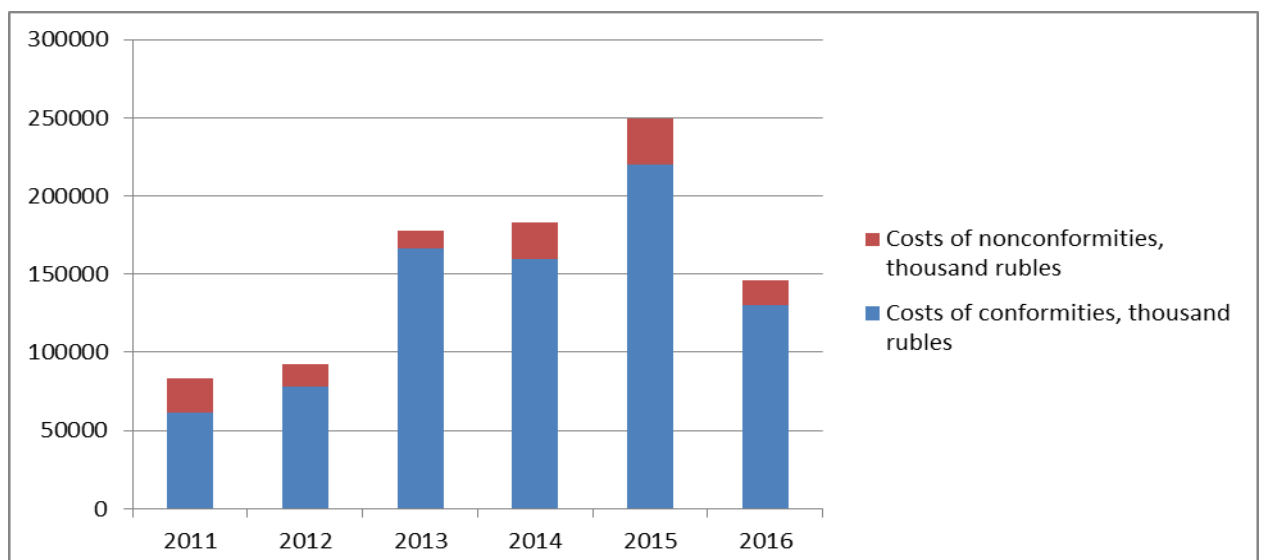
At the present time experts in quality management suggest the following grouping of quality costs, based on Juran-Feigenbaum's approach (Feigenbaum, 1984):

- I. Costs to prevent the possibility of defects, i.e. the costs related to an activity which reduces or completely prevent the risk of defects or losses (costs of preventive measures or preventive costs)
- II. Costs for control, i.e. the costs of determination and confirmation of the attained level of quality (current costs)
- III. Internal costs for defects. The costs incurred within the enterprise, when the specified quality level is not reached, i.e. before the product was sold (the internal losses).
- IV. External costs for defects. The costs incurred outside the enterprise, when the specified quality level is not reached, i.e. after the product was sold (the external losses).

However, due to the peculiarities of the accounting of quality costs at the analyzed enterprise, the costs are grouped in accordance with the classification based on P. Crosby's approach. (Crosby, 1979; Ryzhenko, 2004). P. Crosby added the costs that A. Feigenbaum placed to the groups I and II, into the first category. The costs that A. Feigenbaum placed to the groups III and IV, were included into the second category.

Analyzing the annual dynamics of the amount of quality costs of the enterprise under research for the years 2011-2016, one can note its growth up to 2015 and a significant decline in 2016 (pic.1). The increase in quality costs from 2011 to 2015 is associated with acquisition of new equipment, carrying out experimental works on introduction of new technologies, new tools, and quality control schemes. Implementation of new technologies and automated quality control systems significantly reduced quality costs in 2016. Also, fluctuations in the dynamics of costs are associated with changes in volume of the output.

**Fig. 1: The amounts of quality costs and their structure at the enterprise for the years 2011-2016**



Source: author's own work

In general, during the specified period the rate of growth of costs at the enterprise was 64%, the average annual rate of growth was 10%. The increase of total costs was mainly due to the increase of costs for preventive measures, appraisal costs and costs for output quality control. This should be noted as a positive trend. Quality cost management also implies that their structure is optimal (Sheshukova, 2012; Shah, 1998; Šatanová, 2015). The experience of foreign industrial enterprises in the sphere of cost management shows the following optimal ratio (Seregin, 2009): 10% - costs of defects elimination (external and internal), 90% - costs of testing and control (estimation) and costs of preventive measures. Costs of nonconformities are on average by 10, 6% in the actual quality costs structure of the enterprise under research. This fact indicates the proximity to the structure of western enterprises.

According to the results of the analysis of quality costs one can deduce positive trends in the dynamics of costs and their structure. In order to reveal the relationship between the groups within the costs structure for estimation of the optimal enterprise budget for quality assurance, the authors used methods of the correlation-regression analysis and the mathematical analysis.

## 2. Research technique

The conceptual framework of the analysis of quality costs is formed by the model which is traditionally used to determine the optimum level of zero-defects when the minimum of quality costs is reached. (Hwang, 1996). It can be represented by the dependence:

$$QRC(x) = CNC(x) + CC(x), \quad (1)$$

where  $QRC(x)$  - is the quality related costs function,  $CNC(x)$  - is the function of costs of nonconformities,  $CC(x)$  - is the function of costs of conformities,  $x$  - is the defect rate of the output product.

Because the dynamics of cost indicators is analyzed for a sufficiently long period of time, it was made a transition from absolute cost indicators to relative indicators in order to overcome the effect of inflation. Therefore, the system of indicators for mathematical modeling includes:

- the share of quality costs in product costs ( $Y$ )
- the share of costs of conformity in quality costs ( $Y1$ )
- the share of costs of nonconformity in quality costs ( $Y2$ )
- the percentage of defects in product costs ( $x$ )

For practical use of the model, it is necessary to estimate the dependence  $Y(x), Y_1(x)$ , with the use of the correlation-regression analysis methods. The function  $Y_2(x)$  is determined from the condition (2)

$$Y_1(x) + Y_2(x) = 1. \quad (2)$$

The type of the function for the dependence description  $Y(x), Y_1(x)$  is taken from elementary functions (polynomial, logarithmical) on the basis of quality criteria of approximation – the coefficient of determination  $R^2$ . The coefficient of determination  $R^2$ , shows which part of the dispersion of the original feature space is explained by the explicative variables that entered the model. The model parameters (constant coefficients) are determined by the least squares method (LS method). As it is assumed that the linear models are built according to the estimated parameters, we use classic hypothesis tests for testing hypothesis of the importance of the equation and parameters. The importance of the equations is verified by using Fisher's F-test. The importance of the parameters is verified by using Student's t-test

Next, on the basis of the constructed function of the share of quality costs in product costs  $Y(x)$ , we define the optimal percentage of defects in product costs  $x_{min}$ , provided that the minimum share of quality costs  $Y_{min}$  will be attained. For this purpose we employ methods of mathematical analysis of determining of extremum of function. We solve equation (3)

$$\frac{dY}{dx} = 0 \Rightarrow x_{min}. \quad (3)$$

Then we find

$$Y_{min} = Y(x_{min}). \quad (4)$$

Based on the values found  $x_{min}, Y_{min}$ , with setting the planned value of product cost of the future period ( $PC$ ), we can estimate rate of quality related costs of the enterprise ( $QRC$ ), and also rate of costs for conformity ( $CC$ ) and nonconformity ( $CNC$ ):

$$QRC = PC \cdot Y(x_{min}) = PC \cdot Y_{min}; \quad (5)$$

$$CC = PC \cdot Y_1(x_{min}); \quad (6)$$

$$CNC = PC \cdot Y_2(x_{min}) = PC \cdot (1 - Y_1(x_{min})). \quad (7)$$

### 3. Calculations of the optimal quality costs and their structure

In order to attain the objectives, the authors chose the following operating rates of PJSC "Montazhkonstrukciya": costs of conformity (costs of preventive measures and control), thousand rubles; costs of nonconformity (internal cost for defects, external costs for defects), thousand rubles; quality costs (costs of conformity, costs of nonconformity), thousand rubles; product cost, thousand rubles. In terms of indicators the authors formed the quarterly dynamics for the years 2011-2016 and calculated the indicators values ( $Y, Y1, Y2, x$ ).

The authors used the correlation-regression analysis and correlation (2) to build dependences  $Y(x), Y1(x), Y2(x)$ . The calculations were carried out with the use of MS Excel. According to the calculations the authors chose regression models of with the largest level of the coefficient of determination  $R^2$  (tab. 1).

**Tab.1: Regression models, evaluation of their importance and quality**

Regression equation (corresponding values $t_{obs.}$ are given within the brackets under the coefficients)	Observed value of Fisher's F-test ( $F_{obs.}$ )	Coefficient of determination $R^2$
$\hat{Y} = 1\,270,69x^2 - 5,58x + 0,04$ $t_{obs.} \quad (6,35) \quad (2,65) \quad (4,25)$ $t_{tabl.}(\alpha = 0,05; \gamma = 21) = 2,08$	18,31	0,56
$\hat{Y1} = -0,05 \cdot \ln(x) + 0,61$ $t_{obs.} \quad (-5,03) \quad (10,64)$ $t_{tabl.}(\alpha = 0,05; \gamma = 22) = 2,07$	25,38	0,72
$\hat{Y2} = 0,05 \cdot \ln(x) + 0,39$ $t_{obs.} \quad (-5,03) \quad (10,64)$ $t_{tabl.}(\alpha = 0,05; \gamma = 22) = 2,07$	25,38	0,72

Source: author's own work

Estimation of the importance of the coefficients was carried out with the help of Student's t-test. The observed values of the t- test for the coefficients  $t_{obs.}$  are given within the brackets under the equation. For all values  $|t_{obs.}| > t_{tabl.}$ , therefore the coefficients of the equations are significantly different from zero. According to Fisher's F-test we can arrive at conclusion about importance of the equation, because  $F_{obs.} > F_{tabl.} = 4,3$  when the significance level is 0,05. Coefficient of determination  $R^2$  of the equations shows a moderate relationship between endogenous and exogenous variables.

On the basis of the dependence  $Y(x)$  we determine the optimal percentage of defects in product costs  $x_{min}$ , provided that the minimum share of quality costs in product costs will be attained. The local minimum point is found from the conditions (3), (4):

$$\frac{dY}{dx} = 0 \Rightarrow (1\,270,69x^2 - 5,58x + 0,04)' = 0 \Rightarrow$$

$$2541,37x - 5,58 = 0 \Rightarrow x_{min} = 0,0022 \text{ (0,22\%)}.$$

In such a case, the share of the budget for quality in product costs is:

$$Y_{min} = 1\,270,69 \cdot 0,0022^2 - 5,58 \cdot 0,0022 + 0,04 = 0,032 \text{ (3,2\%)}.$$

Then, on the basis of the equations described in the table 1 we define shares of costs of conformity and nonconformity for the optimal level of the percentage of defects  $x_{min}$ :

$$\widehat{Y1} = -0,05 \cdot \ln(0,0022) + 0,61 = 0,92 \text{ (92\%)},$$

$$\widehat{Y2} = 1 - \widehat{Y1} = 0,08 \text{ (8\%)}.$$

Therefore, in current conditions at PJSC "Montazhkonstrukciya" the optimal share of the budget for quality is 3,2%, when the percentage of defects in product costs is 0,22%. Whereby 92% of the budget for quality should be accounted for costs of conformity and only 8% should be accounted for costs of nonconformity. The latest value is slightly below the average for foreign industrial enterprises estimated by western experts.

## Conclusion

The research showed that the dynamics of quality costs for the years 2007-2011 is volatile for the analyzed enterprise. At the same time the quality cost structure at the enterprise is close to theoretically optimal. The use of statistical and mathematical modeling methods allowed to get models that enable to determine the level of quality costs ensuring the balance between the benefit by reducing losses from nonconforming products and the losses due to the increase in costs of the product conformity. This is a relevant management problem. The authors specified the optimal cost structure for the enterprise in accordance with management, manufacturing and technical features of the enterprise. The proposed economic and mathematical models will allow to reduce total quality costs and to improve the cost structure in the sphere of quality assurance of the output product, when the proposed measures are implemented. The data obtained through the research produce results which will be used as a guideline to develop a methodology for the total costs analysis of the process. This methodology will help managers to increase efforts to improve the quality of the business processes, products and services.

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