STRESS TESTING OF INTEREST-RATE-DEPENDENT POSITIONS USING PORTFOLIO-SPECIFIC SCENARIOS

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Abstract

This article presents a proposal of an approach to stress testing of interest-rate-dependent portfolios. Stress-test scenarios are suited here to the structure of the portfolio. In this way, severity of a scenario may be increased as compared to standard methods. Scenarios are constructed on the basis of the information about portfolio sensitivity. When constructing a scenario, the direction of changes to selected nodes (vertices) of the interest-rate term structure is determined by the signs of partial sensitivity measures with respect to these nodes. Positive shifts are imposed where the sensitivity is negative and negative ones – where the sensitivity is positive. The shifts themselves are not more extreme than the ones that are used in the industry-standard stress scenarios. The approach is analyzed by an example of a portfolio of positions in IRS contracts. Then, profits or losses obtained under the proposed scenarios are compared with the results of 9 standard stress tests. For the test portfolio, it the proposed scenarios are severe indeed, but two standard ones (parallel downward shift of the whole term structure and a combination of historical minima) came out to be even more severe in this particular case.

Key words: stress testing, stress scenario, yield curve, PV01

JEL Code: G20, G21

Introduction

For stress testing of positions that are exposed to term structure of interest rates, it is an industry standard to use scenarios based on some basic types of yield curve changes (like parallel shift, change of slope, humps). This is a commonly adopted approach in practice of financial institutions, clearing houses and supervisory authorities. It may be based, for example, on historical scenarios. The historical changes of interest rates may be taken straightforwardly (e.g., Jorion, 2009) or used in a historical simulation (e.g., Almeida, Duarte Júnior, & Fernandes, 2004; Loretan, 1997). To that, big market disruptions may be included to the historical stress tests, even if they happened in the periods that are not spanned by the sample of a standard length (Murphy & Macdonald, 2016). These may be also hypothetical scenarios, originating from deterministic or stochastic yield curve models (Abdymomunov & Gerlach, 2014). They can be also obtained from an econometric or structural model of the economy, in which a set of different factors influence interest rates and the stress scenarios are based on extreme but plausible values of these factors. An approach of this kind, albeit for stock prices, not for interest rates, was discussed, for instance, by Golub, Greenberg, and Ratcliffe (2018). And finally, some arbitrary hypothetical disturbances to the yield curve may be introduced by experts.

A broader discussion on stress-test scenario formulation, including both more risk factors within the area of market risk, as well as more types of risk, may address also such problems as market-liquidity issues from a portfolio manager's perspective, liquidity issues from the systemic-risk perspective, clearing-system malfunctions resulting from too large concentrated positions (a special case of liquidity-risk problem), materialization of credit risk, etc. A framework of stress testing that takes into account the potential impact of both interest rate changes and credit risk materialization is discussed, by a stylized bank example, by Drehmann, Sorensen, and Stringa (2010). The question of large concentrated position is taken up, amongst others, by Budding, Cox, and Murphy (2016). Stress testing of liquidity from a system-wide perspective is comprehensively discussed in the BIS report of 2013, containing a survey of both theoretical and practical aspects of this topic (BIS, 2013). Jobst, Ong, and Schmieder (2017) present also some more current findings and observation on liquidity stress-testing. This area of research is newer and less thoroughly investigated than the field of market and credit risk, but it already has a broad literature (see also, e.g.: Geršl, Komárková, & Komárek, 2016; Pagratis, Topaloglou, & Tsionas, 2017; Wong & Hui, 2009). Market liquidity is also investigated from a portfolio-manager perspective (Banks, 2014), where the focus is on potential problems with closing-out of positions in thin-traded assets. In turn, the type of risk that is referred to in our article is just market risk (to be more precise – interest rate risk). And only the influence of some extreme interest-rate changes on the analyzed portfolio is taken into consideration here.

Let us also point out that this article does not concentrate on the source of extreme scenarios, but rather on determining such combination of distortions to the analyzed yield curve vertices (nodes) which makes the resulting scenario especially severe to a given portfolio. The approaches mentioned above, albeit well-grounded in theoretical sense and justified by their empirically-proven practical utility, do not incorporate the information about the structure of the analyzed portfolios. A portfolio may be in fact more sensitive to some other scenarios than the "standard" ones. Our scenarios are designed to be especially severe to a given portfolio.

The proposed procedure is based on sensitivity analysis. A vector of partial PV01 measures for the analyzed yield curve vertices is first determined. The information contained in it is then used twice. First time, only signs of PV01 values are considered. For each vertex, the sign indicates which direction of the vertex change exerts an adverse influence on the portfolio. Then, an extreme change of the vertex is taken. Dependent on the method of stress scenario generation, it may be a historical extremum or an extreme value obtained from a model or it may be pointed by an expert. Here, for the purpose of concept presentation, two alternative approaches are adopted. In the first one, historical extrema are used. The historical sample spans 10 years. An extreme historical change of a given vertex is determined. This is the maximum of absolute values of all changes to this vertex, so the sign of the change is for now disregarded. Dependent on the sign of the sensitivity measure obtained in the previous step, the extreme change is taken with a positive (for a negative PV01 value) or negative sign (for a positive PV01). The second approach consists in using the same value of change as the one used in the standard parallel-shift stress scenario. As compared to the standard parallel shift, the difference is that the distortion is applied with the sign "+" to the vertices of a negative PV01 and with a "–" to the vertices of positive PV01.

On this stage, the vector of stress-test changes is obtained. Then, it may be used for stress-testing of a given portfolio or to compare the impact of the stress scenario on this portfolio vs. some variant of its protection, that is – the same portfolio plus some hedging transaction(s).

To quickly assess the influence of the analyzed scenario on portfolios, interest rate changes from the stress-test scenario are multiplied by corresponding sensitivities. This is the second time when the information about sensitivity may be used. It allows to check if hedging takes the desired effect in a simple way. The stress scenario is suited to the structure of the position to be hedged and, thus, it reflects threats specific to this particular portfolio.

In this article, the aforementioned two types of stress scenarios are compared with some "standard" ones that are widely used by the financial industry. An arbitrary chosen portfolio of position in IRS contracts is taken as an example (the test portfolio). Assuming several stress-test scenarios, losses for the portfolio are calculated for a given day. This is to check, if the scenarios proposed in this article are indeed severe as compared with the typical ones.

1 The portfolios and PV01 estimates

The portfolio taken as an examples of interests-rate-sensitive position is given in **Tab. *1***. Let us assume that the day of analysis (current date) is 16 Nov. 2017.

Tab. 1: Analyzed portfolios

Transaction type	Currency	Index	Date when transaction was concluded	Tenor	Party	Par value
IRS	PLN	WIBOR 6M	2015-11-16	3Y	Buy	100 000 000
IRS	PLN	WIBOR 6M	2015-11-16	4Y	Buy	100 000 000
IRS	PLN	WIBOR 6M	2015-11-16	6Y	Buy	50 000 000
IRS	PLN	WIBOR 6M	2015-11-16	8Y	Buy	50 000 000
IRS	PLN	WIBOR 6M	2015-11-16	10Y	Buy	10 000 000
IRS	PLN	WIBOR 6M	2015-11-16	15Y	Buy	10 000 000
IRS	PLN	WIBOR 6M	2015-11-16	20Y	Buy	10 000 000

The day when the contracts were concluded was 16th Nov 2015, that is – 2 years before the date of analysis.

Tab. *3* and **Tab. *4*** in the Appendix present nodes of the term structure, spot and 6M forward respectively, that are taken into consideration. In the last columns of these tables, also values of partial PV01 measure of the analyzed portfolio are given.

2 Stress-test scenario

The analyzed portfolio may be presented as a stream of cash flows whose present value depends on two yield curves, that is – on the 6M-forwad-rate term structure and on the discount curve.

First, signs of sensitivities to the considered vertices of these two curves are recorded.

As it has already been mentioned, two approaches are adopted then:

Approach 1: The extreme changes of yield curve vertices are defined just as historical extrema for each vertex. From a 10-year sample (between 15 Nov. 2008 and 15 Nov. 2017), absolute values of minima and maxima of daily changes in the rates are taken (minima and maxima of changes, not of the rates themselves). Then, they are given the signs that are opposite to the signs of corresponding PV01 values.

Approach 2: The extreme changes are defined as that of the parallel-shift stress-test scenario, but with signs adjusted to the signs of the corresponding partial PV01 measures (a "–" for a positive PV01 and a "+" for a negative one).

The assumed holding period is 5 days, therefore all daily changes are rescaled to a 5-day scale (a 5-day holding period is a standard assumption for the OTC market).

The extreme changes are added to current quotations of 16th Nov. 2017. The vertices that correspond to positive sensitivity are decreased and the vertices to which the portfolio is negatively sensitive are increased.

The changes that are used to formulate the stress-test scenarios of the Approach 1, for spot and forward yield curve, are given in the last columns of **Tab. *5*** and **Tab. *6*** respectively. The values provided there are already rescaled to a 5-day holding period.

For the Approach 2, the absolute value of all changes is 0,0126 (126,34 b.p.) in the 5-day scale. It is obtained from rescaling of a 56,50 b.p. daily change to the standard holding period. This change was obtained as a one-day shift of the PLN-6M-SWAP-OTC-3YR swap rate quotation on 21st Ostober 2008.

3 How to use the obtained scenario

The scenario may be used to assess a potential extreme loss to the portfolio value and then to compare the loss with the level of margins and reserve funds of an institution.

The stress-test scenario may be put directly to the pricing function or an approximated assessment of loss may be performed. In the second case, partial PV01 measures may be used.

Let the vector with extreme directional changes of spot rates be denoted with the symbol $\Delta\boldsymbol{R}$. This vector is contained by the last column of **Tab. *5*** (to be more precise, this consists of columns 4 and 8, where the column 4 is continued as a column 8 in the layout that is used there). Let the vector of extreme directional changes for forward rate be called a vector $\Delta\boldsymbol{W}$. These are the data from the last column of **Tab. *6*** (actually, columns 4 and 8). Let us, to that, denote the last column of **Tab. *3*** (the one with the values of the PV01 measure in respect of spot rates) with the symbol $\boldsymbol{D_R}$ and the last column of **Tab. *4*** with the symbol $\boldsymbol{D_W}$. Then, the linearly approximated potential loss on the portfolio is:

$$\Delta P \approx \boldsymbol{D_R}^T \cdot \Delta\boldsymbol{R} + \boldsymbol{D_W}^{\mathrm{T}} \cdot \Delta\boldsymbol{W}, \qquad (1)$$

where: $\boldsymbol{D_R}^T$ and $\boldsymbol{D_W}^{\mathrm{T}}$ are transposed vectors of PV01 (rows now), $\Delta\boldsymbol{R}$ and $\Delta\boldsymbol{W}$ are extreme changes of the respective vertices of the corresponding yield curves (columns), "·" is matrix multiplication operator and ΔP is change of value of the analyzed portfolio (scalar).

4 Portfolio performance under stress-test scenarios

The stress test scenarios that are often used in the industry are usually of the following types:

1) Parallel upward shift of the whole yield curve (based on maximum historical change from a reference vertex during the last 10 years);

2) Parallel downward shift of the whole curve (also by the absolute value of the biggest historical change for the reference vertex);
3) Increase of slope (let us say that the shortest short-term rate decreases by ½ of the absolute value of shift from the points 1) and 2), the “right-most” long-term rate increases by the same amount, and the modifications of the vertices in the middle are linearly interpolated between the decrease of the “left-most” short-term one and the “right-most” long-term one);
4) Decrease of slope (the “left-most” short-term rate increases and the “right-most” long-term rate increases by the same absolute amount as in the point 3);
5) Increase of concavity (middle-term rates increase and short-term and long-term ones decrease);
6) Decrease of concavity or increase of convexity (middle-term rates decrease and short-term and long-term ones increase);
7) Historical maxima: for each node, its historical maximum is taken;
8) Historical minima: for each node, its historical minimum is taken.
9) Straightforward historical scenario: actual 5-day changes of the analyzed vertices, calculated on the day on which the maximum change of 3Y 6M swap rate was observed.

Let us compare the profits and losses that the analyzed portfolio would generate under these 9 scenarios, listed with the labels from 1 to 9 in **Tab. *2***, and under the scenarios proposed in this article, listed with labels 10 and 11 (10 for the Approach 1 and 11 for the Approach 2).

Tab. 2: Losses under stress-test scenarios

Stress-test Scenario	Profit (+) of loss (−)
1	13 138 871,74
2	-14 152 834,18
3	-1 845 694,48
4	1 806 787,09
5	337 671,64
6	-439 902,55

Stress-test Scenario	Profit (+) of loss (−)
7	12 680 094,56
8	-13 797 677,21
9	-11 400 570,29
10	-12 181 928,19
11	-12 262 494,10

Note: Scenario #10 is the scenario constructed using the Approach 1 and scenario #11 is obtained from the Approach 2.

As it may be read from **Tab. *2***, the amount of loss that would be incurred on the portfolio is the highest under the scenario 2 (parallel downwards shift). The second biggest loss would be incurred under the scenario 8 (historical minima). The third biggest loss would be incurred under the scenario formulated using the Approach 2 (scenario #11) proposed in this article. The fourth biggest loss is obtained from the Approach 1 (scenario #10). Because of long positions in IRS contracts prevailing in the analyzed portfolio, increases of interest rates are to the advantage of the position holder. Such scenarios are obviously not adverse and hence it is hard to think of them as of stress-test scenarios. Excluding such obviously non-adverse scenarios, the adverse scenarios proposed here (Approaches 1 and 2) are, respectively, the third and fourth most stressful out of 7. This may be a premise to postulate that our scenario-generation method produces good stress test scenarios, but, for the analyzed example, the some of the existing approaches give even more severe scenarios. Thus, it cannot be stated that scenarios obtained by adjusting directions of interest-rate changes to the term structure of sensitivities in the analyzed portfolio must give more adverse scenarios than the comparable standard ones.

Summary

The proposed approaches to stress-test scenario construction works in this sense that it allows to obtain really adverse scenarios even if individual changes of yield-curve nodes used in the procedure are not bigger than those of the “standard” stress-test scenarios. The concept underlying the proposed approaches is adjusting the term structure of interest-rate changes in the stress-test scenario to the information about partial sensitivity measures of the analyzed portfolio. Therefore, for portfolios with a more or less the same number of positive and negative partial sensitivities, especially if both positive and negative ones are evenly scattered throughout the whole term structure of interest rates, our stress-test procedure might be even more effective than for the analyzed example. At the same time, it must be admitted that this type of adjustment does not guarantee a more severe stress scenario than the standard ones based, for example, on the parallel shift or actual historical extrema.

Further analysis should definitely take more portfolios of different compositions into consideration. To that, it would be highly recommended to introduce a benchmark to which the resulting loss would be referred. For instance, the benchmark might be based on margins, deposits and the level of financial means of relevant safety funds. Detailed construction of the benchmark should be adjusted to the type of institution.

Appendix

Tab. 3: Spot curve vertices and respective partial PV01s for the analyzed portfolio

Vertex #	Maturity	Reference index or transaction	Year fraction	Rate (as of 2017-11-16)	Partial PV01 (portf. of Tab. 1)
1	6M	PLN_OIS_OIS_OTC_6M	0,5	1,48%	0,71
2	1Y	PLN_OIS_OIS_OTC_1YR	1	1,54%	-22,32
3	2Y	PLN_1M_SWAP_OTC_2YR	2	1,91%	-93,27
4	3Y	PLN_1M_SWAP_OTC_3YR	3	2,09%	-39,27
5	4Y	PLN_3M_SWAP_OTC_4YR	4	2,35%	-86,01
6	5Y	PLN_3M_SWAP_OTC_5YR	5	2,51%	-86,13
7	6Y	PLN_3M_SWAP_OTC_6YR	6	2,64%	-108,23
8	7Y	PLN_3M_SWAP_OTC_7YR	7	2,74%	-44,79
9	8Y	PLN_3M_SWAP_OTC_8YR	8	2,83%	-54,75
10	9Y	PLN_3M_SWAP_OTC_9YR	9	2,92%	-45,27
11	10Y	PLN_3M_SWAP_OTC_10YR	10	2,99%	10,99
12	12Y	PLN_3M_SWAP_OTC_12YR	12	3,14%	-22,03
13	15Y	PLN_3M_SWAP_OTC_15YR	15	3,31%	-37,62
14	20Y	PLN_3M_SWAP_OTC_20YR	20	3,35%	-56,95

Tab. 4: Forward curve vertices and respective partial PV01s for the analyzed portfolio

Vertex #	Name	Reference index or transaction	Rate (as of 2017-11-16)	Partial PV01 (portf. of Tab. 1)
1	6M spot	PLN_WIBOR_6M	1,81%	16378,59
2	6M in 6M	PLN_6M_FRA_OTC_0612	1,89%	16250,57
3	6M in 12M	PLN_6M_FRA_OTC_1218	2,07%	11210,25
4	6M in 18M	PLN_6M_FRA_OTC_1824	2,27%	11076,57
5	6M in 3Yrs	PLN_6M_SWAP_OTC_3YR	2,21%	6117,34
6	6M in 4Yrs	PLN_6M_SWAP_OTC_4YR	2,38%	5941,50
7	6M in 5Yrs	PLN_6M_SWAP_OTC_5YR	2,53%	3553,98
8	6M in 6Yrs	PLN_6M_SWAP_OTC_6YR	2,66%	3453,93
9	6M in 7Yrs	PLN_6M_SWAP_OTC_7YR	2,76%	1258,97
10	6M in 8Yrs	PLN_6M_SWAP_OTC_8YR	2,85%	1223,09
11	6M in 9Yrs	PLN_6M_SWAP_OTC_9YR	2,93%	792,17
12	6M in 10Yrs	PLN_6M_SWAP_OTC_10YR	3,00%	-346,75
13	6M in 12Yrs	PLN_6M_SWAP_OTC_12YR	3,14%	1843,51
14	6M in 15Yrs	PLN_6M_SWAP_OTC_15YR	3,31%	-334,37
15	6M in 20Yrs	PLN_6M_SWAP_OTC_20YR	3,37%	668,65

Tab. 5: Extreme historical changes for discount curve vertices

Vertex #	Maturity	Sign of PV01	Adjusted directional 1-day extreme change
1	6M	−	0,01990
2	1Y	+	-0,02191
3	2Y	+	-0,01518
4	3Y	+	-0,01641
5	4Y	+	-0,01511
6	5Y	+	-0,01533
7	6Y	+	-0,01806

Vertex #	Maturity	Sign of PV01	Adjusted directional 1-day extreme change
8	7Y	+	-0,01254
9	8Y	+	-0,01175
10	9Y	+	-0,01135
11	10Y	−	0,01135
12	12Y	+	-0,01219
13	15Y	+	-0,01302
14	20Y	+	-0,01448

Tab. 6: Extreme historical changes for 6M forward curve vertices

Vertex #	Maturity	Sign of PV01	Adjusted directional 1-day extreme change
1	6M spot	−	0,01051
2	6M in 6M	−	0,01612
3	6M in 12M	−	0,01308
4	6M in 18M	−	0,01308
5	6M in 3Yrs	−	0,01263
6	6M in 4Yrs	−	0,01096
7	6M in 5Yrs	−	0,01118
8	6M in 6Yrs	−	0,01453

Vertex #	Maturity	Sign of PV01	Adjusted directional 1-day extreme change
9	6M in 7Yrs	−	0,00856
10	6M in 8Yrs	−	0,00854
11	6M in 9Yrs	−	0,00816
12	6M in 10Yrs	+	-0,00814
13	6M in 12Yrs	−	0,00866
14	6M in 15Yrs	+	-0,00950
15	6M in 20Yrs	−	0,01096

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