AVERAGE WAGES AND WAGE QUANTILES IN THE CZECH REPUBLIC

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Abstract

The most-widely used wage characteristic is the average. Average wages are among the basic economic indices, even though they are far from ideal. It is usually observed that about two-thirds of employees do not achieve the average level of wages. In other words, the average is about a 67% quantile. The skewness of the wage distribution is one of the main reasons for this phenomenon: this distribution is skewed towards the positive (right-hand) values. This in turn is caused by high wages; there are not too many of those, but their values are far from the average. As the years go by, the distribution of wages is changed; namely, its skewness grows (that is, more high wages are observed). The question arises of what the behavior is of the quantile measures for such a wage distribution. We will mainly address the issues related to the median, but also the 10% and 90% quantiles, as well as the upper (25%) and lower (75%) quartiles. The data will first be processed for the entire Czech Republic and then various comparisons will take place. We will work with data from the period of 1995-2017.

Key words: average wage, quantile measures, median, quantiles

JEL Code: C530, F470

Introduction

The average is without any doubt the most-widely used wage characteristic. The average wage is very often mentioned in the media and much attention is paid to it. Lately, we have witnessed a steep wage growth, which is a result of the fast-growing economy in almost all economic areas. And the highest wages grow as well, of course. Yet, it is no secret that the average is not an ideal wage measure. Therefore, we should show not only the average but also other wage measures, i.e. mainly quantile measures that are not sensitive to the presence of outliers. In particular monthly wages over 100,000 CZK are major wage outliers. The average is very sensitive to these wages, but not quantile measures. Quantiles can also be used to show the characteristics of variability, skewness and kurtosis as an alternative to classic moment measures. The goal of this article is to analyze the quantile measures of wages in the

CR during the years of 1995 - 2017 and to evaluate their trend over time. Wages and their description (mainly outliers, i.e. high wages) have been the center of attention of several authors. Let's mention at least some elaborations (Malá, 2017; Pacáková et al., 2012; Terek, 2016; Wang et al, 2017). Some authors focus on wage distribution modeling in general (Vrabec at al., 2016) or analyze wage distribution in different branches of the economy (Marek at al., 2016). We will examine these areas later on; for now, we will work with aggregate data for the entire CR.

1 Data and results

We work with data in the form of interval frequency distribution. The interval width is 500 CZK, which is sufficiently detailed for an easy calculation of both moment and quantile measures of wages. The statistical population is also sufficiently large – over 2 million in the last analyzed year. The data came from the firm Trexima (Trexima, 2017). We did not adjust the data for inflation. But it does not matter for our purposes since we are comparing the trend in current prices and are mainly interested in the relationship of values in individual years, and so the comparison would be the same regardless of inflation.

1.1 Basic wage characteristics

Tab. 1 shows basic wage characteristics. The columns have the following meaning:

Average – average wage

D1 – the first decile (a 10% quantile)

Q1 – lower quartile (a 25% quantile)

Median – a 50% quantile

Q3 – upper quartile (a 75% quantile)

D9 – the ninth decile (a 90% quantile)

StandDev – standard deviation.

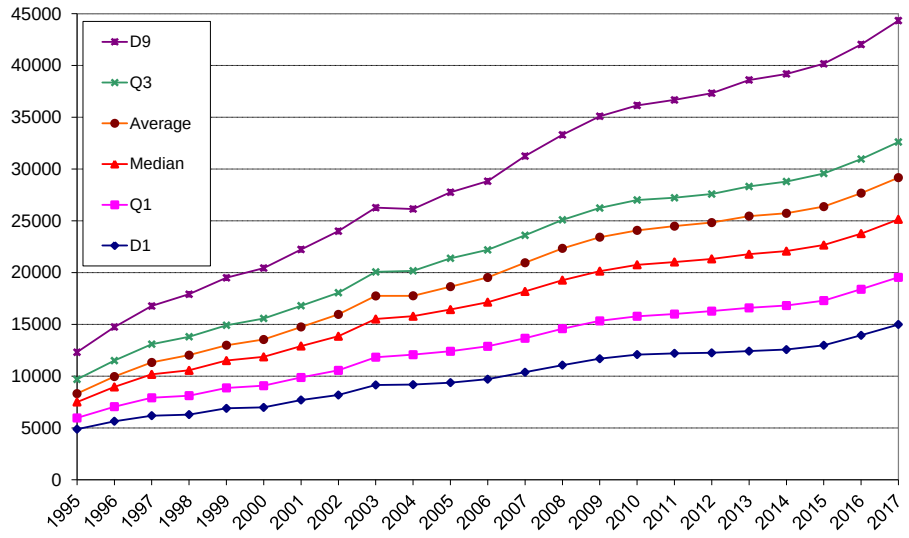
We showed these characteristics in a graph to see their trend better. The graph does not include the standard deviation of wages since it is a variability measure while other characteristics are location measures.

Tab. 1: Basic wage characteristics

Year	Average	D1	Q1	Median	Q3	D9	StandDev
1995	8,311	4,879	5,963	7,500	9,691	12,314	4,133
1996	9,962	5,645	7,047	8,956	11,505	14,748	5,393
1997	11,322	6,178	7,910	10,171	13,083	16,774	6,490
1998	12,026	6,287	8,114	10,563	13,801	17,911	8,261
1999	12,982	6,894	8,859	11,506	14,911	19,499	8,262
2000	13,541	6,981	9,077	11,860	15,570	20,435	9,651
2001	14,743	7,693	9,870	12,901	16,794	22,234	10,372
2002	15,964	8,181	10,564	13,857	18,058	24,003	12,994
2003	17,748	9,143	11,829	15,519	20,070	26,271	13,504
2004	17,759	9,185	12,073	15,789	20,168	26,143	13,062
2005	18,640	9,371	12,403	16,432	21,376	27,754	13,796
2006	19,526	9,710	12,882	17,143	22,192	28,828	17,696
2007	20,953	10,381	13,659	18,185	23,602	31,257	18,055
2008	22,338	11,060	14,583	19,267	25,094	33,306	20,714
2009	23,418	11,681	15,339	20,138	26,241	35,093	19,014
2010	24,077	12,084	15,778	20,753	27,009	36,143	19,316
2011	24,484	12,199	15,996	21,020	27,225	36,677	24,802
2012	24,829	12,255	16,281	21,319	27,583	37,328	20,109
2013	25,448	12,416	16,595	21,779	28,322	38,598	20,564
2014	25,728	12,570	16,821	22,074	28,794	39,182	19,612
2015	26,369	12,978	17,290	22,658	29,566	40,162	19,903
2016	27,668	13,944	18,391	23,757	30,963	42,026	20,478
2017	29,166	14,982	19,547	25,135	32,610	44,334	20,749

Source: Trexima

Fig. 1: Trend in wage characteristics



Source: Own graph

The data in the graph are in compliance with the legend as to the order from the top to the bottom. D9 is the highest value, after that it is Q1, etc. Both the table and the graph show several general facts:

- Wages keep going up over time linearly;
- The wage growth did not really slow down even during the economic crisis;
- The scissors of individual characteristics widen over time – it means, among other things, that the wage variability keeps growing, which is also obvious from the growing standard deviation;
- The ninth wage decile shows the fastest growth, which means that high wages keep growing much faster than other wages.

Now, we will comment on the specific values for the last analyzed year, i.e. the year 2017. The mentioned facts are obvious mostly from Tab. 1.

- The wages of 10% of employees are below 14,982 CZK;
- The wages of 25% of employees are below 19,547 CZK, while the wages of 75% of employees are over 19,547 CZK;
- The wages of 50% of employees are below 25,135 CZK and the wages of 50% of employees are over 25,135 CZK;
- The average is above the median but under a 75% quantile. In fact, the average is approximately a 67% quantile – more (Marek, 2017);
- The wages of 75% of employees do not exceed 32,610 CZK, the wages of 25% of employees are over 32,610 CZK;
- The ninth decile is 44,334 CZK. It means that only 10% of employees have a higher wage than that, while 90% of employees have a lower wage.

1.2 The relationship between quantile characteristics

Tab. 2 provides an interesting view of the trend in quantile characteristics of wages and their mutual relationship.

Again, we can reach several conclusions that are obvious at first sight.

- The ninth decile (thanks to big growth) quickly distances itself both from the first decile (column 1) and the median (column 5). Both these differences show a 6.2% average annual growth rate. This growth is also confirmed by the D9/D1 ratio (column 3). Also, the difference between the median and the first decile (column 6) keeps quickly growing at a 6.1% average annual growth rate.

- We can see a somewhat slower-growing difference between the values of both quartiles (column 2, a 5.6% average annual rate), the median and the first quartile (column 8, a 5.8% average annual rate) and the third quartile and the median (column 7, 5.5% average annual rate);
- Although the difference between the quartiles keeps growing absolutely, their ratio is almost constant over time (column 4). This is also true about the deciles (column 3).

Tab. 2: Differences and ratios of quantile measures

D9-D1	Q3-Q1	D9/D1	Q3/Q1	D9-Median	Median-D1	Q3-Median	Median-Q1
7,436	3,728	2.524	1.625	4,815	2,621	2,192	1,536
9,104	4,458	2.613	1.633	5,792	3,311	2,549	1,909
10,596	5,173	2.715	1.654	6,603	3,994	2,912	2,261
11,624	5,687	2.849	1.701	7,348	4,276	3,238	2,449
12,606	6,052	2.829	1.683	7,994	4,612	3,405	2,647
13,454	6,493	2.927	1.715	8,575	4,879	3,710	2,783
14,541	6,924	2.890	1.701	9,333	5,208	3,893	3,030
15,822	7,494	2.934	1.709	10,147	5,675	4,201	3,293
17,128	8,241	2.873	1.697	10,751	6,377	4,551	3,690
16,957	8,095	2.846	1.670	10,354	6,603	4,379	3,716
18,383	8,973	2.962	1.723	11,322	7,061	4,944	4,029
19,118	9,310	2.969	1.723	11,686	7,432	5,049	4,261
20,876	9,943	3.011	1.728	13,072	7,804	5,417	4,526
22,246	10,510	3.011	1.721	14,040	8,207	5,827	4,683
23,412	10,902	3.004	1.711	14,954	8,458	6,103	4,799
24,059	11,231	2.991	1.712	15,389	8,669	6,255	4,975
24,477	11,229	3.006	1.702	15,656	8,821	6,205	5,024
25,073	11,302	3.046	1.694	16,009	9,064	6,264	5,038
26,182	11,727	3.109	1.707	16,819	9,363	6,543	5,184
26,612	11,973	3.117	1.712	17,108	9,504	6,720	5,254
27,185	12,276	3.095	1.710	17,504	9,681	6,908	5,368
28,083	12,572	3.014	1.684	18,270	9,813	7,206	5,366
29,351	13,063	2.959	1.668	19,199	10,152	7,476	5,588

Source: Own calculations

Now, we will explain the meaning of the characteristics for the last year of our analysis, i.e. the year 2017.

- The difference between the 10% worst paid employees and the 10% best paid employees is getting bigger over time and represents 29,351 CZK;
- The difference between the 25% worst paid employees and the 25% best paid employees (quartile difference) is also getting bigger over time and represents 13,063 CZK;
- The distance of the median from the ninth decile is 19,199 CZK, while the distance of the median from the first decile is only 10,152 CZK;

- The distance of the median from the upper quartile is 7,476 CZK, while the distance of the median from the lower quartile is much smaller – only 5,588 CZK;
- Especially the last two results indicate that growth applies much more to higher wages. Higher wages grow faster than lower wages.

1.3 Other quantile characteristics of wages

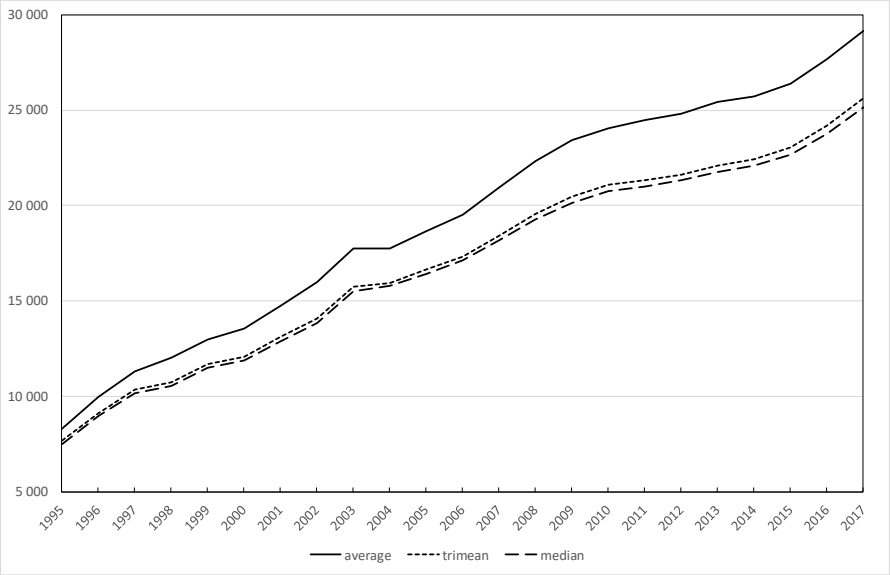
So far, we have focused on quantiles and their mutual relationship. However, other measures derived from quantiles can be used for analyses – more (Cyhelský, 1981).

First, let's mention *the trimean*, which is another characteristic of location calculated as a weighted average of quartiles:

$$\overline{x}_{trimean} = \frac{x_{0.25} + 2x_{.5} + x_{.75}}{4}, \qquad (1)$$

where the median has a double weight against quartiles as we can see in Fig. 2. The values of this characteristic do not differ very much from the median. The difference is really very tiny and thus there is no reason to show this characteristic as another measure of wage location.

Fig. 1: Average, median and trimean



Source: Own graph

Other characteristics include variability measures, especially *the interquartile range:*

$$R = x_{.75} - x_{0.25}, \qquad (2)$$

This value is already in column 2 of Tab. 2. Another used characteristic is *the quartile deviation*:

$$R_2 = \frac{x_{.75} - x_{0.25}}{2}, \qquad (3)$$

We will use *the quantile skewness coefficient* that measures the quantile skewness:

$$S_p = \frac{(x_{1-\mathrm{p}} - x_{0.50}) - (x_{0.50} - x_{\mathrm{p}})}{x_{1-\mathrm{p}} - x_{\mathrm{p}}}, \qquad (4)$$

where 0<p<1 and *p* is usually 0.1 or 0.25.

We will also calculate the quantile measure of kurtosis, the so-called *quantile kurtosis coefficient:*

$$K_p = \frac{x_{\max} - x_{\min}}{x_{1-\mathrm{p}} - x_{\mathrm{p}}}. \qquad (5)$$

However, it may be difficult to determine the minimum or maximum values of this coefficient if we do not have the original data. In such a case, we must estimate them (especially maximum values). Let's look at the results in Tab. 3.

Tab. 3: Quantile measures of variability, skewness and kurtosis

Year	R_2	$S_{0.1}$	$S_{0.25}$	$K_{0.1}$	$K_{0.25}$
1995	1,864	0.295	0.176	20.806	41.496
1996	2,229	0.273	0.144	16.994	34.703
1997	2,587	0.246	0.126	14.600	29.906
1998	2,843	0.264	0.139	13.309	27.204
1999	3,026	0.268	0.125	12.273	25.563
2000	3,246	0.275	0.143	11.499	23.826
2001	3,462	0.284	0.125	10.639	22.345
2002	3,747	0.283	0.121	9.778	20.644
2003	4,121	0.255	0.104	9.032	18.772
2004	4,047	0.221	0.082	9.123	19.112
2005	4,486	0.232	0.102	8.415	17.241
2006	4,655	0.222	0.085	8.092	16.617
2007	4,971	0.252	0.090	7.411	15.559
2008	5,255	0.262	0.109	6.954	14.719
2009	5,451	0.277	0.120	6.608	14.190
2010	5,615	0.279	0.114	6.430	13.775
2011	5,615	0.279	0.105	6.320	13.777
2012	5,651	0.277	0.108	6.170	13.688
2013	5,863	0.285	0.116	5.909	13.192
2014	5,987	0.286	0.122	5.813	12.921
2015	6,138	0.288	0.125	5.691	12.602
2016	6,286	0.301	0.146	5.509	12.305
2017	6,532	0.308	0.144	5.271	11.843

Source: Own calculations

We chose *p* to be 0.1 and 0.25, i.e. decile-based and quartile-based measures.

Based on Tab. 3 we can conclude the following:

- Variability measured based on the variation deviation keeps growing over time, the results are in compliance with classic moment measures (the standard deviation). This quantile measure thus only confirms what we already know from classic measures.
- Skewness is similar at the beginning and end of the analyzed period and the smallest in the middle of the analyzed period (around the year 2006). This does not correspond much to the empirical division of frequency of wages (Marek, 2010) and contradicts the classic moment skewness measures. Therefore, we should be somewhat cautious when using quantile skewness measures.
- Kurtosis keeps diminishing during the entire period of 23 years. This is in line with the empirical division of frequency of wages as well as with the classic moment kurtosis measures. Therefore, the kurtosis measure very well describes the behavior of wage division and can be recommended as a suitable measure.

Conclusion

The article mainly compares, analyses and uses the quantile measures of wages. We can say that the combination of the average, selected quartiles and the first and ninth deciles provide a very good idea about the location of wage distribution. These measures also very well indicate the trend in wages in the CR during the 23 analyzed years. Therefore, the level of wages should not be presented only using the average but also at least the median and potentially also quartiles and the first and ninth deciles. Moreover, with these quantiles it is possible to make rather simple calculations and comparisons that provide very good information about wages in the CR.

The quality and explanatory power of the quantile measures of variability, skewness and kurtosis are much worse than the quality and explanatory power of classic moment measures. Therefore, if we have moment measures, it is not necessary to provide quantile measures. However, if these moment measures cannot be determined for some reason, quantile measures will give us an initial idea about the behavior of the entire distribution. Moreover, they have an unquestionable advantage - they are simple and easy to calculate contrary to classic moment measures.

Acknowledgment

This paper was written with the support of the Czech Science Foundation Project No. P402/12/G097 "DYME – Dynamic Models in Economics".

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