APPLICATION OF TESTS JUC, JCC AND JIND FOR CHECKING THE CORRECTNESS OF THE VALUE-AT-RISK - AN EXAMPLE OF UNITED STATES AND GERMAN CAPITAL MARKETS

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Abstract

The paper concentrates on the problem of proper application of Value-at-Risk. The measure enables to quantify the level of risk, which is related to dynamic increase in the interdependences between whole economies or given markets, especially including financial markets. The problems of risk measurement become especially important during crisis situation, where after occurrence of a particular shock, one can expect successive, often unpredictable shocks, which are very difficult to predict. In this context, the article objective is to assess the quality of Value at Risk measure for measuring risk in the United States and German capital markets. The market of United States was chosen as it is the most developed market of global economy. On the other hand, the German capital markets is the most important market for continental Europe. VaR quality assessment was carried out through the application of backtesting. For this purpose, the results of the binomial LRuc, LRind, LRc tests have been subjected to the interpretation. The analysis was carried out in the period 2000-2012, where the GARCH model with conditional t-student distribution was used to estimate the VaR value.

Key words: Value-at-Risk, backtesting, Juc test, Jcc test, Jind test

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Introduction

The article focuses on the application of the Value at Risk measure that allows the level of risk in financial markets to be measured. This risk is related to the dynamic growth of interdependence between entire economies and specific markets, including financial markets. The existence of linkages between markets has been examined and confirmed by numerous empirical studies (Pietrzak et al., 2017a; 2017b; Vukovic et al., 2017). It should be emphasized that the value of the financial market, including the capital market, in individual countries is constantly growing. This is due to the dynamic development of financial institutions and a systematically increasing capitalization of capital markets, which on the one hand can stimulate enterprise financing and economic growth, but on the other hand increase the level of systemic risk (see Balcerzak *et al*., 2017; Kubiszewska, 2017; Meluzin *et al.*, 2018a; 2018b; 2017; Sosnowski, 2017; Skvarciany *et al.*, 2018).The existing interdependence between markets and the problem of risk measurement are become particularly significant under crisis situations, when after the occurrence of a shock, successive and often unpredictable disturbances in the functioning of markets take place (Maknickiene et al., 2018; Nawrocki, 2018; Śliwiński & Łobza, 2017). This phenomenon is of great importance also from the macroeconomic perspective and affects the economic situation in real sphere.

Therefore, measuring risk related to the functioning of capital markets is becoming an important element of economic management, both in macro and microeconomic terms. The most common measure for evaluating market risk is Value at Risk. This results from the universality of potential applications of the measure, as well as from the rich methodology regarding VaR determination methods and statistical tests (Fałdziński, 2017).

The objective of this article is to assess the quality of the application of the VaR measure in assessing risk on the US and German capital markets. This study covered two major market indices - DJIA and DAX. The analysis was carried out in the time period 2000-2012, where a GARCH class model with the conditional Student's *t* distribution was used to determine VaR. Next, a backtesting procedure was used to assess quality (Fałdziński, 2017), and within this procedure binominal tests were done. The quality of the VaR obtained was checked by means of the Juc, Jind, and Jcc tests (Candelon *et al*., 2011).

1. Value at Risk as a measure of market risk

Value at Risk is a measure of risk that belongs to the group of threat measures. Due to its simplicity of use and interpretative values, VaR is recommended by supervisory bodies to measure the financial risk in various financial institutions. Applying VaR allows measuring the probability of occurrence of the determined loss of an asset (portfolio). In order to establish VaR, the distribution of returns for the selected asset (portfolio) should be determined. Determining distribution is not an easy task, due to the volatility of variance over time. In order to describe a conditional variance for individual assets or indices, a ARMA(p,q)-PARCH(p',q') model may be used, where the conditional variance depends on lagged conditional variances and on the square of returns. The model ARMA(p,q)-PARCH(p',q') (The Power ARCH model, see: Ding et al., 1993) can be defined as follows:

$$y_t = c + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t, \varepsilon_t | F_{t-1} \sim t(0, h_t, \mathrm{v}) \quad (1)$$

$$h_t^\delta = \alpha_0 + \sum_{i=1}^{p'} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q'} \beta_j h_{t-j}^\delta, \quad (2)$$

where $\alpha_0 > 0, \delta \geq 0, \alpha_i \geq 0, i = 1, \dots, p', -1 < \gamma_i < 1, \beta_j \geq 0, j = 1, \dots, q'$.

where y_t is the process of returns, μ_t is the conditional mean of returns, h_t is the conditional standard deviation equation, $c, \varphi_1, \dots, \varphi_p, \theta_1, \dots, \theta_q$ $\alpha_0, \alpha_1, \dots, \alpha_{p'}, \beta_1, \dots, \beta_{q'}$ are the parameters of both the conditional mean and the conditional standard deviation, ε_t is the noise term following the t-distribution with v degrees of freedom.

Value at risk is a measure of risk by which one can assess the probability of occurrence of a specific loss. Value at Risk is the value of the market loss of an asset (portfolio) whose probability of reaching or exceeding it in a given time interval is equal to a given tolerance level. If the quantile of the distribution of returns r_α is determined by the formula:

$$P(r \leq r_\alpha) = \alpha \quad (3)$$

then VaR can be determined as

$$VaR = -r_\alpha C_0, \quad (4)$$

where r is the rate of the formula, C_0 is the original value of the asset (portfolio), and α means the level of tolerance (see :).

In order to determine VaR, first the parameters for the distribution of returns should be established. Since the variance of returns changes over time and there are periods of elevated and significantly reduced volatility in the market, a GARCH class model can be used to estimate the parameters of the distribution of returns (Szumilo et al., 2018). The formula for estimating a one-day VaR, where the results of the GARCH class model parameter estimation are used, is defined as follows:

$$VaR_\alpha^t = \mu_{t+1} + h_{t+1} z_q \quad (5)$$

where μ_{t+1} is the forecast of the conditional mean of returns, h_{t+1} is the forecast of the conditional volatility and z_q is q-quantile of the conditional distribution, and α is known as coverage level.

After estimating Value at Risk, the quality of the measure should be assessed. The VaR measure is tested for two properties: unconditional coverage and independence property. The first one assumes that the expected number of hits (cases when returns are greater than estimated VaR in absolute term) equals the given coverage level. The second one states that the hit process is independent. The most common approach to assessing the quality of VaR is called backtesting, which is based on the examination of hits resulting from the estimated values of the measure. Therefore, in the article binominal tests were used including the unconditional coverage test Juc, the test of independence Jind, and the conditional coverage test Jcc which test both of the properties at once (Candelon *et al*., 2011). These tests are characterized by a greater strength when compared to such binominal tests as LRuc, LRind, and LRcc.

2. Empirical research

Due to the undertaken problem of risk measurement related to the functioning of capital markets, an empirical study on the US and German markets was carried out. For this purpose, the returns of the two major stock indices (the DJIA and DAX indices) were used. The study was conducted in the period commencing on 3 January 2000 and ending on 3 January 2012. The data analysed was obtained from the 'Yahoo Finance' website.

An indispensable element in the procedure of establishing Value at Risk is determining the distribution of returns. The best solution is to describe their volatility using an ARMA-PARCH class model. Therefore, the parameters of the ARMA-PARCH model were estimated for the period 2000-2012 for each market index separately. The initial analysis of the autocorrelation function of returns and the return rate squares for the WIG and DAX indices indicated the model specification in the form of ARMA(p,q)-PARCH(1,1)

$$y_t = c + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t, \varepsilon_t | F_{t-1} \sim t(0, h_t, \mathrm{v}) \quad (6)$$

$$h_t^{\delta} = \alpha_0 + \alpha_1 (|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^{\delta} + \beta_1 h_{t-1}^{\delta}, \quad (7)$$

where y_t is the process of returns, $\mu_t$is the conditional expected value of returns, h_t is the conditional variance equation, $\alpha_0, \alpha_1, \omega_0, \omega_1, \beta_1$ are the parameters of both the conditional expected value and the conditional variance, ε_t is the random component with the Student's t distribution with v degrees of freedom.

The results of the estimation of the ARMA(p,q)-PARCH(1,1) model parameters are presented in Table 1, where the lag order of the ARMA(p,q) model was selected based on the Akaide information criterion (AIC). The estimated parameters, their significance and the statistical properties of the models indicate a satisfactory match with the empirical data. In the case of the conditional mean equation not all the parameters are significant which is due to a selection process based on the AIC. All other parameters proved to be statistically significant at the 5% significance level. The sums of parameters do not exceed the value of one in both models. The estimated parameter v of the *t*-distribution indicate the occurrence of fat tails in the distribution of returns, which justifies the use of this distribution in the estimation procedure.

Tab. 1: The estimation of the ARMA(p,q)-PARCH(1,1) model parameters for DJIA and DAX indexes

Parameter	Estimate	Std. error	p-value	Estimate	Std. error	p-value
	DJIA			DAX		
c	0.0264	0.0144	0.0671	0.0181	0.0207	0.3818
φ_1	0.5078	0.1825	0.0054	0.2417	0.0275	0.0000
φ_2	-0.5616	0.1520	0.0002	-0.8852	0.0297	0.0000
θ_1	-0.5591	0.1834	0.0023	-0.2495	0.0331	0.0000
θ_2	0.5671	0.1542	0.0002	0.8888	0.0301	0.0000
θ_3	0.0014	0.0232	0.9495	0.0157	0.0201	0.4351
α_0	0.0128	0.0026	0.0000	0.0254	0.0037	0.0000
α_1	0.0568	0.0066	0.0000	0.0660	0.0059	0.0000
β_1	0.9999	0.0000	0.0000	0.9999	0.0000	0.0000
γ_1	0.9357	0.0061	0.0000	0.9245	0.0072	0.0000
δ	1.2152	0.1555	0.0000	1.1393	0.1485	0.0000
v	8.1531	1.1239	0.0000	20.726	6.1892	0.0000

Source: own calculations.

Based on the high quality of the estimated ARMA(p,q)-PARCH(1,1) model in the period 2000-2012, a decision was made to apply this model to estimate the Value at Risk. The procedure consisted in estimating 2000 ARMA-PARCH models, where the first model was assessed based on the initial 1000 observations, and the last model based on the last 1000 observations. This means that for each model the observation window was rolling one day forward. Next, on the basis of the ARMA(p,q)-PARCH(1,1) model parameter estimations, 2000 one-day forecasts of the VaR were made based on formula (5). The received set of values was used to complete the backtesting procedure. The results regarding the use of the selected Juc, Jind, Jcc binomial tests are contained in Table 2. For the 99% coverage level neither unconditional coverage property nor independence property is not met for DJIA and DAX. For the 95% coverage level unconditional coverage property is not met for both indices. It should be noted that for both market indexes (DJIA and DAX) the properties of the VaR measure can be considered as correct for 90% coverage level and independence property for 95% coverage level. It indicates that estimates of VaR are failing when we are dealing with high quantiles. We should probably refer to methods which can deal better with high losses. On of such method is extreme value theory (see Fałdziński, 2017).

Tab. 2: Results for Backtests Juc, Jcc, Jind

Index	Test	Statistic	Simulated p-value	Test	Statistic	Simulated p-value
99% coverage level						
DJIA	Juc(p=1)	11.6402	0.002200	Jind(p=1)	0.0243	0.9789
	Jcc(p=2)	16.6751	0.008499	Jind(p=2)	3.6554	0.0176
	Jcc(p=3)	19.8883	0.008599	Jind(p=3)	5.5090	0.0159
	Jcc(p=4)	22.2886	0.010699	Jind(p=4)	5.6312	0.0315
	Jcc(p=5)	24.0892	0.010199	Jind(p=5)	5.7003	0.0437
	Jcc(p=6)	25.3982	0.011399	Jind(p=6)	6.4608	0.0411
DAX	Juc(p=1)	6.4935	0.011299	Jind(p=1)	0.0290	0.9774
	Jcc(p=2)	7.4374	0.031397	Jind(p=2)	0.1600	0.6818
	Jcc(p=3)	7.5593	0.041996	Jind(p=3)	0.1867	0.8116
	Jcc(p=4)	7.5941	0.057794	Jind(p=4)	0.4265	0.7412
	Jcc(p=5)	7.6371	0.070193	Jind(p=5)	0.6549	0.7170
	Jcc(p=6)	7.7108	0.080692	Jind(p=6)	0.7433	0.7455
95% coverage level						
DJIA	Juc(p=1)	6.4473	0.0114	Jind(p=1)	0.0083	0.9050
	Jcc(p=2)	6.7528	0.0364	Jind(p=2)	0.0090	0.9925
	Jcc(p=3)	6.8719	0.0618	Jind(p=3)	0.2379	0.8366
	Jcc(p=4)	6.9695	0.0825	Jind(p=4)	0.2948	0.8786
	Jcc(p=5)	6.9982	0.1014	Jind(p=5)	0.3145	0.9292
	Jcc(p=6)	6.9983	0.1263	Jind(p=6)	0.3604	0.9505
DAX	Juc(p=1)	7.2874	0.0061	Jind(p=1)	0.0082	0.9046
	Jcc(p=2)	7.2985	0.0343	Jind(p=2)	0.8209	0.3261
	Jcc(p=3)	7.3414	0.0557	Jind(p=3)	0.8215	0.4760
	Jcc(p=4)	7.3429	0.0760	Jind(p=4)	1.0393	0.5637
	Jcc(p=5)	7.3870	0.0939	Jind(p=5)	1.1119	0.6276
	Jcc(p=6)	7.4190	0.1155	Jind(p=6)	1.1385	0.7018
90% coverage level						
DJIA	Juc(p=1)	2.2171	0.1282	Jind(p=1)	0.0050	0.7658
	Jcc(p=2)	2.5974	0.2484	Jind(p=2)	0.3326	0.5587
	Jcc(p=3)	2.7682	0.3261	Jind(p=3)	0.4603	0.7226
	Jcc(p=4)	2.8651	0.3959	Jind(p=4)	0.8938	0.6497

	Jcc(p=5)	3.5435	0.3625	Jind(p=5)	2.0223	0.4314
	Jcc(p=6)	4.1740	0.3381	Jind(p=6)	2.4815	0.4135
DAX	Juc(p=1)	2.4224	0.1211	Jind(p=1)	0.0050	0.7652
	Jcc(p=2)	2.5876	0.2469	Jind(p=2)	0.0960	0.7666
	Jcc(p=3)	3.2098	0.2626	Jind(p=3)	1.0162	0.4466
	Jcc(p=4)	3.6132	0.2861	Jind(p=4)	1.2837	0.5170
	Jcc(p=5)	3.6568	0.3482	Jind(p=5)	1.2869	0.6092
	Jcc(p=6)	3.6609	0.4027	Jind(p=6)	1.3297	0.6838

Source: own calculations.

Conclusion

The subject of the article concerns the problem of risk measurement related to the functioning of the US and German capital markets. This risk results from the existing interdependence between the financial markets, whose strength and scope are constantly growing along with the increasing globalization. The problem of risk measurement grows in importance in crisis situations, when after a certain shock successive and often unpredictable disturbances occur in the functioning of the markets. Under such conditions, the build-up of successive market shocks causes that a crisis situation may develop in an unpredictable manner, often leading to a global crisis. The global financial crisis revealed that shocks on capital markets can lead to bankruptcy of financial entities, even of those operating globally.

Value at Risk is a commonly used tool to measure market risk, which stems from the universality of its application. Identification and correct measurement of market risk is an important research problem, and the use of Value at Risk should allow entities to protect against crisis situations. In connection with the issue of risk measurement undertaken in the article, the quality of VaR estimates was tested for the two major stock indices (DJIA and DAX). For the purpose of evaluating quality backtesting was used, where binominal tests were carried out - the Juc, Jind and Jcc tests. The results obtained confirmed the correctness of the VaR measure estimation in the case of both market indices.

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