MARGINAL EXPECTED SHORTFALL: THE CZECH PX INDEX CASE STUDY

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Abstract

The systemic risk is undoubtedly an important concept employed in the framework of modern risk regulatory systems as are Basel III in finance or Solvency II in insurance. This contribution primarily concentrates on a particular quantitative approach to measuring the systemic risk, which seems to be a significant risk in today's financial world (not solely in banks and insurance companies). The marginal expected shortfall measure is based on the well-known concept of the expected shortfall. More specifically, it can be regarded as a conditional version of the expected shortfall in which the global returns exceed a given market drop. We shall demonstrate that the marginal expected shortfall is a useful risk measure when studying the Prague Stock Exchange index and all its constituents. The corresponding modelling scheme is introduced and discussed. It is extended in such a way that one can describe time-varying dependencies using the multivariate GARCH modelling class. Moreover, such an econometric approach enables to forecast the capital shortfall over a potentially long period (e.g. a quarter or half year), which might be appreciated in financial and insurance practice.

Key words: expected shortfall, GARCH, PX index, systemic risk, volatility

JEL Code: C32, C58, C60

Introduction

The systemic risk seems to be a highly significant risk in today's financial world. It has been more frequently regulated since the previous financial crises demonstrated various weaknesses in the global regulatory framework and banks' risk management practices. The paper deals with a specific quantitative approach to measuring the systemic risk, when portfolio scheme is applicable (e.g. particular firms comprising a stock index may present a systemic risk, when aggregate capital drops below a given threshold).

One can examine various quantitative aspects of systemic risk with many references in research and applied literature and calculation outputs in financial practice. Acharya, Engle, and Richardson (2012) developed a simple model in which a group of banks set leverage levels and choose asset positions in a broader economic environment with systemic risk emerging, when aggregate bank capital drops below a given threshold. Billio, Getmansky, Lo, and Pelizzon (2012) proposed several econometric measures of systemic risk to capture interconnectedness among the monthly returns of hedge funds, banks, brokers, and insurance companies. A component expected shortfall approach to systemic risk was suggested in Banulescu and Dumitrescu (2015). Another popular approach to systemic risk is based on the concept of *CoVaR*, which measures changes in the system's Value at Risk, when one particular institution is under financial stress as measured by its own individual Value at Risk (Adrian & Brunnermeier, 2008).

This paper concentrates just on the quantitative aspects of systemic risk, in particular, on measuring the systemic risk in a portfolio context. Principally, there are two ways of evaluating the contribution of a given firm to the overall risk of the system (Benoit, Colletaz, Hurlin, & Pérignon, 2013). The first (supervisory) approach relies on firm-specific information (size, leverage, liquidity, interconnectedness, substitutability, and others) and uses data provided by the financial institution to the regulator. The second approach relies on publicly available market data (stock returns, CDS spreads, and others) since such data are believed to reflect all information about publicly traded firms. In both cases one must apply a suitable measure of the corresponding systemic risk. This contribution primarily focuses on the second described case when applying the marginal expected shortfall *MES* measure, which represents one of the most common systemic risk measures. It can be regarded as a conditional version of the expected shortfall in which the global returns exceed a given market drop (Benoit et al., 2013).

The paper is organized as follows. Section 1 introduces the concept of marginal expected shortfall *MES* and discusses the typical model situation, when one applies this instrument for systemic risk analysis of a firms' portfolio. Section 2 investigates the systemic risk of stocks of key companies composing the index PX of Prague Stock Exchange.

1 Modelling marginal expected shortfall

Marginal expected shortfall *MES* is based on the well-known concept of the expected shortfall *ES* (the *ES* at level α is the expected return in the worst $\alpha \times 100\%$ of the cases). The expected shortfall is usually preferred among risk measures in today's financial practice due to its coherence and other properties giving to it preferences in comparison with classical measures such as Value at Risk (Artzner, Delbaen, Eber, & Heath, 1999).

To describe *MES* in a simple way, we shall use the following model situation (Benoit et al., 2013): Let us consider N firms and denote r_{it} the return of firm i at time t. The corresponding market return r_{mt} at time t is defined as follows:

$$r_{mt} = \sum_{i=1}^{N} w_{it} r_{it}, \tag{1}$$

where w_{it} is the relative market capitalization of firm i at time t. The modification of *ES* to the *MES* is based on conditioning *ES* in which global returns exceed a given market drop $C < 0$:

$$MES_{it}(C) = \mathrm{E}_{t-1}(r_{it} | r_{mt} < C). \tag{2}$$

If the conditional *ES* of the system is formally defined as

$$ES_{mt}(C) = \mathrm{E}_{t-1}(r_{mt} | r_{mt} < C) = \sum_{i=1}^{N} w_{it} \mathrm{E}_{t-1}(r_{it} | r_{mt} < C), \tag{3}$$

then it holds:

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}}. \tag{4}$$

One must understand the symbols $MES_{it}(C)$ and $ES_{mt}(C)$ conditionally at time as $MES_{i,t|t-1}(C)$ and $ES_{m,t|t-1}(C)$, i.e. computed at time t given the information available at time $t-1$. Therefore, the marginal expected shortfall measures the increase in the risk of the system (measured by the *ES*) induced by a marginal increase in the weight of firm i in the system. The higher is *MES* of the firm, the higher is the individual contribution of the firm to the risk of the financial system (Scaillet, 2005).

Various modelling alternatives for the *MES* calculation have been proposed in literature. We shall deal with the *MES* in the framework of the simple econometric model supplemented in such a way that one can model time-varying dependencies using the multivariate DCC-GARCH modelling class. Moreover, such an econometric method enables to estimate the capital shortfall over a potentially long period (e.g. a quarter or half year), which is surely useful in common financial practice (Cipra & Hendrych, 2017).

The final model (e.g. for a whole portfolio of stocks composing an exchange index) can be explained by means of bivariate conditionally heteroskedastic models, which characterize the dynamics of the daily firm and market returns. To be more specific, let r_{mt} and r_{it} denote the market and *i*-th firm log returns on day *t*. The bivariate process of the daily market and *i*-th firm returns is modeled as

$$\begin{aligned} r_{mt} &= \sigma_{mt}\varepsilon_{mt}, \\ r_{it} &= \sigma_{it}\varepsilon_{it} = \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\zeta_{it}, \end{aligned} \tag{5}$$

where the shocks (ε_{mt}, ζ_{it}) are independent and identically distributed over time with zero mean, unit variance and zero covariance. A mutual independence of these shocks is not assumed. On the contrary, there are reasons to believe that extreme values of disturbances ε_{mt} and ζ_{it} interact (when the market is in its tail, the firm disturbances may be even further in the tail if there is a serious risk of default).

The stochastic specification is completed by a description of the two conditional standard deviations and the conditional correlation. These quantities can be formulated as follows: Applying the principles of conditional heteroskedasticity, the volatility is modeled by means of the simplest threshold GJR-GARCH(1,1) model as

$$\begin{aligned} \sigma_{mt}^2 &= \omega_m + \alpha_m r_{m,t-1}^2 + \gamma_m r_{m,t-1}^2 I_{m,t-1}^- + \beta_m \sigma_{m,t-1}^2, \\ \sigma_{it}^2 &= \omega_i + \alpha_i r_{i,t-1}^2 + \gamma_i r_{i,t-1}^2 I_{i,t-1}^- + \beta_i \sigma_{i,t-1}^2 \end{aligned} \tag{6}$$

with $I_{m,t}^- = 1$ for $r_{mt} < 0$ and 0 otherwise, $I_{i,t}^- = 1$ for $r_{it} < 0$ and 0 otherwise. According to the applied threshold modification, the model can cover the leverage effect, i.e. the tendency of volatility to increase more after observing negative news rather than positive ones.

The time-varying correlations are captured by using the *asymmetric dynamic conditional correlation ADCC*(1,1) modelling scheme (Engle, 2009). Let $\mathbf{R}_t$ denote the time-varying correlation matrix of the market and firm return. We shall assume that

$$\mathbf{R}_t = \mathrm{var}_{t-1}\begin{pmatrix} r_{mt} \\ r_{it} \end{pmatrix} = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t = \begin{pmatrix} \sigma_{mt} & 0 \\ 0 & \sigma_{it} \end{pmatrix}\begin{pmatrix} 1 & \rho_{it} \\ \rho_{it} & 1 \end{pmatrix}\begin{pmatrix} \sigma_{mt} & 0 \\ 0 & \sigma_{it} \end{pmatrix}, \tag{7}$$

where

$$\mathbf{R}_t = diag(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t\, diag(\mathbf{Q}_t)^{-1/2} \tag{8}$$

with

$$\mathbf{Q}_t = \mathbf{\Omega}_Q + \mathbf{A}_Q^{\mathrm{T}}\boldsymbol{\varepsilon}_{t-1}^{*}\boldsymbol{\varepsilon}_{t-1}^{*\,\mathrm{T}}\mathbf{A}_Q + \mathbf{B}_Q^{\mathrm{T}}\mathbf{Q}_{t-1}\mathbf{B}_Q + \mathbf{C}_Q^{\mathrm{T}}\boldsymbol{\varepsilon}_{t-1}^{**}\boldsymbol{\varepsilon}_{t-1}^{**\,\mathrm{T}}\mathbf{C}_Q, \tag{9}$$

where the matrix operator *diag*(•) creates a diagonal matrix by extracting diagonal elements of the input matrix, $\mathbf{\Omega}_Q$ is a (2×2) symmetric positive definite intercept matrix, $\mathbf{A}_Q$, $\mathbf{B}_Q$, and $\mathbf{C}_Q$

denote the (2×2) matrices of parameters, $\boldsymbol{\varepsilon}_t^* = diag(\mathbf{D}_t)^{-1/2}(r_{mt}, r_{it})^{\mathrm{T}}$ are "degarched" financial returns, and finally $\boldsymbol{\varepsilon}_t^{**} = \min(0, \boldsymbol{\varepsilon}_t^*)$. All cross products of $\boldsymbol{\varepsilon}_t^{**}$ elements will be nonzero only if both multiplied components are negative. Therefore, the model allows that dynamic correlations may be different for negative financial returns from the ones for positive ones. Note that the matrix $\mathbf{Q}_t$ is symmetric and positive definite by construction. For simplicity, one may assume that the matrices $\mathbf{A}_Q$, $\mathbf{B}_Q$, and $\mathbf{C}_Q$ are diagonal. Particularly, we shall put $\mathbf{A}_Q = \alpha_Q \mathbf{I}$ with some $\alpha_Q \geq 0$, $\mathbf{B}_Q = \beta_Q \mathbf{I}$ with some $\beta_Q \geq 0$, and $\mathbf{C}_Q = \gamma_Q \mathbf{I}$ with some $\gamma_Q \geq 0$, where $\mathbf{I}$ denotes the (2×2) identity matrix. Model estimation, properties, and other related issues are studied in literature, e.g. by Engle (2009) and in the works cited therein.

From the practical viewpoint, the model should enable to construct predictions in order to find the future capital shortfall. The one-period ahead *MES* can be expressed as a function of volatility, correlation, and tail expectation of the standardized innovations distribution:

$$\begin{aligned} MES^1_{i,t-1}(C) &= \mathrm{E}_{t-1}(r_{it} | r_{mt} < C) = \sigma_{it} \mathrm{E}_{t-1}(\varepsilon_{it} | \varepsilon_{mt} < C/\sigma_{mt}) = \\ &= \sigma_{it}\rho_{it}\mathrm{E}_{t-1}(\varepsilon_{mt} | \varepsilon_{mt} < C/\sigma_{mt}) + \sigma_{it}\sqrt{1-\rho_{it}^2}\,\mathrm{E}_{t-1}(\zeta_{it} | \varepsilon_{mt} < C/\sigma_{mt}), \end{aligned} \tag{10}$$

where the conditional probability of a systemic event is defined as

$$\mathrm{P}^1_{t-1}(C) = \mathrm{P}_{t-1}(r_{mt} < C) = \mathrm{P}(\varepsilon_{mt} < C/\sigma_{mt}). \tag{11}$$

The conditional tail expectation $\mathrm{E}_{t-1}(\zeta_{it}|\varepsilon_{mt} < C/\sigma_{mt})$ in the expression (10) captures the tail spillover effects from the financial system to the financial institution (firm) that are not captured by the conditional correlation. Assuming that innovations ε_{mt} and ζ_{it} are i.i.d., the nonparametric estimates of the tail expectations in (10) can be obtained as (Chen, 2008):

$$\begin{aligned} \hat{\mathrm{E}}_{t-1}(\varepsilon_{mt} | \varepsilon_{mt} < C/\sigma_{mt}) &= \frac{\sum_{\tau=1}^{T} \varepsilon_{m\tau} \cdot I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}{\sum_{\tau=1}^{T} I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}, \\ \hat{\mathrm{E}}_{t-1}(\zeta_{it} | \varepsilon_{mt} < C/\sigma_{mt}) &= \frac{\sum_{\tau=1}^{T} \zeta_{i\tau} \cdot I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}{\sum_{\tau=1}^{T} I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}, \end{aligned} \tag{12}$$

where T denotes the sample size and $I[\bullet]$ is the binary indicator of an event •. Alternatively, nonparametric kernel methods might be considered to estimate the tail expectations. Finally, the threshold $C < 0$ characterizing the systemic event is given, e.g. as the unconditional or conditional Value at Risk of r_{mt}.

Alternatively, the simple historical one-period ahead *MES* can be used as a simple benchmark (Brownlees & Engle, 2012). It is given directly as

$$MES_{i,t-1}^{HIST}(C) = \frac{\sum_{\tau=t-M}^{t-1} r_{i\tau} \cdot I[r_{m\tau} < C]}{\sum_{\tau=t-M}^{t-1} I[r_{m\tau} < C]}, \tag{13}$$

where the positive integer M is the width of moving window. This estimator is inspired by the financial practice, where various rolling window averages are commonly used.

Note that different modelling strategies would be introduced in order to predict *MES*. Particularly, distinct modelling specifications for calculating volatilities and correlations given by the formulas in (6)-(9) could be considered. For calculating multi-period predictions of *MES*, consult e.g. Cipra and Hendrych (2017) or Brownlees and Engle (2012).

2 Marginal expected shortfall: The Czech PX index case study

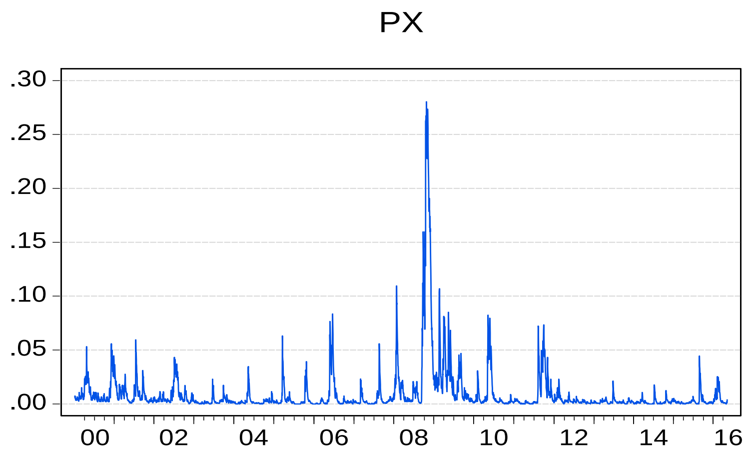
This section presents a case study of the Prague Stock Exchange (PX) index constituents. In order to calculate the marginal expected shortfall for each involved firm, we shall implement a (partly) modified estimation method originally considered by Brownlees and Engle (2012). More precisely, we shall follow the modelling framework introduced in Section 1. The conditional volatilities σ_{mt} and σ_{it} are modeled by the GJR-GARCH(1,1) scheme given by (6). The time-varying conditional correlations ρ_{it} are described by means of the asymmetric DCC model fully specified by (7)-(9). The model is estimated in two consecutive steps by applying quasi maximum likelihood similarly as it is performed in Engle (2009). Given estimated the above-mentioned quantities, the marginal expected shortfall and the conditional probability of a systemic event are then computed by (10), (11), and (13), respectively.

We shall analyze the panel of companies constituting the PX index basis. The panel contains fourteen firms, which were incorporated into the PX index basis according to their market capitalization as of the end of June 2008. This panel is unbalanced in that sense that not all companies have been continuously traded during the sample period. We extracted the daily logarithmic returns from January 6, 2000 to May 9, 2016. The full list of institutions involved in our study is reported in Cipra and Hendrych (2017).

Figure 1 displays the estimated conditional probability of systemic event *POS* as it was given in (11). Point out that the threshold C was set as the 1% unconditional Value at Risk of the PX index log returns. At the first sight, the estimated POS is evidently very high during the financial crisis. Figure 2 shows the estimated one-step ahead *MES* evaluated according to (10) or (13) with the rolling window width $M = 250$. Accordingly, all one-step ahead *MES* values calculated by (10) are also excessive during the financial crisis period. Nevertheless, high one-step ahead *MES* values occur also in other time instants, see e.g. *O2* (the company split observed in 2015) or *Unipetrol* (in 2005 and 2006). It should be highlighted that the *Philip Morris* returns indicate the lowest average one-step ahead *MES* overall, i.e. this comparation shows the firm stability. Finally, the *MES* computed by (13) evidently does not represent a suitable measure of systemic risk. For instance, as can be seen from Figure 2, during the years 2012-2015 the historical one-period ahead *MES* predictions are mostly zero due to absence of any systemic event declared by the threshold *C*. It is in sharp contrast to the *MES* forecasts obtained by (10). Information delivered by such an analysis of *MES* can be applied e.g. by investors to optimize their portfolio or by regulators to initiate a relevant action. One should remind that some drawbacks of the Czech stocks market (e.g. low liquidity) might bring some distortions into the previous results.

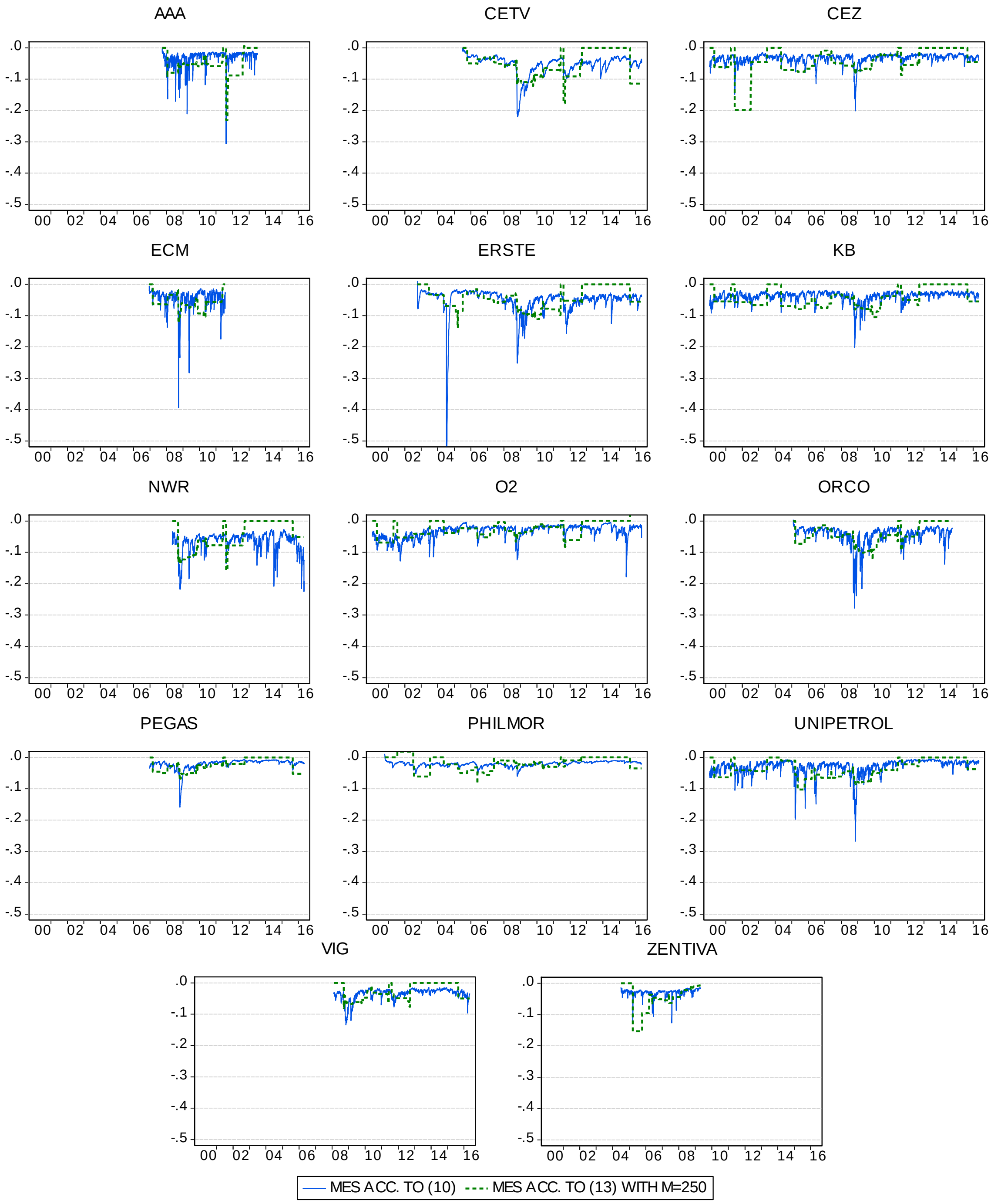
As the methodology used is concerned, Brownlees, Engle, and Kelly (2011) jointly with other works cited therein explored the performance of volatility forecasting by exercising it on a wide range of domestic and international equity indices and exchange rates. It was concluded that the simplest GARCH specification similar to our approach is the most often the best forecaster of future risk when studying various financial asset classes with different volatility regimes.

Fig. 1: Estimated probability of a systemic event POS of the PX index



Source: Authors (by Eviews 8.0)

Fig. 2: MES estimates calculated by (10) and (13) for the PX index constituents



Source: Authors (by Eviews 8.0)

Conclusion

The paper was focused on a special quantitative approach to the systemic risk, which seems to be a significant type of risk in today's financial world. We have slightly modified the common modelling framework used for estimating the marginal expected shortfall and the conditional probability of systemic event occurrence. In particular, we have considered the GJR-GARCH(1,1)-ADCC(1,1) model, which respects the character of financial returns more properly. The introduced modelling strategy has enabled to forecast the capital shortfall (over a potentially long time horizon), which could be useful in financial practice (e.g. for portfolio managers, regulators, brokers, etc.). This methodology was applied in Section 2 in the case study of the portfolio of PX index constituents with discussion of obtained results. One could identify that the conditional probability of systemic event was excessively high during the financial crisis period and that the *MES* of individual firms reflected both the market situation and key firm's events (e.g. splits, acquisitions, restructurings, etc.). It was shown that the rolling-window *MES* evaluated according to (13) is not competitive with the preferred approach (10), which calculates the marginal expected shortfall using more sophisticated econometric models of financial returns.

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