SAMPLING INSPECTION BY ATTRIBUTES – ADVANTAGES AND DISADVANTAGES

Jindřich Klůfa

Abstract

For application of the acceptance sampling plans for inspection by attributes, unlike the acceptance sampling plans for inspection by variables, there are no assumptions. Moreover, inspection procedure for inspection by attributes is simpler than for inspection by variables. On the other hand, the sample size in acceptance sampling plans for inspection by variables is always less than the sample size in acceptance sampling plans for inspection by attributes, i.e. using the acceptance sampling plans for inspection by variables we check a smaller number of items. Also the inspection costs for the acceptance sampling plans for inspection by variables may be less than the inspection costs for the acceptance sampling plans for inspection by attributes. This problem shall be solved in present paper for the acceptance sampling plans when the reminder of rejected lots is inspected which protect the consumer against the acceptance of a bad lot using the average outgoing quality limit.

Key words: Acceptance sampling by attributes, acceptance sampling by variables, inspection costs

JEL Code: C44, C80

Introduction

According to the way of quality control products the acceptance sampling is divided into

- acceptance sampling by attributes (each inspected item is classified as either good or defective)
- acceptance sampling by variables (for each inspected item we obtain the measurement of a quality characteristic).

The inspection procedure for acceptance sampling by attributes is as follows: Draw a random sample of n items (single sampling). Accept the lot if

the number of defective items in the sample is less or equal to c, where c is the acceptance number. Therefore, for decision accept or reject the lot we must find the acceptance plan (n, c). For application of the acceptance sampling plans for inspection by attributes there are no assumptions – see e.g. (Klůfa, 2015).

The inspection procedure for acceptance sampling by variables is as follows: Draw a random sample of n items (single sampling) and compute $\bar{x}$ and s. Accept the lot if

$$\frac{U - \bar{x}}{s} \geq k, \quad \text{or} \; \frac{\bar{x} - L}{s} \geq k, \tag{1}$$

where k is a critical value, U is an upper specification limit, L is a lower specification limit – see e.g. (Kaspříková and Klůfa, 2015). We have determine the sample size n and the critical value k, i.e. the single sampling acceptance plan (n,k). For application of the acceptance sampling plans for inspection by variables we assume that measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s.

1 The average outgoing quality limit plans

Under the assumption that each inspected item is classified as either good or defective (inspection by attributes) Dodge and Romig consider sampling plans which minimize the mean number of items inspected per lot of process average quality, assuming that the reminder of rejected lots is inspected

$$I_s = N - (N - n) \cdot L(\bar{p}; n, c) \tag{2}$$

under the condition

$$\max_{0<p<1} AOQ(p) = p_L \tag{3}$$

(AOQL single sampling plans), where N is the number of items in the lot (the given parameter), $\bar{p}$ is the process average fraction defective (the given parameter), p_L is the average outgoing quality limit (the given parameter, denoted AOQL), n is the number of items in the sample $(n < N)$, c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c), $L(p)$ is the operating characteristic (the probability of accepting a submitted lot with fraction defective p), $AOQ(p)$ is average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was p). Condition (3) protects the consumer against the acceptance of a bad lot, average outgoing quality is is less or equal to p_L (the chosen value) for each fraction defective p before inspection. The AOQL plans for inspection by attributes are extensively tabulated – see (Dodge and Romig, 1998).

The corresponding AOQL plans for inspection by variables were introduced in (Klůfa, 1997). These plans minimize the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables

$$I_m = N - (N - n) \cdot L(\bar{p}; n, k) \tag{4}$$

under the condition (3), i.e. the same condition as used Dodge and Romig. Under assumptions from introduction, for the given parameters p_L, N, $\bar{p}$ we must determine the acceptance plan (n, k) for inspection by variables, minimizing the function I_m in formula (4) under the condition (3). Solution of this problem is in the paper **(**Klůfa**,** 1997). Calculation of these plans when the non-central t distribution is used for the operating characteristic is considerably difficult. This problem was solved in (Klůfa, 2014). Similar problems are solved in (Chen, 2016), (Yazdi and Fallahnezhad, 2017), (Balamurali, Azam and Aslam, 2016), (Kaspříková and Klůfa, 2011), (Wang, 2016), (Klůfa, 2013), (Yazdi, Nezhad, Shishebori et al., 2016), (Aslam, Azam and Jun, 2016), (Wang and Lo, 2016).

2 Comparisson of the AOQL plans

The sample size in acceptance sampling plans for inspection by variables is always less than the sample size in acceptance sampling plans for inspection by attributes. On the other hand the cost of inspection of one item by variables c_m^* is usually greater than the cost of inspection of the same item by attributes c_s^*, i.e. usually is

$$c_m = \frac{c_m^*}{c_m^*} > 1. \tag{5}$$

For the comparison of the AOQL single sampling plans for inspection by variables with the corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view we must estimate the parameter c_m defined by formula (5) in each real situation. Let us define

$$s = 100\left(1 - \frac{I_m}{I_s} c_m\right) = 100\left(1 - \frac{I_m c_m^*}{I_s c_s^*}\right) \tag{6}$$

Since $I_m c_m^*$ is the mean cost of inspection by variables and $I_m c_s^*$ is the mean cost of inspection by attributes, the parameter s represents the percentage of savings in inspection cost when sampling plan for inspection by variables is used instead of the corresponding plan for inspection by attributes. Therefore, if c_m is statistically estimated and

$$s > 0,$$

then the AOQL plans for inspection by variables are more economical than the corresponding Dodge-Romig AOQL plans for inspection by attributes, if c_m is statistically estimated and

$$s < 0,$$

then Dodge-Romig AOQL plans for inspection by attributes are more economical than the corresponding AOQL plans for inspection by variables.

The percentage of savings in inspection cost when sampling plan for inspection by variables is used instead of the corresponding plan for inspection by attributes s depends on input characteristics of the acceptance sampling p_L (the average outgoing quality limit), N (the lot size), $\bar{p}$ (the process average fraction defective) and c_m (the fraction of the cost of inspection of one item by variables and the cost of inspection of the same item by attributes), i.e. s is a function of four variables

$$s = s(p_L, N, \bar{p}, c_m). \tag{7}$$

Some values of the function (7) for chosen parameters p_L, N, $\bar{p}$ and c_m are in Table 1, Table 2, Table 3 and Table 4.

Example 1: The average outgoing quality limit was chosen 0.0025, i.e. $p_L = 0.0025$. The process average fraction defective is $\bar{p} = 0.0015$ and $c_m = 1.4$ (the cost of inspection of one item by variables is higher by 40% than the cost of inspection of one item by attributes). For inspection a lot with 1000 items (the lot size N=1000) we shall look for the AOQL plan for inspection by attributes and the AOQL plan for inspection by variables. Furthermore we shall compare these plans from economical point of view.

The AOQL plan for inspection by attributes we find in (Dodge and Romig, 1998). For given parameters $p_L = 0.0025, \overline{p} = 0.0015, N = 1000$ we have

$$n = 130,\ c = 0.$$

For these parameters we shall compute the AOQL plan for inspection by variables - see Klůfa (2014)

$$n = 76,\ k = 2.5518.$$

For $c_m = 1.4$ the percentage of savings in inspection cost when sampling plan for inspection by variables is used instead of the corresponding plan for inspection by attributes is (see Table 1)

$$s = 44.$$

From this result it follows that under the same protection of consumer the AOQL plan for inspection by variables (76, 2.5518) is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (130,0). Since $s = 44$, it can be expected approximately **44% saving of the inspection cost**.

Tab. 1: Values of the function s for $p_L = 0.0025, \mathrm{c_m} = 1.4$

$\overline{p} \backslash N$	100	500	1000	4000	10000	50000	100000
0.000125	58	68	69	73	80	79	85
0.000250	52	62	65	73	76	79	80
0.000375	47	58	62	73	73	80	78
0.000500	44	55	59	73	72	83	78
0.000625	40	52	57	68	71	75	76
0.000750	37	50	55	65	69	76	75
0.000875	34	47	54	62	69	76	76
0.001000	31	44	51	61	69	78	76
0.001125	30	41	50	59	65	71	72
0.001250	27	40	48	58	62	69	72
0.001375	26	37	47	55	61	68	71
0.001500	23	34	44	54	59	68	71
0.001625	22	33	43	52	57	65	66
0.001750	20	30	40	50	54	64	66
0.001875	17	29	38	48	51	62	65
0.002000	16	26	36	45	48	61	64
0.002125	15	24	30	38	44	52	57
0.002250	13	22	27	36	40	48	51
0.002375	12	20	24	31	36	41	45
0.002500	10	17	22	27	30	33	34

Source: Own calculation

Tab. 2: Values of the function s for $p_L = 0.0025, \mathrm{c_m} = 1.8$

$\overline{p}\backslash N$	100	500	1000	4000	10000	50000	100000
0.000125	46	59	60	66	75	73	80
0.000250	39	51	55	66	69	73	75
0.000375	32	46	51	66	66	75	71
0.000500	28	42	48	66	64	78	71
0.000625	23	39	44	59	62	68	69
0.000750	19	35	42	55	60	69	68
0.000875	15	32	41	51	60	69	69
0.001000	12	28	37	50	60	71	69
0.001125	10	24	35	48	55	62	64
0.001250	6	23	33	46	51	60	64
0.001375	5	19	32	42	50	59	62
0.001500	1	15	28	41	48	59	62
0.001625	-1	14	26	39	44	55	57
0.001750	-3	10	23	35	41	53	57
0.001875	-6	8	21	33	37	51	55
0.002000	-8	5	17	30	33	50	53
0.002125	-10	3	10	21	28	39	44
0.002250	-12	-1	6	17	23	33	37
0.002375	-13	-3	3	12	17	24	30
0.002500	-15	-6	-1	6	10	14	15

Source: Own calculation

Tab. 3: Values of the function s for $p_L = 0.0075, \mathrm{c_m} = 1.4$

$\overline{p}\backslash N$	100	500	1000	4000	10000	50000	100000
0.000375	45	50	52	66	64	71	69
0.000750	38	45	51	59	59	64	65
0.001125	33	43	52	55	59	61	68
0.001500	29	41	52	54	62	64	73
0.001875	24	40	47	54	55	61	61
0.002250	22	38	44	54	54	61	59
0.002625	19	37	40	54	54	64	61
0.003000	16	37	37	55	55	66	64
0.003375	13	33	34	45	51	55	58
0.003750	12	30	33	44	50	57	55
0.004125	9	26	30	43	50	58	55
0.004500	8	23	29	41	48	59	58
0.004875	6	20	26	37	43	50	52
0.005250	3	17	24	34	40	48	51
0.005625	2	15	22	33	37	48	51
0.006000	1	13	20	30	34	48	51
0.006375	-1	10	17	26	29	38	41
0.006750	-2	8	15	22	24	34	36
0.007125	-4	5	10	17	19	29	30
0.007500	-5	2	8	13	12	20	20

Source: Own calculation

Tab. 4: Values of the function s for $p_L = 0.0075, c_m = 1.6$

$\bar{p}\backslash N$	100	500	1000	4000	10000	50000	100000
0.000375	38	42	46	62	58	66	65
0.000750	30	38	44	54	54	58	60
0.001125	23	34	46	49	54	55	63
0.001500	18	33	46	47	57	58	70
0.001875	14	31	39	47	49	55	55
0.002250	10	30	36	47	47	55	54
0.002625	7	28	31	47	47	58	55
0.003000	4	28	28	49	49	62	58
0.003375	1	23	25	38	44	49	52
0.003750	-1	20	23	36	42	50	49
0.004125	-4	15	20	34	42	52	49
0.004500	-6	12	18	33	41	54	52
0.004875	-7	9	15	28	34	42	46
0.005250	-10	6	14	25	31	41	44
0.005625	-12	2	10	23	28	41	44
0.006000	-14	1	9	20	25	41	44
0.006375	-15	-2	6	15	18	30	33
0.006750	-17	-6	2	10	14	25	26
0.007125	-18	-9	-2	6	7	18	20
0.007500	-20	-12	-6	1	-1	9	9

Source: Own calculation

Remark 1: For given parameters $p_L = 0.0075, \bar{p} = 0.006, N = 100, c_m = 1.6$ is (see Table 4) $s = -14.$ In this case Dodge-Romig AOQL plan for inspection by attributes is more economical than the corresponding AOQL plan for inspection by variables.

Conclusion

From the results of numerical investigations it follows (see also Table 1, Table 2, Table 3 and Table 4):

- ❖ when lot size N increases, then saving of the inspection cost s increases (using the AOQL plan for inspection by variables instead of the corresponding plan for inspection by attributes).
- ❖ when the process average fraction defective $\bar{p}$ increases, then saving of the inspection cost s decreases (using the AOQL plan for inspection by variables instead of the corresponding plan for inspection by attributes).

- ❖ when the fraction of the cost of inspection of one item by variables and the cost of inspection of one item by attributes c_m increases, then saving of the inspection cost s decreases (using the AOQL plan for inspection by variables instead of the corresponding plan for inspection by attributes).

In many situations under the same protection of consumer the AOQL plans for inspection by variables are more economical than the corresponding Dodge-Romig AOQL attribute sampling plans – see e.g. Example 1.

Acknowledgment

The paper was processed with contribution of long term support of scientific work on Faculty of Informatics and Statistics, University of Economics, Prague (IP 400040).

References

Aslam, M., Azam, M., Jun, CH. (2016). Multiple dependent state repetitive group sampling plan for Burr XII distribution. *Quality Engineering*, 28(2), 231-237.

Balamurali, S., Azam, M., Aslam, M. (2016). Attribute-variable Inspection Policy for Lots Using Resampling Based on EWMA. *Communications in Statistics-Simulation and Computation*, 45(8), 3014-3035.

Chen, C. H. (2016). Optimum process mean setting with specified average outgoing quality limit protection for variable single sampling plan. *Journal of Statistics & Manegement Systems*, 19(3), 499-508.

Dodge, H. F., Romig, H. G. (1998) *Sampling Inspection Tables: Single and Double Sampling.* John Wiley.

Kaspříková, N., Klůfa, J. (2011). *Calculation of LTPD Single Sampling Plans for Inspection by Variables and its Software Implementation.* In: *5th International Days of Statistics and Economics*, 266-276. ISBN 978-80-86175-86-7.

Kaspříková, N., Klůfa, J. (2015). AOQL Sampling Plans for Inspection by Variables and Attributes Versus the Plans for Inspection by Attributes. *Quality Technology and Quantitative Management*, 12(2), 133-142.

Klůfa, J. (1997). Dodge-Romig AOQL single sampling plans for inspection by variables. *Statistical Papers*, 38, 111-119.

Klůfa, J. (2013). *Comparison of Entrance Examination in Mathematics.* In: *10th International Conference on Efficiency and Responsibility in Education*, 270-275.

Klůfa, J. (2014). *Dodge-Romig AOQL Sampling Plans for Inspection by Variables - Optimal Solution.* In: *17th International Conference on Enterprise and the Competitive Environment, Procedia Economics and Finance*, 12, 302-308.

Klůfa, J. (2015). Economic aspects of the LTPD single sampling inspection plans. *Agricultural Economics-Zemedelska Ekonomika*, 61(7), 326-331.

Klůfa, J. (2016). Economic efficiency of the AOQL single sampling plans for the inspection by variables. *Agricultural Economics-Zemedelska Ekonomika*, 62(12), 550-555.

Wang, F. K. (2016). A Single Sampling Plan Based on Exponentially Weighted Moving Average Model for Linear Profiles. *Quality and Reliability Engineering International*, 32(5), 1795-1805.

Wang, F. K., Lo, S. C. (2016). Single Mixed Sampling Plan Based on Yield Index for Linear Profiles. *Quality and Reliability Engineering International*, 32(4), 1535-1543.

Yazdi, A. A., Nezhad, M. S. F., Shishebori, D. et al. (2016). Development of an Optimal Design for Conforming Run Length Sampling Methods in the Presence of Inspection Errors. *Journal of Testing and Evaluation*, 44(5), 1885-1891.

Yazdi, A. A., Fallahnezhad, M. S. (2017). Comparison between count of cumulative conforming sampling plans and Dodge-Romig single sampling plan. *Communications in Statistics-Theory and Methods*, 46(1), 189-199.

Contact

Jindřich Klůfa
University of Economics in Prague, Department of Mathematics
W. Churchill Sq. 4, 13067 Prague 3
klufa@vse.cz