EXPLORING RELATIONSHIP BETWEEN DEVELOPED AND DEVELOPING FINANCIAL MARKETS BY WAVELETS

Milan Bašta – Peter Molnár – Karel Helman

Abstract

We use wavelet analysis to study the relationship between developed and developing financial markets. Developed markets are represented by the U.S. stock market and developing markets are represented by the Brazilian, Hong Kong and Indian stock market. Wavelet analysis enables us to explore the relationship such as the strength of the comovement, lead/lag relationship, etc. on a scale by scale basis (and possibly also as a function of time) and thus provides us with a more detailed insight into the nature of the relationship compared to traditional analysis. We find that the relationship between the developed and developing markets is time varying. During the first part of our sample, from 2007 to 2012, there is a strong comovement on long scales between the U.S. stock market and stock markets in Brazil, Hong Kong and India. Moreover, the U.S. stock market is following the developing markets, and particularly the Brazilian stock market. However, this relationship disappears after 2012. The results of the analysis could be important, among others, for investment strategy planning, risk management, portfolio allocation and understanding the transmission mechanisms among different markets.

Key words: developed markets, developing markets, time series, wavelets

JEL Code: C40, E32, G15

Introduction

We study the relationship between developed and developing financial markets by exploring the comovement of the representative stock market indices. The developed market is represented by an American stock market index, whereas Brazilian, Hong Kong's and Indian stock market indices are used as representatives of the developing markets.

Relationships in the financial market can potentially vary over time (i.e. can be different during a financial crisis and after it) and can exhibit different characteristics at different scales (i.e. can be different in the short, medium and long run). Since wavelet analysis is capable of analyzing relationships between time series as a function of time and scale, we will make use of it in this paper.

Exploring the relationship between the stock markets has been studied extensively[1] and a detailed review of the literature is beyond the scope of this paper. We therefore briefly discuss only those papers that use wavelet techniques similar to ours. Gallegati (2005) studies the comovement between developed stock markets, represented by stock market in the U.S. and EU, and stock markets in MENA countries (Egypt, Israel, Jordan, Morocco and Turkey). Graham and Nikkinen (2011) investigate the relationship between the Finnish stock market and stock markets of other countries and find comovement with developed markets across all frequencies, but comovement with emerging markets only at low frequencies. Aloui and Hkiri (2014) examine the short term and long term dependencies between stock market returns for the Gulf Cooperation Council Countries (Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and the United Arab Emirates) during the period 2005–2010 and find that changes in the patterns of comovement are frequent. Sharkasi et al. (2006) use wavelets to study the reaction of stock markets to crashes and find that developed markets respond to crashes differently to emerging ones in the sense that emerging markets may take up to two months to recover while major markets take less than a month. Gallegati (2012) studies the comovement between the U.S. stock market and the stock markets in Canada, Japan, UK, France, Germany, Italy, Brazil, and China before and after the financial crisis, during the period from 2003 to 2008. For the U.S.-Brazil and U.S.-China pairs he finds significant changes in comovement, which he interprets as a contagion. His results indicate that developed markets are in general closely integrated and moving together, whereas the comovement between developed and developing markets changes over time.

We therefore study the comovement between the U.S. stock market and stock markets in Brazil, Hong Kong and India and indeed find that this relationship changes over time. We find a strong comovement between developed and developing markets at long time scales during the period 2007 – 2012. Moreover, developing stock markets lead the U.S. stock market in this period. However, we find no such relationship in the period 2013 – 2017.

The paper is organized as follows. Section 1 gives an introduction to the continuous wavelet transform, wavelet coherence and further related measures. Section 2 introduces the data used in the analysis. Section 3 provides, interprets and discusses the results. Conclusions are given in the Conclusion.

1 Continuous wavelet transform and wavelet coherence

[1] See e.g. Lyócsa, Výrost and Baumöhl (2012), Baumöhl and Lyócsa (2014), Výrost, Lyócsa and Baumöhl (2015).

Based on Torrence and Compo (1998) and Grinsted et al. (2004) we assume the Morlet wavelet function defined as

$$\psi_0(\eta) = \pi^{-1/4} \exp(i\omega_0 \eta) \exp\left(-\frac{1}{2}\eta^2\right) \quad (1)$$

where the dimensionless frequency ω_0 is set to 6 (Grinsted et al., 2004) and where η is a dimensionless time. Further, assuming an input time series $\{X_t: t = 0, ..., N-1\}$ of length N, the continuous wavelet transform of $\{X_t\}$ at time t and at scale $s > 0$ is defined as (Grinsted et al., 2004)

$$W_{t,s}^X = \frac{1}{\sqrt{s}} \sum_{k=0}^{N-1} X_k \psi_0^* \left(\frac{k-t}{s}\right), \quad (2)$$

where $\psi_0{}^*(.)$ is the complex conjugate of $\psi_0(.)$.

Given two input time series $\{X_t: t = 0, ..., N-1\}$ and $\{Y_t: t = 0, ..., N-1\}$, the wavelet coherence between $\{X_t\}$ and $\{Y_t\}$ at time t and scale s is defined as (Grinsted et al., 2004)

$$R_{t,s}^2 = \frac{\left|S\left(\frac{1}{s}W_{t,s}^{XY}\right)\right|^2}{S\left(\frac{1}{s}\left|W_{t,s}^X\right|^2\right)S\left(\frac{1}{s}\left|W_{t,s}^Y\right|^2\right)}, \quad (3)$$

where

$$W_{t,s}^{XY} = W_{t,s}^X W_{t,s}^{Y*}, \quad (4)$$

where the * symbol in the superscript stands for complex conjugation. The S operator in Equation 3 is a smoothing operator defined as

$$S(W_{t,s}) = S_{scale}(S_{time}(W_{t,s})), \quad (5)$$

where

$$S_{time}(W_{t,s})\Big|_s = \left(W_{t,s} * c_1^{-\frac{t^2}{2s^2}}\right)\Bigg|_s, \quad (6)$$

$$S_{scale}(W_{t,s})\Big|_t = \left(W_{t,s} * c_2\Pi(0.6s)\right)\Big|_t, \quad (7)$$

where c_1 and c_2 are normalization constants and Π is the rectangle function, the * operator denoting convolution in Equation 6 and Equation 7.

Grinsted et al. (2004) define the local phase as

$$\theta_{t,s} = \operatorname{atan2}\left(\operatorname{Im}(W_{t,s}^{XY}), \operatorname{Re}(W_{t,s}^{XY})\right), \qquad (8)$$

where $\operatorname{Im}(W_{t,s}^{XY})$ and $\operatorname{Re}(W_{t,s}^{XY})$ denote the imaginary and real part of $W_{t,s}^{XY}$ and where atan2(. , .) is the two-argument arctangent function.

1.1 Strength of the relationship and time delay as a function of time and scale

Wavelet coherence $R^2_{t,s}$ can be considered as a local (in time and scale) squared correlation between $\{X_t\}$ and $\{Y_t\}$ and can thus be used to assess the strength of the relationship between time series at time t and scale s. Wavelet coherence close to zero (blue colour in figures below) suggests a weak relationship, whereas wavelet coherence close to one (red colour in figures below) suggests a strong relationship.

To assess the significance of wavelet coherence, we follow the procedure outlined by Grinsted et al. (2004). Namely, we generate several realizations of two independent stationary AR(1) processes, the parameters of the processes being estimated making use of $\{X_t\}$ and $\{Y_t\}$. For each generated pair we calculate the wavelet coherence employing Equation 3. Consequently, the wavelet coherence for the original time series $\{X_t\}$ and $\{Y_t\}$ is assumed to be statistically significant (at significance level 0.05) if its value if above the 95th percentile of the simulated distribution. Statistically significant regions will be depicted by bold contours in figures below.

Local phase $\theta_{t,s}$ informs us about the phase (and consequently about the time delay) between the time series. In figures below, $\theta_{t,s}$ will be evaluated only for those times and scales with the highest values of $R^2_{t,s}$, and will be depicted by arrows. If $\theta_{t,s}$ is 0, the time series are in phase (no time delay is present) and the arrow will be pointing to the right. If $\theta_{t,s}$ is $\pi/2$, then $\{X_t\}$ leads $\{Y_t\}$ by $\pi/2$ (in phase) and the arrow will be pointing up. If $\theta_{t,s}$ is $-\pi/2$, then $\{Y_t\}$ leads $\{X_t\}$ by $\pi/2$ (in phase) and the arrow will be pointing down, etc.

Since $R^2_{t,s}$ and $\theta_{t,s}$ are functions of time and scale, it is possible to explore the strength of the relationship, the phase and time delay as a function of time and scale.

The figures presented below have been produced making use the R software (R Core Team, 2014) and the biwavelet R package (Gouhier, 2015).

2 Data

We use four stock market indices. Specifically, the American S&P 500 index represents a stock market index of a developed market, whereas the Brazilian Bovespa index, the Hong Kong's Hang Seng index and the India Nifty 50 index are representatives of stock market indices of developing markets. The data have been downloaded from finance.yahoo.com and cover the period from 17 September, 2007 through 1 April, 2017. Data prior to 17 September, 2007 are not used in the analysis since the data for the India Nifty 50 index prior to 17 September, 2007 are not available on finance.yahoo.com and we want the same period to be covered by all the four indices.

To study the relationship between the developed and developing markets, the following pairs of indices are explored: 1.) S&P 500 and Bovespa, 2.) S&P 500 and Hang Seng and 3.) S&P 500 and Nifty 50. The natural logarithm of the indices is plotted in Figure 1 (S&P 500 and Bovespa), Figure 2 (S&P 500 and Hang Seng) and Figure 3 (S&P 500 and Nifty 50). When studying the relationship within the pair, log returns of the indices are used, being defined as

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \log(P_t) - \log(P_{t-1}), \qquad (9)$$

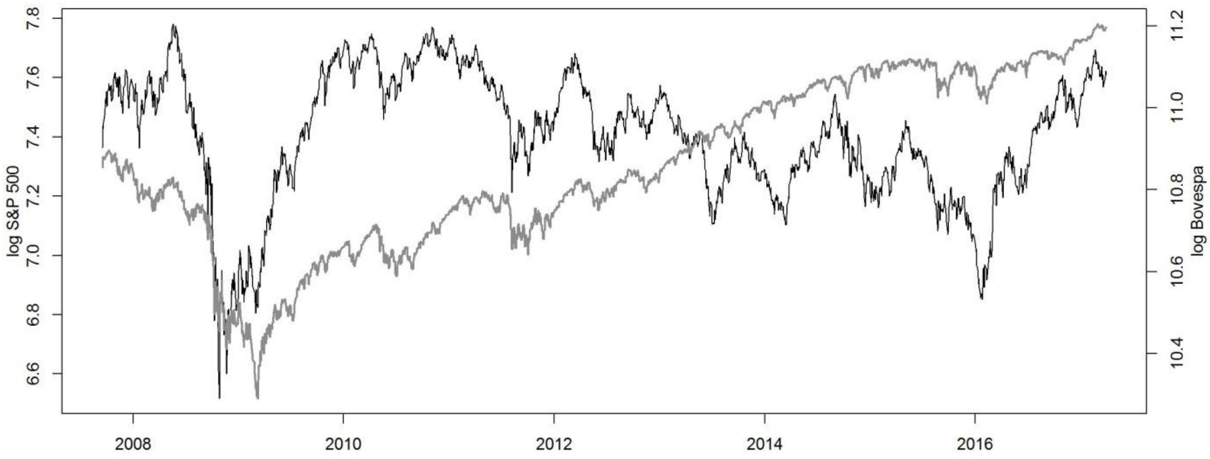
where P_t is the index value at time t.

We are interested in several characteristics of the relationships: the strength of the relationship, lead/lag patterns in the relationship, how the relationship changes over time and how it depends on scale. As mentioned above, these characteristics can be easily captured by exploring $R^2_{t,s}$ and $\theta_{t,s}$ as a function of time t and scale s. It should be stressed that an apparent leading behaviour (up to several hours) could possibly be observed between the indices, being the result of different trading hours in the U.S., Brazil, Hong Kong and India. Consequently, we will not be interested in lead/lag patterns which are equal or less than 1 day.

3 Results

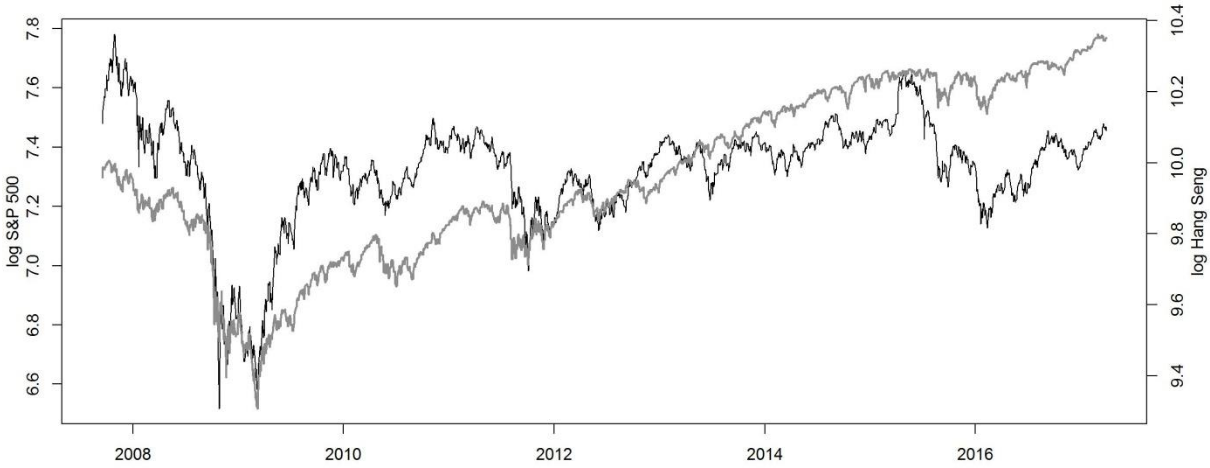
The results for the three different pairs are presented in Figure 4 (S&P 500 and Bovespa), Figure 5 (S&P 500 and Hang Seng) and Figure 6 (S&P 500 and Nifty 50). Time is depicted on the horizontal axis, period P, given as $P = 1.03s$ (Grinsted et al., 2004), is plotted on the vertical axis. The value of $R^2_{t,s}$ is captured in colour (see the colour bar on the right). The value of $\theta_{t,s}$ is captured by the direction of the arrows as explained in Section 1.1.

Fig. 1: Log S&P 500 (gray, left axis) and log Bovespa (black, right axis)



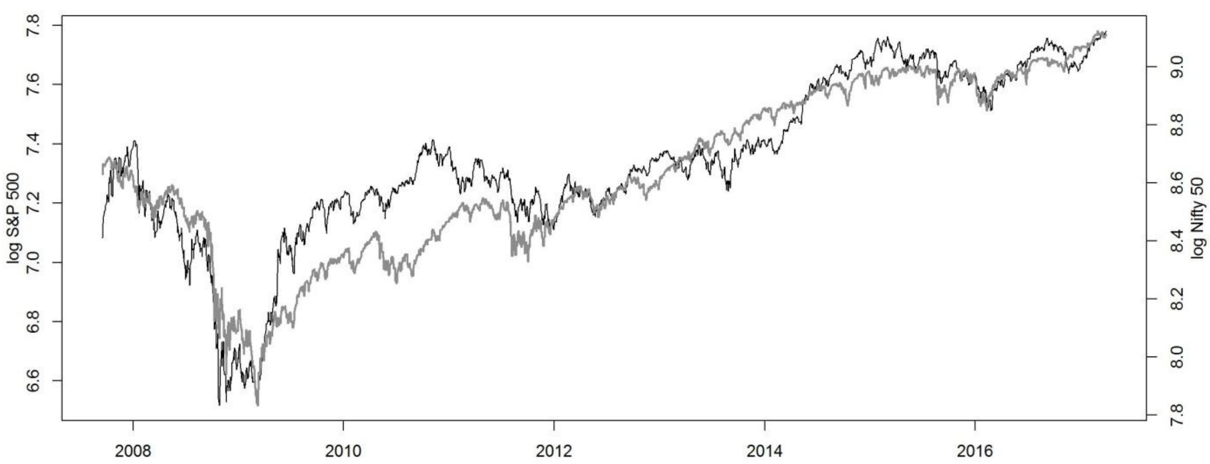
Source: Own construction

Fig. 2: Log S&P 500 (gray, left axis) and log Hang Seng (black, right axis)



Source: Own construction

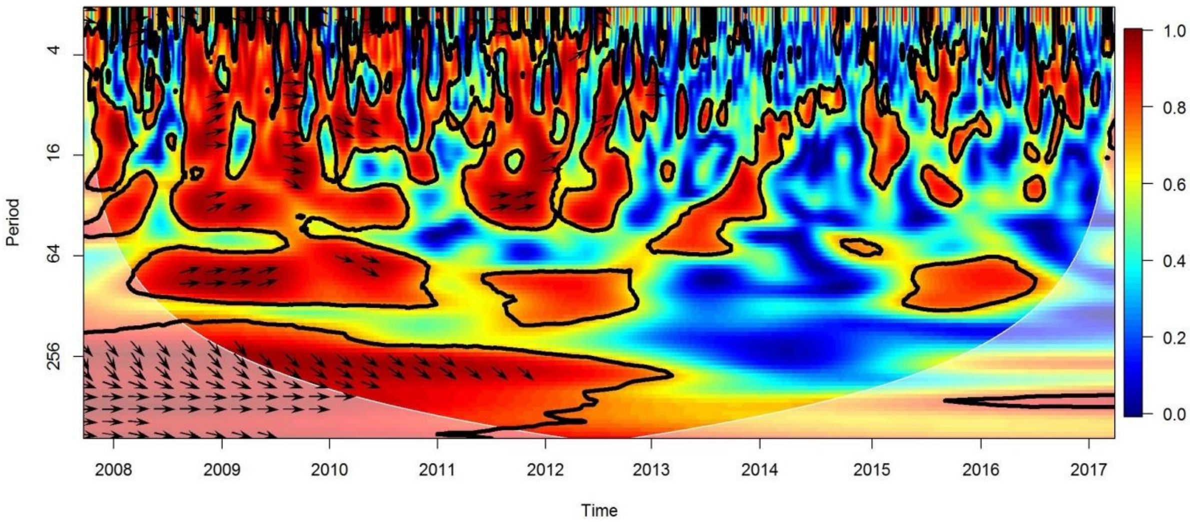
Fig. 3: Log S&P 500 (gray, left axis) and log Nifty 50 (black, right axis)



Source: Own construction

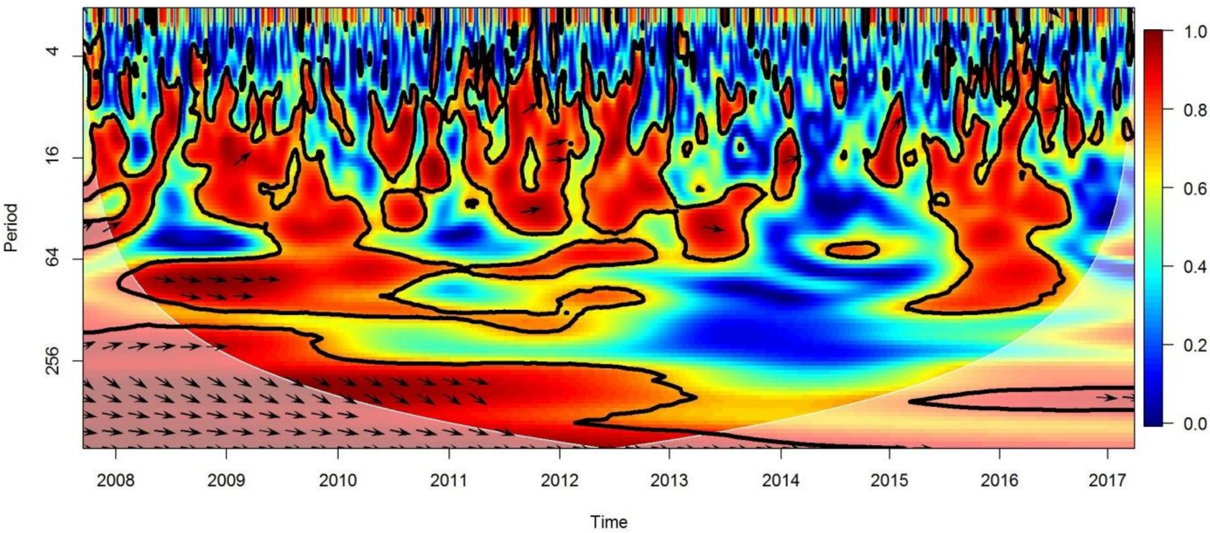
Since the wavelet coefficients are obtained by convolving the input time series with the Morlet wavelet (see Equation 2), boundary effects arise which influence the coefficients as well as the wavelet coherence at the boundaries. The cone of influence is such an area in the plots (whose edge is depicted by the white contours near the left and right boundary) where the wavelet coefficients are influenced to a "larger" degree by the boundary effects. Results in the cone of influence have to be interpreted with caution.

Fig. 4: Wavelet coherence between log returns of S&P 500 and log returns of Bovespa



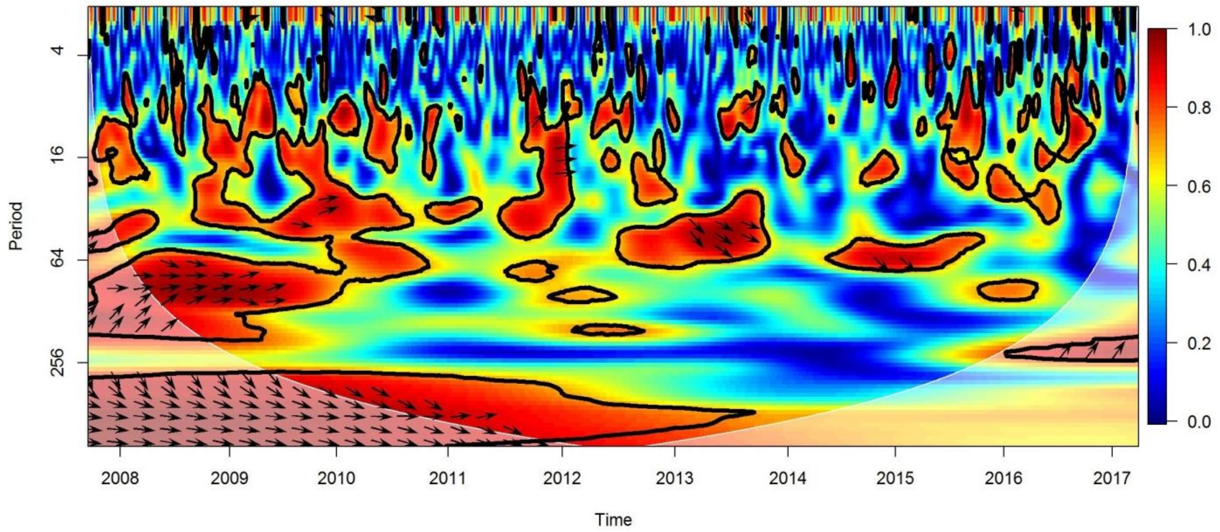
Source: Own construction

Fig. 5: Wavelet coherence between log returns of S&P 500 and log returns of Hang Seng



Source: Own construction

Fig. 6: Wavelet coherence between log returns of S&P 500 and log returns of Nifty 50



Source: Own construction

We can clearly observe that the relationship between S&P 500 and Bovespa was rather strong at all scales in the period from 2007 through 2012 but got much weaker at all scales after 2012. It is also interesting to note that Bovespa was leading S&P 500 at large scales round year 2009. This can be discerned also from Figure 1, where the long-run bottom of the Bovespa index occurs prior to the long-run bottom of the S&P 500 index.

The relationship between S&P 500 and Hang Seng seems to be rather stable over time (with a slight weakening of the relationship after 2012) and can be characterized by a rather strong relationship at medium and large scales and not so strong relationship at short scales. Analogously to the first pair (S&P 500 and Bovespa), Hang Seng seems to slightly lead S&P 500 round the year 2009.

Concerning the relationship between S&P 500 and Nifty 50, it is very similar to the relationship between S&P 500 and Hang Seng in the sense that a stronger relationship occurs at medium and large scales compared to short scales with a slight weakening of the relationship happening after 2012.

In general, the comovement between the U.S. stock market and either of the considered developing markets is quite similar. The conclusion that there is much more comovement at longer time scales than on shorter time scales is similar to the finding of Graham and Nikkinen (2011), who find that the comovement of the Finnish market and emerging markets is confined to long-term fluctuations. The reason why we observe much stronger comovement during the first half of our sample is the financial crisis. During this time period, all the stock indices plunged due to the financial crisis and rose afterwards. After the year 2012, there were no major events that would force the stock indices to move together, and the indices were evolving in dependence on the economic development of their respective countries.

Conclusion

We have investigated the relationship between developed stock markets, represented by the United States, and developing stock markets, represented by Brazil, Hong Kong and India. We have utilized wavelet techniques which allow us to capture the time-varying aspect of the relationship as well as to analyse this relationship at various time scales.

We find that the relationship between developed and developing markets is indeed time-varying, and this result holds for all the considered time scales. On short time scales, the relationship is rather weak. On the contrary, developed and developing markets exhibit strong common movements on longer time scale in the period 2007 – 2012. However, this relationship disappears afterwards. The likely reason, why developed and developing markets were moving together from 2007 to 2012 and not afterwards, is the financial crisis in the first part of our sample. Financial crisis hits all the stock markets, causing them plunge and grow subsequently. On the contrary, in recent years, stock markets have been influenced mostly by the economic development of their respective countries.

Interestingly, developing stock markets were leading the U.S. stock market during the period 2007 – 2012, and this lead-lag relationship was the strongest in the case of the Brazilian market. However, this effect disappeared after the year 2012.

References

Aloui, C., & Hkiri, B. (2014). Co-movements of GCC emerging stock markets: New evidence from wavelet coherence analysis. *Economic Modelling*, *36*, 421-431.

Baumöhl, E., & Lyócsa, Š. (2014). Volatility and dynamic conditional correlations of worldwide emerging and frontier markets. *Economic Modelling*, *38*, 175-183.

Gallegati, M. (2005). A wavelet analysis of MENA stock markets. *Department of Economics Università Politecnica delle Marche, Italia*.

Gallegati, M. (2012). A wavelet-based approach to test for financial market contagion. *Computational Statistics & Data Analysis*, *56*(11), 3491-3497.

Gouhier, T. (2015). biwavelet: Conduct univariate and bivariate wavelet analyses. Version: 0.17.11.

Graham, M., & Nikkinen, J. (2011). Co-movement of the Finnish and international stock markets: a wavelet analysis. *The European Journal of Finance*, *17*(5-6), 409-425.

Grinsted, A., Moore, J. C., & Jevrejeva, S. (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear processes in geophysics*, *11*(5/6), 561-566.

Lyócsa, Š., Výrost, T., & Baumöhl, E. (2012). Stock market networks: The dynamic conditional correlation approach. *Physica A: Statistical Mechanics and its Applications*, *391*(16), 4147-4158.

R Core Team (2014). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL: http://www.R-project.org/.

Sharkasi, A., Crane, M., Ruskin, H. J., & Matos, J. A. (2006). The reaction of stock markets to crashes and events: A comparison study between emerging and mature markets using wavelet transforms. *Physica A: Statistical Mechanics and its Applications*, *368*(2), 511-521.

Torrence, C., & Compo, G. P. (1998). A practical guide to wavelet analysis. *Bulletin of the American Meteorological society*, *79*(1), 61-78.

Výrost, T., Lyócsa, Š., & Baumöhl, E. (2015). Granger causality stock market networks: Temporal proximity and preferential attachment. *Physica A: Statistical Mechanics and its Applications*, *427*, 262-276.

Contact

Milan Bašta
Department of Statistics and Probability, University of Economics in Prague
nám. W. Churchilla 1938/4, 130 67 Praha 3
milan.basta@vse.cz

Peter Molnár
UiS Business School, University of Stavanger
4036 Stavanger, Norway
peto.molnar@gmail.com

Karel Helman
Department of Statistics and Probability, University of Economics in Prague
nám. W. Churchilla 1938/4, 130 67 Praha 3
helmank@vse.cz