THE USE OF L-MOMENTS IN TESTING NORMALITY

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Abstract

Normality is one of the basic assumptions in statistics. Testing normality is a fundamental step before using statistical methods based on assumption of normality. Many approaches to testing normality have been proposed (parametric or nonparametric). If the assumption is not fulfilled, the methods could lead to inaccurate inferences. Parametric tests are based on estimates of the theoretical moments, disadvantage of these tests is the sensitivity of sample moments on outliers. Sample moments are inappropriate estimates of the theoretical moments in that situation and also in case of the small samples. There is a high probability of incorrect rejection of the hypothesis that sample comes from normal distribution. It is recommended to use more robust measures instead of sample moments. L-moments are used in this paper instead of sample moments. The aim of this paper is to compare the accuracy of tests of normality based on L-moments with classical parametric and nonparametric tests like Jarque-Bera or Kolmogorov-Smirnov. The comparison is performed for various situations (different sizes of a sample, various number of outliers in a sample and various distance of outliers from other observations). Tests are compared in terms of type I error. All computations are performed in R.

Key words: tests of normality, tau34-squared test, L-moments, Monte Carlo simulation

JEL Code: C12, C15, C63

Introduction

Testing normality is the one of the fundamental step in statistics. A lot of models or methods are based on an assumption of normality. Parametric and nonparametric tests can be used for testing normality of a dependent variable or residuals in a regression analysis etc. Nonparametric tests are often based on a comparison of the empirical cumulative distribution function with the theoretical cumulative distribution function of the Normal distribution. The most used tests are the Kolmogorov-Smirnov which provides inaccurate inferences in some cases, so its modification the Lilliefors-corrected Kolmogorov-Smirnov test is preferred. Parametric tests are generally based on a comparison between the sample moments and the theoretical moments of the normal distribution (skewness or kurtosis). The Jarque-Bera, the

D´Agostino and the Shapiro-Wilk tests are the most commonly used and well-known parametric tests. Disadvantages of parametric tests based on the sample moments are sensitivity of the sample moments to outliers and limitations of the value of the sample skewness and the sample kurtosis. It was derived that the sample skewness is lower or equal $\sqrt{n}$ and the sample kurtosis is lower or equal $n+3$. Outliers in a sample lead to bias estimates of the theoretical moments which could cause type I error (rejection null hypothesis when it is true). The type I error increases with a growing number of outliers in a sample. It is recommended to use more robust measures instead of the sample moments or robust tests of normality.

1 Test of normality based on L-moments

1.1 L-moments

L-moments have been defined by (Hosking, 1990). They are robust alternative to describe distribution of a random variable. L-moments are linear combinations of differences of the expectations of ordered statistics. Assume a real-valued random variable X, the L-moments are generally defined by formula

$$\lambda_r = r^{-1}\sum_{j=0}^{r-1}(-1)^j\binom{r-1}{j}E\left(X_{r-j:r}\right). \tag{1}$$

where $X_{r-j:r}$ is $r-j$ ordered statistic of a sample size r and $r \geq 1$. Measures of skewness and kurtosis based on L-moments are L-skewness $\left(\tau_3 = \frac{\lambda_3}{\lambda_2}\right)$ and L-kurtosis $\left(\tau_4 = \frac{\lambda_4}{\lambda_2}\right)$. There are no two distribution with the same L-moments. L-moments have lower variance and are robust measures of distribution than the conventional moments. Comparing of them was made for example in (Hosking, 1992) The L-moments are estimated by sample L-moments. The sample L-moments are based on an ordered random sample and they are derived as

$$\hat{\lambda}_r = \frac{1}{r}\binom{n}{r}^{-1}\sum_{i=1}^{n}\left[\sum_{j=0}^{r-j}(-1)^j\binom{r-1}{j}\binom{i-1}{r-1-j}\binom{n-i}{j}\right]x_{i:n}. \tag{2}$$

1.2 Tau34-squared test

The tau34-squared test was presented in (Coble & Harri, 2011). In this study, Coble and Harri compared power of this test with others tests of normality. Null hypothesis of the test assumes that a random variable has Normal distribution. L-skewness of Normal distribution is equal 0 and L-kurtosis is equal to 0.1226. This test is based on the normalized transformation of L-skewness and L-kurtosis. The sample L-skewness and the sample L-kurtosis have an approximal Normal distribution. The test statistic is

$$\tau_{3,4}^2 = Z(\tau_3)^2 + Z(\tau_4)^2, \tag{3}$$

where

$$Z(\tau_3) = \frac{\hat{\tau}_3}{\sqrt{0.1866/n \ + 0.8/n^2}} \tag{4}$$

and

$$Z(\tau_4) = \frac{\hat{\tau}_4}{\sqrt{0.0883/n \ + 0.68/n^2 + 4.9n^3}}. \tag{5}$$

$Z(\tau_3)$ and $Z(\tau_4)$ have approximated standard Normal distribution. $\tau_{3,4}^2$ has approximated Chi-squared distribution with two degrees of freedom. The study in (Coble & Harri, 2011) proved that the tau34-squared test has good power compared with other normality tests.

2 Tests of normality

We compare the tau34-squared test with other tests of normality (parametric and nonparametric). We consider parametric tests the Jarque-Bera, the Robust Jarque-Bera, the D´Agostino, the Shapiro-Wilk tests and nonparametric tests the Kolmogorov-Smirnov and the Lilliefors-corrected Kolmogorov-Smirnov tests.

The Kolmogorov-Smirnov test is based on a comparison of the theoretical and the empirical distribution function. Its disadvantage is a requirement of knowledge of the theoretical distribution. The Lilliefors-corrected test is based on a simulation of distribution of its test statistic for selected theoretical distribution (parameters are estimated from a random sample).

The Shapiro-Wilk test compares an ordered random sample with a Normal distribution quantiles for probability $P_i = i/n$. Test statistic of the D´Agostino test is linear combination of the normalized transformation of the sample skewness and the sample kurtosis. The test statistic has approximately chi-squared distribution with two degrees of freedom. The Jarque-Bera test is based on normalized transformation of the sample skewness and the sample kurtosis too. A robust modification of the Jarque-Bera test is based on the robust estimate sample estimates.

More information about these tests can be found for example in (Jarque & Bera, 1980), (D´Agostino, Belanger, & D´Agostino Jr., 1990), (Shapiro & Wilk, 1965), (Gel & Gastwirth, 2008) etc.

3 Simulation

3.1 Algorithm of simulation

Study in this article is based on Monte Carlo simulations with 30,000 iterations (it is sufficient for this simulation). The aim is to compare the accuracy of normality tests based on L-moments with classical parametric and nonparametric tests mentioned in Section 2. The accuracy of tests is measured for different sizes of samples (10, 20, 50 and 100 observations), various number of outliers (depends on size of samples) and various distance of outliers from the expected value of the standard Normal distribution. A criterion of accuracy is a ratio of correctly evaluated tests which can be easily transformed into a type I error. There is no possibility to test the power of tests which is the most useful criterion in hypothesis testing. A random sample is drawn from the standard Normal distribution. In this study, an outlier is defined as an observation with absolute value larger then 3σ. The algorithm of simulation can be described by following steps:

1. Generate a random sample from the standard Normal distribution without outliers.
2. Generate outliers with suitable distance from the expected value.
3. Replace the largest observations in the sample by generated outliers.
4. Use considered tests of normality and save results of the tests.
5. Repeat steps 1-4.
6. Estimate the ratio of correctly evaluated tests for each test of normality.

The distance of outliers in step 2 is generated from the Uniform distribution where the parameters are bounds of a considered interval (for example 3 and 3.5). All computations are performed in R. We use packages (Asquith, 2016) and (Gavrilov & Pusev, 2014).

3.2 Results

Table 1 contains results of simulations for samples with 10 observations. Column *out* represents number of outliers in a generated sample and column *distance* represents distance of outliers compared to the expected value of the distribution. Others columns contain ratios of correctly evaluated tests (ratio of tests with not rejected null hypothesis against all tests). Acronyms in header of Table 1 are *tau34* is for the tau34-squared test, *jb* is for the Jarque-Bera test, *rjb* is for the Robust Jarque-Bera test, *sw* is for the Shapiro-Wilk test, *da* is for the D´Agostino test, *ks* is for the Kolmogorov-Smirnov test and *lks* is an acronym for the Lilliefors-corrected tests. This header is used in all tables below.

The ratio of correctly evaluated tests decreases with increasing distance of outliers compared to the expected value for all tests. The Kolmogorov-Smirnov test has ratio around

0.95 and 0.96 for all considered situations which is suspicious and results of this test should not be compared with others (we observe this in all tables in the study, for all sizes of samples). Development of ratio with increasing number of outliers has interesting development for all parametric tests excluding the Shapiro-Wilk test. The ratio increases and all these tests have rejected none of 30,000 null hypotheses in situation where three outliers exclude the tau34-squared test but its values are close to others. It is caused by high sensitivity of the sample moments and the sample L-moments (which seem little more robust than the sample moments).

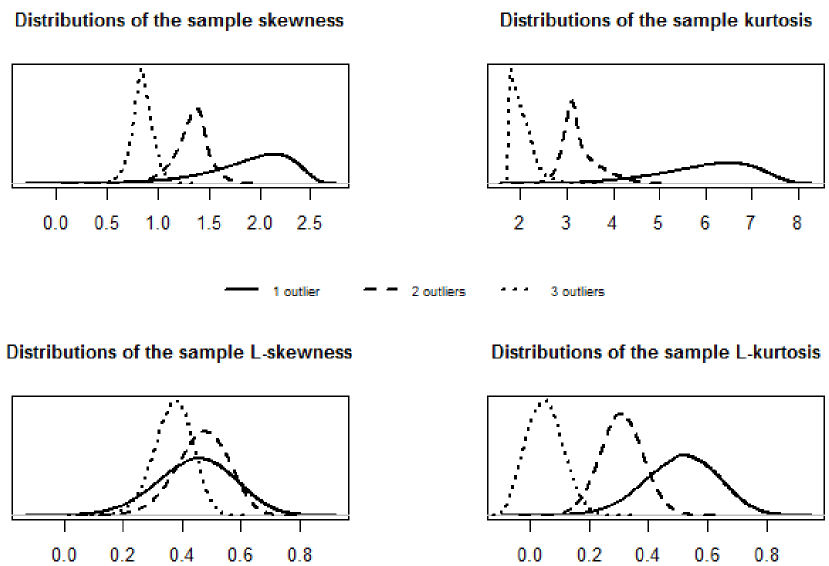
Tab. 1: Ratio of correctly evaluated tests of normality, 10 observations

out	*distance*	*tau34*	*jb*	*rjb*	*sw*	*da*	*ks*	*lks*
1	3-3.5	0.59	0.51	0.52	0.67	0.53	0.95	0.79
1	3.5-4	0.43	0.32	0.34	0.50	0.34	0.95	0.67
1	4-4.5	0.29	0.17	0.19	0.34	0.19	0.95	0.54
1	4.5-5	0.17	0.08	0.09	0.20	0.09	0.95	0.41
2	3-3.5	0.79	0.90	0.81	0.49	0.81	0.96	0.54
2	3.5-4	0.67	0.82	0.69	0.31	0.69	0.96	0.37
2	4-4.5	0.55	0.73	0.56	0.18	0.56	0.96	0.24
2	4.5-5	0.42	0.62	0.42	0.09	0.42	0.96	0.15
3	3-3.5	0.92	1.00	1.00	0.34	1.00	0.96	0.45
3	3.5-4	0.89	1.00	1.00	0.20	1.00	0.96	0.31
3	4-4.5	0.84	1.00	1.00	0.10	1.00	0.96	0.19
3	4.5-5	0.79	1.00	1.00	0.05	1.00	0.96	0.11

Source: Own calculations

Figure 1 shows estimated distribution of the sample moments and the sample L-moments. It is obvious that the expected value of estimates converges to the value of theoretical moments. Values in Table 1 shows that the nonparametric Lilliefors-corrected test has smallest type I error (the highest ratio of correctly evaluated tests). The best parametric test is the Shapiro-Wilk test.

Fig. 1: Distribution of sample moments and sample L-moments



Source: Own calculations

Table 2 contains results of simulations for samples with 20 observations. We can observe the same problem like in previous case, the ratio of correctly evaluated parametric tests increases with increasing number of outliers. The ratio of the Jarque-Bera, the Robust Jarque-Bera and the D´Agostino test increases since there are 3 outliers in a sample. The "limit" for the tau34-squared test is observed when a sample has 4 outliers. It seems that the tau34-squared test correctly evaluates the highest ratio from all parametric tests. But the Lilliefors-corrected test has the best results, like in previous case with 10 observations.

Tab. 2: Ratio of correctly evaluated tests of normality, 20 observations

out	*distance*	*tau34*	*jb*	*rjb*	*sw*	*da*	*ks*	*lks*
1	3-3.5	0.76	0.51	0.55	0.71	0.60	0.95	0.90
1	3.5-4	0.59	0.23	0.28	0.45	0.34	0.95	0.82
1	4-4.5	0.41	0.07	0.10	0.22	0.14	0.95	0.71
1	4.5-5	0.25	0.01	0.03	0.08	0.04	0.95	0.56
2	3-3.5	0.49	0.46	0.41	0.37	0.42	0.95	0.62
2	3.5-4	0.26	0.20	0.17	0.14	0.18	0.95	0.41
2	4-4.5	0.11	0.06	0.05	0.04	0.05	0.95	0.22
2	4.5-5	0.03	0.01	0.01	0.01	0.01	0.95	0.10
3	3-3.5	0.47	0.60	0.48	0.18	0.44	0.95	0.35
3	3.5-4	0.25	0.35	0.23	0.04	0.20	0.95	0.16
3	4-4.5	0.10	0.15	0.08	0.01	0.07	0.95	0.06
3	4.5-5	0.03	0.05	0.02	0.00	0.01	0.95	0.02
4	3-3.5	0.60	0.83	0.72	0.08	0.63	0.95	0.19
4	3.5-4	0.38	0.66	0.49	0.01	0.40	0.95	0.07
4	4-4.5	0.21	0.46	0.28	0.00	0.20	0.95	0.02
4	4.5-5	0.09	0.27	0.13	0.00	0.08	0.95	0.00
5	3-3.5	0.68	0.98	0.97	0.03	0.94	0.96	0.12
5	3.5-4	0.51	0.95	0.93	0.00	0.87	0.96	0.03
5	4-4.5	0.34	0.89	0.84	0.00	0.74	0.96	0.01
5	4.5-5	0.20	0.81	0.72	0.00	0.59	0.96	0.00

Source: Own calculations

Next case is a sample with 50 observations, results are shown in Table 3. The so-called breakdown point of parametric tests can be found in samples with six (the Jarque-Bera, the Robust Jarque-Bera) or eight outliers (the tau34-squared and the D´Agostino). But there is no situation where tests reject none null hypotheses. The Table 3 shows that the tau34-squared test provides the best results compared to others parametric tests. The ratio of other parametric tests decreases more rapidly than the ratio of the tau34-squared test. For example, if we consider a sample with one outlier with distance 4.5-5 from the expected value, the tau34-squared test correctly evaluates 60 % of all samples and other parametric tests only few percent. It seems that the tau34-squared test is the best option of all considered parametric tests. But the Lilliefors-corrected test provides the best results compared to all considered parametric and nonparametric tests. Its ratio of correctly evaluated tests decreases much more slowly than tau34-squared test´s ratio with increasing number of outliers and increasing distance of them.

Tab. 3: Ratio of correctly evaluated tests of normality, 50 observations

out	*distance*	*tau34*	*jb*	*rjb*	*sw*	*da*	*ks*	*lks*
1	3-3.5	0.92	0.71	0.76	0.86	0.83	0.95	0.95
1	3.5-4	0.85	0.30	0.39	0.56	0.54	0.95	0.93
1	4-4.5	0.75	0.05	0.11	0.21	0.22	0.95	0.90
1	4.5-5	0.60	0.00	0.01	0.03	0.05	0.95	0.84
2	3-3.5	0.72	0.38	0.40	0.45	0.49	0.95	0.88
2	3.5-4	0.45	0.06	0.08	0.10	0.14	0.95	0.75
2	4-4.5	0.21	0.00	0.01	0.01	0.02	0.95	0.54
2	4.5-5	0.07	0.00	0.00	0.00	0.00	0.95	0.30
3	3-3.5	0.43	0.23	0.21	0.16	0.26	0.95	0.69
3	3.5-4	0.14	0.02	0.02	0.01	0.04	0.95	0.40
3	4-4.5	0.03	0.00	0.00	0.00	0.00	0.95	0.15
3	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.04
4	3-3.5	0.23	0.17	0.13	0.04	0.14	0.95	0.43
4	3.5-4	0.04	0.01	0.01	0.00	0.01	0.95	0.14
4	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.02
4	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
5	3-3.5	0.13	0.15	0.09	0.01	0.09	0.95	0.21
5	3.5-4	0.01	0.01	0.00	0.00	0.00	0.95	0.04
5	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
5	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
6	3-3.5	0.09	0.15	0.08	0.00	0.07	0.95	0.09
6	3.5-4	0.01	0.01	0.00	0.00	0.00	0.95	0.01
6	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
6	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
7	3-3.5	0.07	0.16	0.08	0.00	0.06	0.95	0.04
7	3.5-4	0.01	0.02	0.00	0.00	0.00	0.95	0.00
7	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
7	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
8	3-3.5	0.07	0.19	0.10	0.00	0.06	0.95	0.01
8	3.5-4	0.00	0.02	0.01	0.00	0.00	0.95	0.00
8	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
8	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
9	3-3.5	0.08	0.22	0.14	0.00	0.07	0.95	0.00
9	3.5-4	0.01	0.03	0.01	0.00	0.00	0.95	0.00
9	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
9	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
10	3-3.5	0.09	0.26	0.22	0.00	0.10	0.00	0.00
10	3.5-4	0.01	0.04	0.03	0.00	0.01	0.00	0.00
10	4-4.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	4.5-5	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Source: Own calculations

Table 4 contains results of simulations for samples with 100 observations. The so-called breakdown point of parametric tests is not identified here. It seems that a sample needs more than 10 % of outliers to bias an estimate of the sample moments or the sample L-moments. All considered tests provide good results in case with one outlier with distance 3-3.5 from the expected value. The ratio decreases rapidly with increasing distance for all parametric tests excluding the tau34-squared test. The ratios of the Jarque-Bera, the Robust Jarque-Bera, the Shapiro-Wilk and the D´Agostino tests have an exponential decrease. These tests of normality are not useful when a sample has three and more outliers (all located on one side, on the left or on the right from the expected value of distribution). Even if the tau34-squared test provides the best results of parametric tests, the nonparametric Lilliefors-corrected test provides better results. Its ratio decreases slowly with increasing number of outliers and outliers distance.

Tab. 4: Ratio of correctly evaluated tests of normality, 100 observations

out	*distance*	*tau34*	*jb*	*rjb*	*sw*	*da*	*ks*	*lks*
1	3-3.5	0.96	0.91	0.92	0.95	0.94	0.95	0.95
1	3.5-4	0.93	0.52	0.63	0.75	0.77	0.95	0.95
1	4-4.5	0.89	0.10	0.20	0.29	0.43	0.96	0.94
1	4.5-5	0.83	0.00	0.02	0.03	0.13	0.95	0.92
2	3-3.5	0.89	0.58	0.62	0.68	0.73	0.95	0.94
2	3.5-4	0.76	0.09	0.13	0.14	0.27	0.95	0.90
2	4-4.5	0.54	0.00	0.01	0.00	0.03	0.96	0.82
2	4.5-5	0.31	0.00	0.00	0.00	0.00	0.95	0.66
3	3-3.5	0.73	0.29	0.31	0.27	0.42	0.96	0.89
3	3.5-4	0.41	0.01	0.02	0.01	0.06	0.96	0.75
3	4-4.5	0.14	0.00	0.00	0.00	0.00	0.95	0.49
3	4.5-5	0.03	0.00	0.00	0.00	0.00	0.95	0.21
4	3-3.5	0.49	0.14	0.14	0.07	0.21	0.95	0.79
4	3.5-4	0.14	0.00	0.00	0.00	0.01	0.95	0.48
4	4-4.5	0.02	0.00	0.00	0.00	0.00	0.95	0.17
4	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.03
5	3-3.5	0.27	0.07	0.06	0.01	0.09	0.95	0.60
5	3.5-4	0.03	0.00	0.00	0.00	0.00	0.95	0.22
5	4-4.5	0.00	0.00	0.00	0.00	0.00	0.96	0.03
5	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
6	3-3.5	0.12	0.04	0.03	0.00	0.04	0.96	0.39
6	3.5-4	0.01	0.00	0.00	0.00	0.00	0.95	0.07
6	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
6	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
7	3-3.5	0.05	0.02	0.01	0.00	0.02	0.96	0.21
7	3.5-4	0.00	0.00	0.00	0.00	0.00	0.95	0.02
7	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
7	4.5-5	0.00	0.00	0.00	0.00	0.00	0.96	0.00
8	3-3.5	0.02	0.01	0.01	0.00	0.01	0.95	0.10
8	3.5-4	0.00	0.00	0.00	0.00	0.00	0.95	0.00
8	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
8	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
9	3-3.5	0.01	0.01	0.00	0.00	0.00	0.95	0.04
9	3.5-4	0.00	0.00	0.00	0.00	0.00	0.95	0.00
9	4-4.5	0.00	0.00	0.00	0.00	0.00	0.96	0.00
9	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
10	3-3.5	0.00	0.01	0.00	0.00	0.00	0.95	0.01
10	3.5-4	0.00	0.00	0.00	0.00	0.00	0.95	0.00
10	4-4.5	0.00	0.00	0.00	0.00	0.00	0.95	0.00
10	4.5-5	0.00	0.00	0.00	0.00	0.00	0.95	0.00

Source: Own calculations

Conclusion

Tau34-squared test mentioned in (Coble & Harri, 2011) provides good results in situation when a random sample generated from the standard normal distribution has several outliers. It is the best from all considered parametric tests except in case of a sample with 10 observations. Shapiro-Wilk test has the higher ratio of correctly evaluated tests than tau34-squared test in that case. In other cases, tau34-squared test is more accurate than another considered parametric test. If we compare this test with nonparametric Lilliefors-corrected Kolmogorov-Smirnov test then the nonparametric test should be used instead of tau34-squared test. This simulation shows that the sample L-moments are more robust than the sample moments which are used in test statistics in parametric tests.

Parametric tests tau34-squared, Jarque-Bera, Robust Jarque-Bera and D´Agostino for sample with 10 observations reject less null hypotheses with increasing number of outliers which is caused by sensitivity of the sample moments and L-moments to number of outliers. Even the sample L-moments are more robust than the sample moments.

The ratio of correctly evaluated tests of Kolmogorov-Smirnov test is equal for all considered cases. It seems that this test is not affected by presence of outliers in a sample. We assume that this is unexpected behaviour and needs more detailed analysis.

Although the tau34-squared test provides good results compared to parametric test, it is not sufficient due to better results of Lilliefors-corrected Kolmogorov-Smirnov test. Parametric test based on TL-moments (robust version of the L-moments) could provide better results than the nonparametric. But this test has not yet been created. It could be a task for a future research.

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