ECONOMIC EFFICIENCY EVALUATION OF LTPD VARIABLES SAMPLING PLANS BASED ON EWMA STATISTIC

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Abstract

The LTPD plans for acceptance sampling which minimize the mean inspection cost per lot of the process average quality when the remainder of the rejected lots is inspected were originally designed by Dodge and Romig for the inspection by attributes. The sampling plans for the inspection by variables were then proposed and it has been shown that such plans are in many situations in practice more economical than the corresponding attributes sampling plans. It has been shown that it is possible to achieve further savings when EWMA based statistic is used. The design of the recently introduced EWMA statistic based rectifying LTPD variable sampling plans minimizing the mean inspection cost per lot of process average quality is recalled. The measure for assessing the comparative economic efficiency of the plans, which may be used for getting a guidance for selecting the most appropriate type of the sampling plan to be used, is proposed and an evaluation of the economic characteristics of the plans is shown.

Key words: inspection cost, variable sampling plan, optimization

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Introduction

Sampling inspection is one of the quality control tools used in industry to help keep the quality of the products at satisfactory level while at the same time having the cost in control. When using acceptance sampling inspection, a decision on whether the lot of items is to be accepted or rejected is based on results of inspecting a sample of items from the lot.

There are several ways how acceptance sampling schemes may be classified. One such classification is according to whether an item is inspected by attributes, i.e. just classified as either good or defective (nonconforming) or by variables. Sampling plans for inspection by variables in many cases allow obtaining the same level of the protection as the corresponding sampling plans for the inspection by attributes while using a lower sample size. The basic notions of the variables sampling plans are addressed in (Jennett and Welch, 1939).

The LTPD sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed by Dodge and Romig for the inspection by attributes. Plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of rejected lots is inspected by attributes) were then proposed and it has been shown that these plans are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. The LTPD plans for the inspection by variables and attributes have been introduced in (Klůfa, 1994), using a sort of an approximate calculation of the plans. Exact plans, using the non-central t distribution in calculation of the operating characteristic, have been reported in (Klůfa, 2010) and implemented in (Kaspříková, 2012). The operating characteristics used for these plans are discussed in (Jennett and Welch, 1939) and (Johnson and Welch, 1940). It has been shown that these plans are in many situations superior to the original attribute sampling plans and similar results have been obtained for the AOQL plans – see the analysis is discussed in (Kaspříková and Klůfa, 2015). The recent development of acceptance sampling plans (see e.g. (Aslam et al., 2015)) includes designs of plans, which make use of the EWMA statistic. Using the EWMA statistic enables some savings in the cost of inspection as it allows using information on the quality of the previous lots.

With the aim of obtaining further savings in the cost of inspection, the new LTPD plans for the inspection by variables and attributes, designed to use the EWMA statistics, have been proposed in (Kaspříková, 2015). Using an economic model similar to the model used in (Kaspříková and Klůfa, 2015), a measure for assessing the comparative economic efficiency of the plans, which may be used for getting a guidance for selecting the most appropriate type of the sampling plan to be used, is proposed in this paper and an evaluation of the economic characteristics of the plans is shown. The structure of the paper is as follows: the LTPD plans for the inspection by attributes are recalled first, then the design of the recently introduced EWMA statistic based rectifying known sigma LTPD variable sampling plans minimizing the mean inspection cost per lot of the process average quality is recalled and finally the economic efficiency measure is introduced and the analysis of the economic performance of the plans is provided.

1 Attributes inspection plans

For the case that each inspected item is classified as either good or defective (the acceptance sampling by attributes), Dodge and Romig (1998) consider sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\bar{p}; n; c) \tag{1}$$

under the condition

$$L(p_t; n; c) \le \beta, \tag{2}$$

where *L(p, n, c)* is the operating characteristic (the probability of accepting a submitted lot with proportion defective *p* when using plan *(n, c)* for acceptance sampling), *N* is the number of items in the lot (the given parameter), $\bar{p}$ is the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, $P_t = 100 p_t$ is the lot tolerance per cent defective, denoted LTPD), *n* is the number of items in the sample *(n<N)*, *c* is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than *c*).

Condition (2) provides a guarantee for the consumer that lots of unsatisfactory quality level, with proportion defective p_t are going to be accepted only with specified probability β (consumer's risk). The value $\beta = 0.1$ is used for the consumer's risk in Dodge and Romig (1998).

2 Variables inspection plans

The LTPD plans for the inspection by variables and attributes have been designed in (Kaspříková, 2015) under the following assumptions:

The measurements of a single quality characteristic *X* are independent, identically distributed normal random variables with unknown parameter μ and known parameter σ^2. For the quality characteristic *X* there is given either an upper specification limit *U* (the item is defective if its measurement exceeds *U*), or a lower specification limit *L* (the item is defective if its measurement is smaller than *L*).

The inspection procedure is as follows:

Draw a random sample of *n* items from the lot and compute sample mean $\bar{x}$ and the statistic *T* at time *t* as $T_t = \lambda \bar{x} + (1 - \lambda) T_{t-1}$, where λ is a smoothing constant between 0 and 1.

The values of the smoothing constant over 0.5 give more weight to the sample in the current lot. Accept the lot if

$$\frac{U - T_t}{\sigma} \geq k, \text{ or } \frac{T_t - L}{\sigma} \geq k. \quad (3)$$

Suppose that c_s^* is the cost of inspection of one item by attributes and c_m^* is the cost of inspection of one item by variables and that the sample is inspected by variables. Then the inspection cost per lot with proportion defective p, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is $n \cdot c_m^*$ with probability $L(p,n,k)$ and $\left[n \cdot c_m^* + (N-n) \cdot c_s^*\right]$ with probability $\left[1 - L(p;n,k)\right]$.

The mean inspection cost per lot of process average quality $\bar{p}$ is therefore

$$C_{ms} = n \cdot c_m^* + (N-n) \cdot c_s^* \cdot \left[1 - L(\bar{p};n,k)\right]. \quad (4)$$

Dividing (4) by c_s^* gives the objective function

$$I_{ms} = n \cdot c_m + (N-n) \cdot \left[1 - L(\bar{p};n,k)\right], \quad (5)$$

where $c_m = c_m^* / c_s^*$ is the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes (this parameter has to be estimated in each real situation, it is usually $c_m > 1$). Note that both the function $I_{ms} = C_{ms} / c_s^*$ and the function C_{ms} have a minimum for the same acceptance plan (n,k). Therefore, we shall look for the acceptance plan (n,k) minimizing (5) instead of (4) under the condition

$$L(p_t;n;k) = \beta. \quad (6)$$

Setting the value of c_m to 1 can be used in situations, when both sample and the remainder of rejected lots are inspected by variables. Acceptance sampling by variables can thus be considered just as a special case of acceptance sampling by variables and attributes. Then instead of I_{ms} we may use notation I_m and setting c_m = 1 in (5) we obtain

$$I_m = N - (N-n) \cdot L(\bar{p};n,k), \quad (7)$$

i. e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables.

The task to be solved is to determine plan (n,k) minimizing (5) under the condition (6) for given values of input parameters N, c_m, p_t and $\bar{p}$.

The operating characteristic is (see e.g. (Aslam et al., 2015))

$$L(p;n,k) = \Phi((u_{1-p} - k)A), \tag{8}$$

where

$$A = \sqrt{\frac{n(2-\lambda)}{\lambda}}. \tag{9}$$

The function Φ in (8) is a standard normal distribution function and u_{1-p} is a quantile of order $1 - p$ (the unique root of the equation $\Phi(u) = 1 - p$).

When the operating characteristic is in the form (8), if $\beta = 0.1$ we get the solution of the equation (6) for k as

$$k = \frac{u_{0.9}}{\sqrt{\frac{(2-\lambda)n}{\lambda}}} + u_{1-p_t}, \tag{10}$$

where $u_{0.9}$ is a quantile of order 0.9 of the standard normal distribution. Inserting formula (10) for k into the I_{ms} function, we obtain a function of one variable n

$$I_{ms}(n) = n \cdot c_m + (N-n) \cdot \alpha(n), \tag{11}$$

where $\alpha(n) = 1 - L(\bar{p}, n, k(n))$ is the producer's risk (the probability of rejecting a lot of process average quality $\bar{p}$). So we search for the sample size n minimizing (11).

3 Economic efficiency measure and calculations of the plans

Let's calculate the LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes in an example below. The task will be solved using the operating characteristic given by (8). The resulting sampling plan will be evaluated with regard to economic characteristics and compared with the corresponding Dodge-Romig plan in (Dodge and Romig, 1998).

Example. We consider a lot of N = 1000 in the acceptance procedure. Lot tolerance proportion defective is given to be p_t =0.01 and the consumer's risk $\beta = 0.1$. It is known that average process quality is $\bar{p}$ = 0.001. A cost of inspecting an item by variables is known to be five times higher than the cost of inspecting an item by attributes, so parameter c_m equals 5.

Find LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes, using the operating characteristic given by (8) and the EWMA statistic with smoothing constant 0.92.

The plan can be calculated using the LTPDvar package (Kaspříková, 2012) for the R software (R Core Team, 2016). The solution based on the operating characteristic given by (8), is n = 16, k = 2.622053. For the values of input parameters given in our problem, there is plan (205, 0) for acceptance sampling by attributes in (Dodge and Romig, 1998). Let us compare plans (n=16, k = 2.622053) and (n=205, c=0) with regard to the economic efficiency.

For the comparison of the LTPD sampling plans for the inspection by variables (or by variables and attributes) and the corresponding Dodge-Romig LTPD sampling plans for inspection by attributes with regard to the economic point of view we will use parameter e, defined as

$$e = \frac{I_{ms}}{I_s} \cdot 100. \tag{12}$$

The expression $(1-e)$ then represents the percentage of savings in mean inspection cost per lot of process average quality when sampling plan for inspection by variables and attributes is used in place of the corresponding plan for inspection by attributes.

Let us denote plan for inspection by variables and attributes as (n_1, k) and the corresponding plan for inspection by attributes as (n_2, c) then it is

$$e = \frac{n_1 \cdot c_m + (N - n_1) \cdot \left[1 - L(\bar{p}, n_1, k)\right]}{N - (N - n_2) \cdot L(\bar{p}, n_2, c)} \cdot 100. \tag{13}$$

Since for (n_1, k) = (16, 2.622053) and (n_2, c) = (205, 0), we get

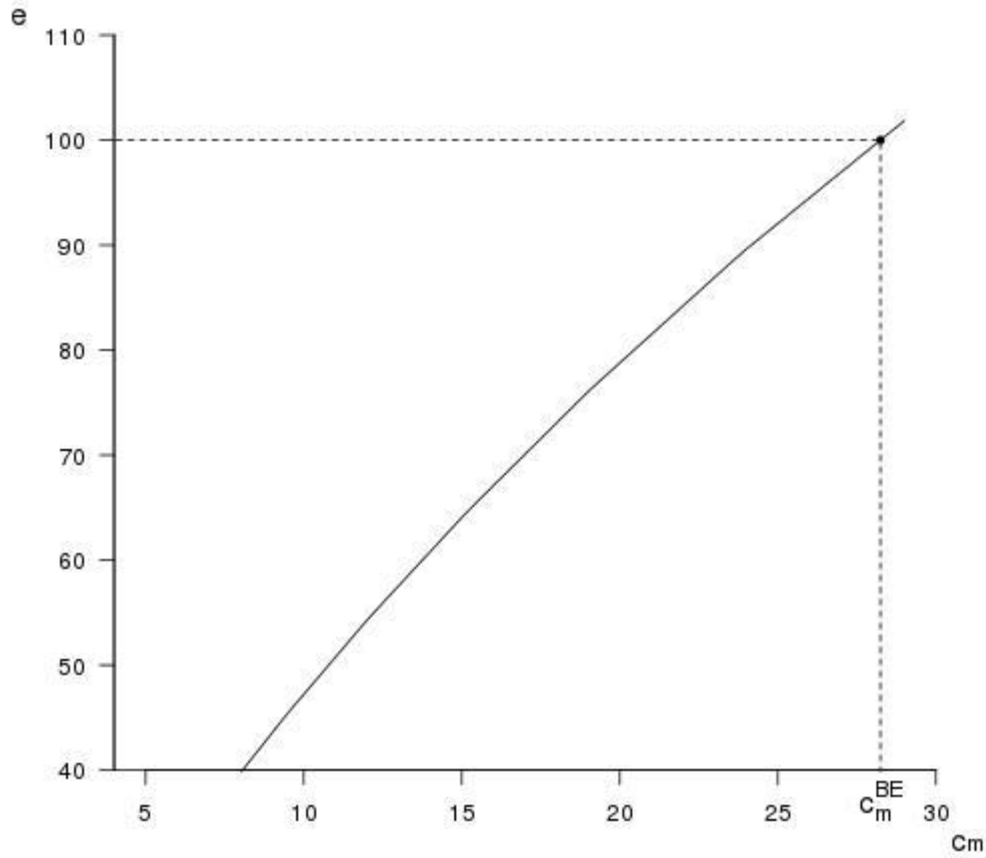
$$e = 26.9,$$

it can be expected that over 70% savings in inspection cost can be achieved using the plan EWMA based statistic plan (16, 2.622053) for the inspection by variables and attributes in place of the corresponding Dodge-Romig plan.

Now consider the situation, when c_m equals 10. Then the solution based on the operating characteristic given by (8) gives plan (13, 2.654403) and the value of the parameter e is 46.3. In cases when c_m is even higher, the resulting value of the parameter e is greater. For example the optimal plan obtained for c_m equal to 20 results in $e = 77.6$, and $c_m = 25$ leads to $e = 90.8$. When c_m is as high as 29, the parameter e is over 100.

We define c_m^{BE} to be such value of the parameter c_m for which the parameter e just equals 100. For the situation considered in the example above, the value of the parameter e in response to the c_m values is shown in Figure 1. The c_m^{BE} value in our case equals 28.2, as shown in Figure 1. Since the ratio of the unit cost of inspection by variables to the unit cost of inspection by attributes may not be known precisely in some cases in practice, the break-even value of this parameter may be used to guide the decision about which plan to use. In case that the c_m^{BE} value is high, then the variables sampling plans may seem preferable.

Fig. 1: Comparative economic efficiency assessment



Source: the figure has been produced by the author in R software

Conclusion

It has been shown that the LTPD plans for the inspection by variables and attributes minimizing the mean inspection cost per lot of process average quality, which were designed to use the EWMA statistics in the decision procedure, may bring significant savings in the inspection cost. A measure for assessing the comparative economic efficiency of the plans, which may be used for getting a guidance for selecting optimal sampling plan to be used, has been discussed.

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