CHANGE POINT TREND ANALYSIS OF GNI PER CAPITA IN SELECTED EUROPEAN COUNTRIES AND ISRAEL

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Abstract

The main goal of this study is to present and apply recently developed nonparametric change point detection and estimation techniques for confirming significant changes in short-term and long-term trends of living standards in selected European countries and Israel. We aim also to conduct the comparative change point analysis of living standards trends between OECD average, selected countries and Israel. The common index of living standards is the Gross National Income (GNI). Thus, GNI adjusted for purchasing power parity (PPP) per capita as well as the real GNI in local (national) currency converted into U.S. dollars (utilizing the Atlas Method) per capita were investigated for the considered countries over a period of last few decades. The change point analysis reveals a complex pattern of change. In particular, we found significant changes in short-term trends of living standards in most considered countries. The growth of living standards in considered countries has decreased last years in comparison with several previous years. However, we did not found significant changes in long-term trends of living standards. The results were supported also by traditional statistical methods.

Key words: Change point analysis, Nonparametric testing, Living standards trends, Atlas method

JEL Code: C14, C12, N10

Introduction

The standard of living includes many factors such as income, quality and availability of employment, class disparity, poverty rate, quality and affordability of housing, hours of work required to purchase necessities, gross domestic product, inflation rate etc. The living standards are closely related to quality of life and often used to compare geographic areas or distinct points in time. Most popular index for comparing living standards is income-measures ability of citizens to satisfy their needs and wants, hence living standards. Gross National

Income (GNI) is the value added of all resident producers, essentially income earned. Limitations here are the use of the exchange rate, it is hard to get an idea of real prices for utilities and food in the country as they differ across nations with the exchange rate. Therefore, it is common to use the Atlas Method to convert the real GNI in local (national) currency into U.S. dollars utilizing the Atlas conversion factor. The Atlas conversion factor for any year is the average of a country's exchange rate for that year and its exchange rates for the two preceding years, adjusted for the difference between the rate of inflation in the country and international inflation; the objective of the adjustment is to reduce any changes to the exchange rate caused by inflation. In addition, it is acceptable also to use the real GNI adjusted for purchasing power parity (PPP) - when exchange rates are equalized to the price of identical goods and services in different economies. This a measure of the real, inflation adjusted purchasing power of the people in a country. Since some countries have more population than others we use in our study two aforementioned indexes (GNI, Atlas Method and GNI, PPP) per capita that provides us with a much more accurate representation of the purchasing power of the average individual in the economy. The main goal of this study is to present and apply recently developed nonparametric change point detection and estimation techniques for confirming significant changes in short-term and long-term trends of living standards in selected European countries and Israel. The study also aims to conduct the comparative change point analysis of living standards trends between OECD average, selected countries and Israel. Utilizing presented change point techniques we studied annual growth tendencies of the considered indexes over a period of last few decades. The obtained results were supported also by traditional statistical methods. This paper proceeds as follows. In Section 1, we introduce the considered data and the change point detection and estimation methodology. In Section 2, a step by step application of the proposed change point techniques is outlines, as well as traditional statistical tools for two representative examples. This Section also presents final computational results of the proposed statistical analysis for all the data considered, accompanied by short comments. The final section concludes the paper.

1 Data and Methodology

1.1 Database

The following two indicators related to Economy and Growth topic of the World Bank open data were used in this study: GNI per capita, Atlas method (current US$) and GNI per capita, PPP (current international $). These indicators were considered for the selected countries over a period of 1980-2013.

1.2 Methodology: The Change Point Detection and Estimation Techniques

The issues of the so-called retrospective AMOC (at most one change) change point problem arise in experimental and mathematical sciences (epidemiology, quality control, etc.). Various biostatistical and engineering applications cause to consider different forms of the classical change point problem, i.e., detection and estimation of a single change point in the distributions of a sequence of independent random variables. These problems are directly connected to process capability and also important in education, economics, climatology and other fields (James et al., 1987; Gombay and Horvath, 1994, Gurevich and Vexler, 2005, 2010, Gurevich et al., 2011). Let $X_1, X_2, ..., X_n$ be a sample of independent one-dimensional observations. In the formal context of hypotheses testing, the detection change point problem is to test for

$$H_0\text{, the null: } X_1, X_2, ..., X_n \sim F_0 \quad \text{versus}$$

(1)

$$H_1\text{, the alternative: } X_i \sim F_1, X_j \sim F_2,\ i = 1,...,\nu - 1,\ j = \nu,...,n,$$

where F_0, F_1, F_2 are distribution functions that correspond to density functions f_0, f_1, f_2. The unknown parameter ν, $1 \le \nu \le n$ is called a change point. Following certain applied aspects of quality control studies, the literature assumes mostly the function F_1 is equal to F_0 (e.g., James et al., 1987; Gombay and Horvath, 1994). Gurevich and Vexler (2010) stated a general case when F_1 can be different from the null distribution F_0. In accordance with the statistical literature, the problem (1) has been investigated in parametric, semiparametric or nonparametric forms, depending on assumptions made on the distribution functions F_0, F_1 and F_2. In the parametric case of (1), the distribution functions F_0, F_1 and F_2 are assumed to have known forms that can contain certain unknown parameters (e.g., Gurevich, 2007; Gurevich and Vexler, 2010). In the semiparametric case of (1), assumptions regarding the distribution functions F_0, F_1 and F_2 are only used to evaluate values of some parameters involving in the test statistics construction (e.g., Gurevich, 2006; Gurevich and Vexler, 2010). Thus, in this case, the null distribution of test statistics does not depend of the assumptions related to the functions F_0, F_1 and F_2. In the nonparametric case of (1), the functions F_0, F_1, F_2 are assumed to be completely unknown (e.g., Bhattacharyya and Johnson, 1968; Sen and Srivastava, 1975; Pettitt, 1979; Gombay, 2001; Gurevich, 2009).

The parametric and semiparametric cases of the change point problem (1) have been dealt with extensively in both the theoretical and applied literature. However, we would like to avoid assumptions regarding distributions of considered observations. Therefore, in this study we concentrate on the nonparametric form of the problem (1).

When the problem (1) is stated nonparametrically in the context of one-dimensional independent random observations $X_1, X_2, ..., X_n$, there is no universal powerful methodology for this subject. In this case, the common components of change point detection policies have been proposed to be based on signs and/or ranks and/or U-statistics (e.g., Bhattacharyya and Johnson, 1968; Pettitt, 1979; Gombay, 2001; Gurevich, 2006, 2009; Gurevich and Vexler, 2010). In addition, relatively recently proposed empirical likelihood methodology was also successfully applied to the change point problem (1) (e.g., Vexler and Gurevich, 2010).

Bhattacharyya and Johnson (1968) considered the problem (1) with the unknown distributions $F_0(x) = F_1(x)$, $F_2(x) = F_1(x-\beta)$, where $\beta > 0$ is unknown and suggested rejecting H_0 for large values of the statistics

$$J_1 = \sum_{k=2}^{n} M_{k-1,n-k+1} \qquad (2)$$

or

$$J_2 = \sum_{k=2}^{n} U_{k-1,n-k+1}, \qquad (3)$$

where $M_{k-1,n-k+1}$ the number of observations among the last $n\text{-}k+1$ that exceeds the median of all n observations. That is, $M_{k-1,n-k+1}$ is the statistic of the median test for two samples of size k-1 and $n\text{-}k+1$, $U_{k-1,n-k+1} = \sum_{i=1}^{k-1} \sum_{j=k}^{n} I(X_i \le X_j)$, ($I(\cdot)$ is the indicator function) is the Mann-Whitney statistic for two samples of size k-1 and $n\text{-}k+1$. For the same problem, without any analysis, Sen and Srivastava (1975) suggested to reject H_0 for large values of the statistics

$$D_1 = \max_{2 \le k \le n} \left\{ \left[M_{k-1,n-k+1} - \frac{n-k+1}{2} \right] \Big/ \left[\frac{(k-1)(n-k+1)}{4(n-1)} \right]^{\frac{1}{2}} \right\}, \qquad (4)$$

$$D_2 = \max_{2 \le k \le n} \left\{ \left[U_{k-1,n-k+1} - \left((k-1)(n-k+1)\right)/2 \right] \Big/ \left[(k-1)(n-k+1)(n+1)/12 \right]^{\frac{1}{2}} \right\}. \qquad (5)$$

Considering the same problem, Pettitt (1979) proposed rejecting H_0 for large values of the statistic

$$K = 2\max_{2\le k\le n}\left\{U_{k-1,n-k+1} - (k-1)(n-k+1)/2\right\}. \tag{6}$$

Thus, the statistics K and D_2 have a similar structure. Gombay (2001) studied the asymptotic behavior of U-statistics. Gurevich (2009) analyzed the problem (1), when $F_0 = F_1$, F_2 are unknown and for all x, $F_2(x) \le F_1(x)$ (that is, after a possible change the observations are stochastically larger than before the change). The author modified the statistics J_2, D_2 and K (given by (3), (5) and (6), respectively), and suggested to reject H_0 for large values of the statistics

$$MK = \sum_{k=2}^{n}\left(U_{k-1,n-k+1} - (k-1)(n-k+1)/2\right), \tag{7}$$

$$MD = \sum_{k=2}^{n}\frac{U_{k-1,n-k+1} - (k-1)(n-k+1)/2}{\sqrt{(k-1)(n-k+1)(n+1)/12}}. \tag{8}$$

Since the consideration of the operator sum instead of the operator max in definition of statistics (7) and (8) allows evaluating an accurate asymptotic $(n \to \infty)$ behavior of these statistics under the null hypothesis H_0, these modifications were introduced to present the following asymptotic results

$$\lim_{n\to\infty} P_{H_0}\left(\frac{MK}{S1} > x\right) = 1 - \Phi(x), \tag{9}$$

$$\lim_{n\to\infty} P_{H_0}\left(\frac{MD}{S2} > x\right) = 1 - \Phi(x), \tag{10}$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution, $-\infty < x < \infty$,

$$S1 = \sqrt{\frac{n+1}{12}\left(\sum_{k=2}^{n}(k-1)(n-k+1) + 2\sum_{k=2}^{n}\sum_{r=1}^{n-k+1}(k-1)(n-k+1-r)\right)},$$

$$S2 = \sqrt{(n-1) + 2\sum_{k=2}^{n}\sum_{r=1}^{n-k+1}\sqrt{\frac{(k-1)(n-k+1-r)}{(n-k+1)(k-1+r)}}}.$$

Monte Carlo experiments presented in Gurevich (2009) showed that the rate of convergence of the asymptotic results (9), (10) is fast and these results provide accurate approximations for a level of significance of the tests based on the statistics (7), (8) for sample sizes commonly observed in practice. Moreover, a Broad Monte Carlo study presented in Gurevich and Vexler (2010) confirms that the nonparametric change point tests based on the statistics J_1, J_2, D_1, D_2, K, MK, MD are powerful and robust for various stochastically ordered alternatives. For $\nu \approx n/2$ it seems that the test based on the statistics K and MK have a higher power than that based on the statistics D_2 and MD, but for ν that is close to edges ($\nu \approx 1, \nu \approx n$) this property is reversed. When the two-sided statement (after a possible change the observations are stochastically larger or stochastically smaller than before the change) is assumed, the absolute values of the statistics (7), (8) as well as the absolute values under the operator max in the statistics (4)-(6) should be considered.

When H_0 is rejected, the issue of estimating the unknown parameter ν can be stated. While the literature on the change point relies mainly on testing the hypotheses (1), rather scant work has been done on the problem of estimating the change point ν. Gurevich and Raz (2010) considered several nonparametric change point estimators as the maximizing point of the statistics (5), (6). They conducted a broad Monte Carlo study comparing the behavior of these estimators and investigating their properties. Simulation results presented in Gurevich and Raz (2010) confirm the efficiency of the proposed estimators even for small and average sample sizes. However, the performance of the estimators based on the statistic (5) seems to be slightly better than that based on the statistic (6). Thus, for both of one-sided alternative (when one assumes the observations after a possible change are stochastically larger or smaller than before the change) and two-sided alternative the authors have recommended the following estimator based on the statistic (5):

$$\hat{\nu} = \arg\max_{2 \le k \le n} \left\{ \left| U_{k-1,n-k+1} - (k-1)(n-k+1)/2 \right| / \left[(k-1)(n-k+1)(n+1)/12 \right]^{\frac{1}{2}} \right\}. \qquad (11)$$

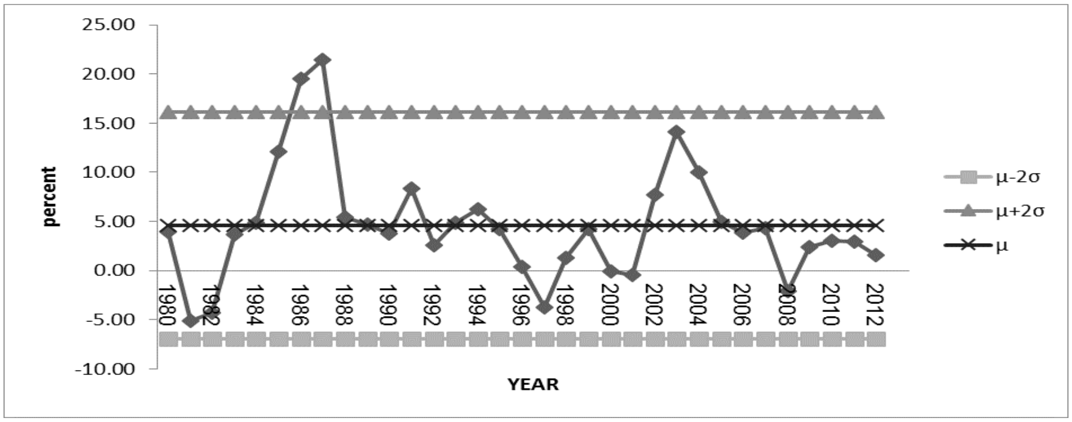
2 Application

For the sake of the presentation's simplicity we firstly introduce in this Section the detailed analysis of OECD’s annual growth of GNI per capita, Atlas method (current US$) between the years 1980-2012 (long term) and 2002-2012 (short term). After that, we present the final results (without all intermediate computational details) of the similar analysis for the rest of the data.

2.1 An analysis of OECD’s annual growth of GNI per capita, Atlas method

The following Figure 1 depicts the OECD’s annual growths (in percent) of GNI per capita, Atlas method (current US$) between the years 1980-2012, indicating their sample mean (μ) and standard deviation (σ). The change point tests described in Section 1.2 for a two-sided alternative based on statistics (7), (8) were applied. The following values of the test statistics were straightforwardly obtained: $MK / S1 = -0.87$, $MD / S2 = -1.35$, where $S1$, S2 are defined in equation (10). Therefore, by (9)-(10), two sided p-values of the tests based on statistics (7), (8) are equal to 0.38, 0.18, respectively. Since both of p-values are larger than 0.05, the conclusion is that there was not a significant change in distribution of the observations.

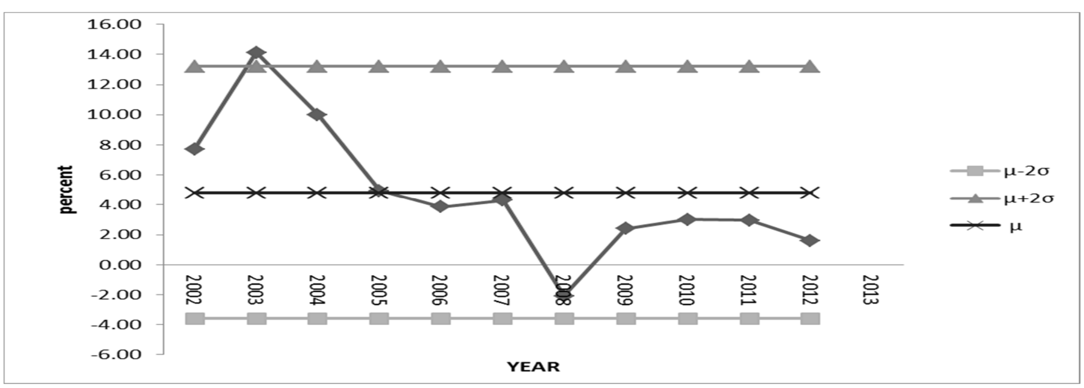
Fig. 1: OECD's annual growth of GNI per capita, Atlas method (1980-2012)



Source: own research.

Similarly, the OECD's annual growths (in percent) of GNI per capita, Atlas method (current US$) between the years 2002-2012 are presented in Figure 2. For these observations, the following values of the test statistics were straightforwardly obtained: $MK/S1 = -2.67$, $MD/S2 = -2.62$. By equations (9)-(10), one sided p-values of the tests based on statistics (7),

<u>Fig. 2: OECD's annual growth of GNI per capita, Atlas method (2002-2012)</u>



Source: own research.

(8) are equal to 0.0038, 0.0044, respectively. Consequently, since the both of p-values are less than 0.05 the conclusion is that there was a significant change in the distribution of the

observations. Straightforwardly, using the equation (11), $\hat{\nu} = 2008$ was obtained for the considered data. Applying the Wilcoxon test and the Student's t-test for two samples of observations (the considered OECD's annual growths in the years 2002-2007 and 2008-2012), the p-values of the tests are found to be less than 0.05. Thus, the final conclusion is that at 2008 there was a change in OECD's annual growths of GNI per capita and these annual growths after the change are significantly smaller than before the change. The following Tables 1, 2 present final results (without all computational details) of the analysis outlined above for annual growths (in percent) of the GNI per capita, Atlas method and GNI per capita, PPP for several considered countries over indicated time periods.

Country	Long term (1980-2012)				Short term (2002-2012)			
	Year of CP	Total mean	Mean before CP	Mean after CP	Year of CP	Total mean	Mean before CP	Mean after CP
Israel	-	5.93	-	-	2008	6.43	7.12	5.60
OECD members	-	4.58	-	-	2008	4.81	7.49	1.58
Turkey	-	6.13	-	-	2008	11.45	18.23	3.30
Greece	-	4.35	-	-	2008	6.03	14.08	-3.63
Slovak Republic	-	10.43	-	-	2008	10.73	19.05	0.76
Czech Republic	-	-	-	-	2008	10.46	19.02	0.19

Table 1: Average annual growth of GNI per capita, Atlas method (Total mean), estimated change point (year of CP), average annual growth of GNI per capita, Atlas method from the beginning of the period to year of CP (Mean before CP) and from year of CP to end of the period (Mean after CP)
Source: own research.

Table 1 demonstrates that there was not a change in long-term trend of annual growth of GNI per capita, Atlas method in all cosidered countries. However, at 2008 there was a significant change in short-term trend and annual growth of GNI per capita, Atlas method in all cosidered countries has significantly decreased last years comparing with several previous years.

Table 2: Average annual growth of GNI per capita, PPP (Total mean), estimated change point (year of CP), average annual growth of GNI per capita, PPP from the beginning of the period to year of CP (Mean before CP) and from year of CP to end of the period (Mean after CP)

Country	Medium term (1990-2012)			
	Year of CP	Total mean	Mean before CP	Mean after CP
Israel	-	3.08	-	-
OECD members	2007	3.77	4.40	2.01

Turkey	-	7.16	-	-
Greece	2008	3.08	4.57	-2.26
Slovak Republic		5.54	-	-
Czech Republic	2008	3.96	5.07	0.64

Source: own research.

Table 2 shows that there was not a significant change in medium-term trend of annual growth of GNI per capita, PPP in Israel, Turkey and Slovak Republic. However, the annual growth of GNI per capita, PPP in Greece, Czech Republic and OECD members has significantly decreased. Results of the similar change point analysis of GNI per capita, Atlas method (current US$) and GNI per capita, PPP (current international $) in Israel relative to France, OECD members, USA, Canada and Germany are presented in following Tables 3,4.

Table 3. Annual GNI per capita, Atlas method

Country		**Long term (1980-2013)**			
		Year of CP	Total mean	Mean before CP	Mean after CP
Israel relative to	France	1996	0.62	0.55	0.68
	OECD members	1996	0.70	0.64	0.74
	USA	1991	0.47	0.40	0.51
	Canada	1993	0.61	0.50	0.68
	Germany	1998	0.60	0.54	0.66

Source: own research.

Analyzing the results provided in Table 3 it can be concluded that between the years of 1991 to 1998 the annual GNI per capita, Atlas method in Israel has significantly increased with respect to the considered countries.

Table 4. Annual GNI per capita, PPP

Country		**Medium term (1990-2013)**			
		Year of CP	Total mean	Mean before CP	Mean after CP
Israel relative to	France	-	0.81	-	-
	OECD members	-	0.83	-	-
	USA	-	0.57	-	-
	Canada	-	0.73	-	-
	Germany	-	0.75	-	-

Source: own research.

Table 4 demonstrates that there was not a significant change in medium term trend of the annual GNI per capita, PPP in Israel with respect to the considered countries.

Conclusion

A recently developed nonparametric change point detection and estimation methods have been presented. The step by step application of this technique has been outlined as well as traditional statistical tools to study a trend behavior of the GNI per capita, Atlas method (current US$) and GNI per capita, PPP (current international $) in Israel and other selected countries. The presented analysis confirms the practical applicability of the change point methodology that can be useful for forecasting a short-term and long-term economic growth.

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