SAMPLING INSPECTION BY VARIABLES VERSUS SAMPLING INSPECTION BY ATTRIBUTES

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Abstract

The sample size for inspection by variables is less than the corresponding sample size for inspection by attributes. On the other hand, the cost of inspecting an item by variables is usually greater than the cost of inspecting the item by attributes. In this paper we shall compare Dodge-Romig LTPD plans (the remainder of rejected lots is inspected) for inspection by attributes with the corresponding LTPD plans for inspection by variables (both the sample and the remainder of rejected lots is inspected by variables) from economical point of view. The LTPD plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig LTPD plans for inspection by attributes. A criterion for deciding if inspection by variables should be considered instead of inspection by attributes (for given input parameters - the lot tolerance proportion defective, the lot size and the process average proportion defective) is suggested in this paper.

Key words: Acceptance sampling, inspection by variables and attributes, cost of inspection.

JEL Code: C44, C80

Introduction

In a book written by Dodge and Romig (Dodge and Romig, 1998) single sampling plans *(n,c)* are considered which minimize the mean number of items inspected per lot of process average quality, assuming that the remainder of rejected lots is inspected

$$I_s = N - (N - n) \cdot L(\bar{p}; n, c) \tag{1}$$

under the condition

$$L(p_t, n, c) = \beta, \tag{2}$$

(LTPD single sampling plans), where N is the number of items in the lot (the given parameter), $\bar{p}$ is the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, $P_t = 100 p_t$ is the lot tolerance per cent defective, denoted LTPD), n is the number of items in the sample *(n<N, n=?)*, *c* is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than *c, c=?*), *L* is the operating characteristic (*L(p, n, c)* is the probability of accepting a submitted lot with proportion defective *p* when using plan *(n, c)* for acceptance sampling).

Dodge-Romig single sampling inspection plans were introduce under the assumption that each inspected item is classified as either good or defective (inspection by attributes – see e.g. (Hald, 1981)). Condition (2) protects the consumer against the acceptance of a bad lot: the lots with the lot tolerance proportion defective p_t (the chosen parameter) are accepted with probability β (consumer's risk). These plans with $\beta = 0.1$ were extensively tabulated – see (Dodge and Romig, 1998).

The LTPD plans for inspection by variables with the same protection of consumer were introduced in (Klůfa, 1994). These plans are in many situations (under the same protection of consumer) more economical than the corresponding Dodge-Romig attribute sampling plans – see (Klůfa, 1994). Exact calculation of LTPD plans for inspection by variables when the non-central t distribution is used for the operating characteristic is considerably difficult. This problem was solved in (Klůfa, 2010). Similar problems are solved in (Klůfa, 1997), (Klůfa, 2008), (Chen and Chou, 2001), (Kaspříková and Klůfa, 2011), (Loster and Pavelka, 2013), (Wilrich, 2012), (Aslam et al. 2013), (Kaspříková and Klůfa, 2015), (Balamurali et al. 2014), (Ho et al. 2012), (Klůfa, 2014) – the average outgoing quality limit plans (AOQL plans).

1 LTPD plans for inspection by variables

The problem to find LTPD plans *for inspection by variables* has been solved under the following assumptions:

Measurements of a single quality characteristic *X* are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic *X* is given either an upper specification limit *U* (the item is defective if its measurement exceeds *U*), or a lower specification limit *L* (the item is defective if its measurement is smaller than *L*). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s.

The inspection procedure is as follows:

(1) Draw a random sample of n items and compute

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i , \quad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \tag{3}$$

(2) Compute $\frac{U-\bar{x}}{s}$ for an upper specification limit, or $\frac{\bar{x}-L}{s}$ for a lower specification limit.

(3) Accept the lot if

$$\frac{U-\bar{x}}{s} \geq k , \quad \text{or} \quad \frac{\bar{x}-L}{s} \geq k . \tag{4}$$

We have determine the sample size n and the critical value k. As well as Dodge and Romig we shall look for the acceptance plan (n,k) minimizing the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables

$$I_m = N - (N-n)\cdot L(\bar{p}; n,k) \tag{5}$$

under the condition

$$L(p_t, n, k) = \beta \tag{6}$$

(LTPD single sampling plans for inspection by variables).

The LTPD plans for inspection by variables (all items from the sample are inspected by variables, remainder of rejected lots is inspected by variables) were created by author of this paper - see (Klůfa, 1994). Exact calculation of LTPD plans for inspection by variables when the non-central t distribution is used for the operating characteristic $L(p,n,k)$ is considerably difficult. This problem was solved in (Klůfa, 2010). Now we shall study economical aspects of these plans.

2 Economical aspects of the LTPD plans by variables

For the comparison of the LTPD single sampling plans for inspection by variables with the corresponding Dodge-Romig LTPD plans for inspection by attributes from economical point of view we introduce parameter E defined by relation (see (1) and (5))

$$E = \frac{I_m}{I_s} 100. \tag{7}$$

Let us denote

$$c_m = c_m^* / c_s^* , \tag{8}$$

where c_s^* is the cost of inspection of one item by attributes, c_m^* is the cost of inspection of one item by variables. For the comparison of these plans the parameter c_m (the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes) must be estimated in each real situation. Usually c_m is greater than 1. Let us denote

$$\varepsilon = Ec_m = 100(I_m c_m^*)/(I_s c_s^*), \tag{9}$$

where $I_m c_m^*$ is the mean cost of inspection by variables and $I_m c_s^*$ is the mean cost of inspection by attributes. Therefore, if c_m is statistically estimated and

$$\varepsilon < 100, \tag{10}$$

then the LTPD plans for inspection by variables are more economical than the corresponding Dodge-Romig LTPD plans for inspection by attributes (difference $(100-\varepsilon)$ is saving of the inspection cost).

Example 1. Let $N = 500, p_t = 0.01, \overline{p} = 0.0015$ and $c_m = 1.5$ (the cost of inspection of one item by variables is higher by 50% than the cost of inspection of one item by attributes). We shall look for the LTPD plan for inspection by variables. Furthermore we shall compare this plan and the corresponding Dodge-Romig LTPD plan for inspection by attributes from economical point of view.

For given parameters $N = 500, p_t = 0.01, \overline{p} = 0.0015$ we shall compute the LTPD plan for inspection by variables - see (Klůfa, 2010)

$$n = 86,\ k = 2.6185$$

and $E = 44$. Corresponding LTPD plan for inspection by attributes we find in (Dodge and Romig, 1998). For given parameters $N = 500, p_t = 0.01, \overline{p} = 0.0015$ we have

$$n = 180,\ c = 0.$$

For $c_m = 1.5$ the economical parameter ε is

$$\varepsilon = Ec_m = 66.$$

From this result it follows that under the same protection of consumer the LTPD plan for inspection by variables (86, 2.6185) is more economical than the corresponding Dodge-Romig LTPD attribute sampling plan (180,0). Since $\varepsilon = 66\%$, it can be expected approximately **34% saving of the inspection cost**.

Economic efficiency measured by ε is a function of four variables p_t, N, $\bar{p}$ and c_m. This function for given p_t, N, $\bar{p}$ is a linear function of one variable c_m. Since (see (7)) $E > 0$, the function ε is increasing. Let us define

$$c_m^L = 100 / E. \tag{11}$$

If $c_m = c_m^L$, then $\varepsilon = 100$. If $c_m < c_m^L$, then $\varepsilon < 100$, i.e. the LTPD plans for inspection by variables are more economical than the corresponding Dodge-Romig LTPD attribute sampling plans. On the other hand, if $c_m > c_m^L$, then inspection by attributes is better.

Example 2. Let $N = 1000, p_t = 0.01, \bar{p} = 0.0015$. We shall determine c_m^L (a limit value of parameter c_m).

For given parameters $N, p_t, \bar{p}$ we shall compute (see (Klůfa, 2010)) the parameter $E = 32$. Therefore (see (11)) $c_m^L = 3.1$, i.e. the LTPD plan for inspection by variables is more economical than the corresponding Dodge-Romig LTPD attribute sampling plan when the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes $c_m < 3.1$. The other values of the parameter c_m^L are in Table 1.

Tab. 1: Values of the parameter c_m^L for $p_t = 0.01$

$\bar{p} \backslash N$	100	500	1000	4000	10000	50000	100000
0.001000	2.0	2.6	3.4	4.5	4.5	7.7	6.3
0.001250	1.9	2.4	3.2	4.0	4.3	4.8	5.0
0.001500	1.8	2.3	3.1	3.8	4.2	4.5	4.5
0.001750	1.7	2.2	3.0	3.7	4.2	4.5	4.5
0.002000	1.6	2.0	2.9	3.7	4.3	4.8	4.8
0.002250	1.5	2.0	2.5	3.4	3.8	4.2	4.3
0.002500	1.5	1.9	2.4	3.4	3.7	4.2	4.3
0.002750	1.4	1.8	2.3	3.3	3.7	4.3	4.5
0.003000	1.4	1.7	2.2	3.3	3.7	4.5	5.0
0.003250	1.4	1.6	2.1	3.0	3.4	3.8	4.0
0.003500	1.3	1.6	2.0	2.9	3.4	3.8	3.8
0.003750	1.3	1.5	1.9	2.8	3.3	3.8	4.0
0.004000	1.3	1.4	1.8	2.7	3.4	4.0	4.2
0.004250	1.3	1.4	1.7	2.6	3.1	3.6	4.0
0.004500	1.2	1.3	1.7	2.6	3.0	3.6	4.2
0.004750	1.2	1.3	1.6	2.4	3.0	3.6	4.3
0.005000	1.2	1.3	1.5	2.4	3.0	3.8	4.8

Source: Own construction

Conclusion

The LTPD plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig LTPD plans for inspection by attributes (see e.g. Example 1). The new parameter c_m^L for deciding if inspection by variables should be considered instead of inspection by attributes (for given input parameters - the lot tolerance proportion defective p_t, the lot size N and the process average proportion defective $\bar{p}$) has been suggested in this paper. If the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes c_m is less than c_m^L, using the LTPD plans for inspection by variables, we can achieve significant savings. These savings will be greater for large lot size N and small process average fraction defective $\bar{p}$ (see e.g. Table 1).

Acknowledgment

This paper was processed with contribution of long term institutional support of research activities by Faculty of Informatics and Statistics, University of Economics, Prague.

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