USING HIGH-FREQUENCY POWER-VARIATION ESTIMATORS IN THE BAYESIAN ESTIMATION OF STOCHASTIC-VOLATILITY JUMP-DIFFUSION MODELS

Milan Fičura – Jiří Witzany

Abstract

Power-variation estimators calculated from high-frequency returns are now widely used for the non-parametric estimation of volatility and jumps in the financial time-series. The paper presents a new methodology of how to utilize these estimators in the Bayesian estimation (using a complex MCMC algorithm) of the parameters and latent state variables of a Stochastic-Volatility Jump-Diffusion (SVJD) model with self-exciting jumps. Two newly developed models are presented: the SVJD-RV model using the realized variance and the SVJD-RV-Z model using in addition to that the Z statistics (used often for the non-parametric jump identification). The models are applied to the past history of the EUR/USD time series. The results indicate superior properties of the new models especially in their ability to model jump clustering when compared to the standard SVJD model (i.e. not utilizing the power-variation estimators). The strongest clustering effects were found by the SVJD-RV-Z model.

Key words: Stochastic volatility, Self-Exciting jumps, Realized volatility, Power-variation estimators, Bayesian inference

JEL Code: C11, C14, C22

Introduction

The ability to accurately model and forecast volatility and jumps in the financial time series is of primary importance in many areas of finance such as option pricing, portfolio construction, risk premium modelling, VaR estimation or quantitative trading. In the last two decades there occurred two breakthroughs in the field of volatility and jumps modelling resulting in two highly successful and intriguing classes of models. The first are the Stochastic-Volatility Jump-Diffusion (SVJD) models, modelling volatility and jumps as latent processes, and the second are the realized variance (RV) models utilizing high-frequency returns and the asymptotic theory of power-variations. The purpose of this paper is to show how these two classes of models could be combined in order to model volatility and jumps more efficiently.

In the SVJD models the volatility and jumps are modelled as latent (unobservable) processes whose values have to be estimated as latent state variables. This is relatively difficult, but manageable through the use of Bayesian simulation techniques such as MCMC sampling (first proposed by Jacquier et al., 1994). In the recent years, with the increase of the computational power and the development of more advance estimation algorithms (MCMC, EMM and Particle filters, etc.) the SVJD models surged in popularity and started to include more advanced features such as jumps in volatility (Eraker et. al 2003), correlations between the markets (Witzany 2013) or self-exciting jumps (Fulop, Li and Yu, 2015).

The second class of volatility models (i.e. the realized volatility models) utilize high-frequency returns and the asymptotic theory of power variations in order to construct non-parametric estimators of the latent volatility and jumps, thus enabling their direct modelling through traditional time series models (for example ARFIMA). The name of the class comes from the first of these estimators, the realized variance (RV), proposed by Andersen et. al (1998). RV is defined for a given frequency as the sum of the squared returns on some higher frequency and it should asymptotically converge to the quadratic variation of the underlying price process. In order to estimate the continuous stochastic variance it is necessary to construct an estimator converging to the continuous component of the quadratic variance which is true for the bipower variation (Barndorff-Nielsen and Shephard, 2004). The statistically significant jumps can then be estimated by normalizing the differences between the realized variance and bipower variation through the integrated quarticity, as was done by Andersen et al (2007) to construct the so called Z variable for jump identification.

Considering the combination of SVJD models with the realized variance models, it has been pursued in the past (Takahashi et al. 2009) but to our knowledge only for the models without jumps (i.e. SV models, not SVJD). As the jumps play an important role in the financial time-series dynamics, we present a methodology of how to extend the previous SV-RV models to incorporate jumps (SVJD-RV model) as well as to utilize the information from the Z statistics (SVJD-RV-Z model). The application is performed on a 17 year history of the EUR/USD time series and it is shown that the new models possess superior properties.

The paper is organized as follows. In the first chapter the non-parametric power-variation estimators are described. In the next section the SVJD-RV and SVJD-RV-Z models are presented, as well as the MCMC estimation procedure. In the fourth chapter the application of the models to the EUR/USD time series is performed and the results are compared with benchmarks (SVJD and the purely nonparametric estimates). In the conclusion the results are summed up and further areas of research are proposed.

1 Power-variation estimators of volatility and jumps

One of the advantages of the power-variation estimators of volatility and jumps is that they are non-parametric and model-free, meaning that they are theoretically valid for a wide range of asset price processes. For the purpose of exposition there is further assumed that the logarithmic asset price follows the following generally defined stochastic process:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + j(t)dq(t), \tag{1}$$

where $p(t)$ is the logarithm of the asset price, $\mu(t)$ is the instantaneous drift rate, $\sigma(t)$ is the instantaneous volatility, $W(t)$ is a Wiener process, $j(t)$ is a process determining the jump sizes and $q(t)$ is a counting process determining the time of jump occurrences.

Omitting the variability of the stochastic drift rate $\mu(t)$, it is possible to express the *quadratic variation* of the price process over the period between $t-1$ and t as follows:

$$QV(t) = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{t-1 \le s < t} \kappa^2(s), \tag{2}$$

where $\kappa(t) = j(t)I[q(t)=1]$ and $I(.)$ is the indicator function. So the quadratic variation, measuring the overall variability of the price process during a given period of time, consists of two components. The first term in (2), called *Integrated Variance*, corresponds to the continuous variability of the price process, while second term, called *Jump Variance*, corresponds to the discontinuous variability (i.e. to the impact of jumps). So we can write:

$$QV(t) = IV(t) + JV(t), \tag{3}$$

where $IV(t)$ is the integrated variance and $JV(t)$ is the jump variance.

The quadratic variation and its components are directly unobservable and have to be estimated. The most widely used power-variation estimator of quadratic variation is the realized variance (Andersen and Bollerslev 1998).

Denoting $r(t,\Delta)$ as the logarithmic return between $t-\Delta$ and t, we can define RV:

$$RV(t,\Delta) = \sum_{j=1}^{1/\Delta} r^2(t-1+j\Delta,\Delta), \tag{4}$$

and it holds that $RV(t,\Delta) \to QV(t)$ as $\Delta \to 0$.

The realized variance is theoretically an unbiased and consistent estimator of the quadratic variance of the underlying process. Nevertheless in real-life applications the Δ may not go sufficiently close to zero as the microstructure-noise effects present on ultra-high frequencies may cause the estimator to be biased. Therefore we use the 15-minute frequency.

Next it is necessary to decompose the quadratic variation into the integrated variance and jump variance. For this purpose a wide range of non-parametric estimators converging to the integrated variance has been constructed, the most well-known is the *bipower variation* (Barndorff-Nielsen and Shephard 2004). It is defined as follows:

$$BV(t,\Delta)=\frac{\pi}{2}\sum_{j=2}^{1/\Delta}\left|r(t-1+j\Delta,\Delta)\right|\left|r(t-1+(j-1)\Delta,\Delta)\right|, \tag{5}$$

and it holds that $BV(t,\Delta)\rightarrow IV(t)$ as $\Delta\rightarrow 0$

Finally, the jump variance could theoretically be estimated as the difference between realized variance and bipower variation. This would be however a very inaccurate approach due to the finiteness of the sampling frequency in the calculation of $RV(t,\Delta)$ and $BV(t,\Delta)$ which causes the resulting $JV(t)$ estimates be very noisy, indicating jumps on almost every day and sometimes acquiring even negative values which is for the real $JV(t)$ variable impossible.

In order to accurately estimate the statistically significant jumps even in the presence of noise it is necessary to normalize the differences between realized variance and bipower variation by using the integrated quarticity: $IQ(t)=\int_{t-1}^{t}\sigma^4(s)ds$, which can be consistently estimated (even in the presence of jumps) through the realized tri-power quarticity:

$$TQ(t,\Delta)=\frac{\pi^{3/2}}{4}\Gamma\left(\frac{7}{6}\right)^{-3}\sum_{j=3}^{1/\Delta}\left|r(t-1+j\Delta,\Delta)\right|^{4/3}\left|r(t-1+(j-1)\Delta,\Delta)\right|^{4/3}\left|r(t-1+(j-2)\Delta,\Delta)\right|^{4/3}, \tag{6}$$

as it holds that $TQ(t,\Delta)\rightarrow IQ(t)$ as $\Delta\rightarrow 0$.

By using $RV(t,\Delta)$, $BV(t,\Delta)$ and $TQ(t,\Delta)$, it is possible to define the variable $Z(t,\Delta)$ (see Andersen et. al 2007), following a standard normal distribution as long as the underlying process does not contain any jumps:

$$Z(t,\Delta)=\frac{\left[RV(t,\Delta)-BV(t,\Delta)\right]RV(t,\Delta)^{-1}}{\sqrt{\left[(\pi/2)^2+\pi-5\right]max\left\{1,TV(t,\Delta)BV(t,\Delta)^{-2}\right\}}}. \tag{7}$$

Large values of $Z(t,\Delta)$ are indicating that a possible jump occurred during the given time period. The statistically significant jumps can be estimated by using the appropriate quantiles:

$$EJV(t,\Delta)=I\left\{Z(t,\Delta)>\Phi(\alpha)^{-1}\right\}\left[RV(t,\Delta)-BV(t,\Delta)\right], \tag{8}$$

where $EJV(t,\Delta)$ is the estimated jump variance, $I\{.\}$ is the indicator function and $\Phi(\alpha)^{-1}$ is the quantile function of the standard normal distribution.

2 The SVJD model using power-variation estimators

For the sake of brevity only a discretized version of our SVJD models will be presented. The discretization was performed by the Eulers method with the assumption that at most one jump can happen during any given time period *t*, which will in our case correspond to one day.

The model assumes that the logarithmic returns of the asset follow equation:

$$r(t) = \mu + \sigma(t)\varepsilon(t) + J(t)Q(t), \tag{9}$$

where $r(t)$ is the daily logarithmic return defined as $r(t) = p(t) - p(t-1)$, where $p(t)$ is the logarithm of the closing price, μ is the mean daily return, $\sigma(t)$ is the stochastic volatility, $\varepsilon(t) \sim N(0,1)$ is a standard normal white noise, $J(t) \sim N(\mu_J, \sigma_J)$ corresponds to the jump sizes and $Q(t) \sim Bern[\lambda(t)]$ is a Bernoulli distributed jump indicator with intensity $\lambda(t)$.

The variance will be modelled through the log-variance model. So we denote $V(t) = \sigma^2(t)$ and $h(t) = \log(V(t))$ and the variable $h(t)$ follows an AR(1) process:

$$h(t) = \alpha + \beta h(t-1) + \gamma \varepsilon_V(t), \tag{10}$$

where $h(t)$ is the logarithm of the conditional variance, $\alpha = (1-\beta)\theta$ is the constant, θ is the long-term log-variance, β is the autoregressive coefficient, γ is the volatility of the log-variance and $\varepsilon_V(t) \sim N(0,1)$ is a standard normal white noise, which is in our case uncorrelated with $\varepsilon(t)$, as the model will be applied to the currency market.

The dynamics of the jump component is modelled using the self-exciting Hawkes process, in which the jump intensity, defined as $Pr[Q(t) = 1] = \lambda(t)$, follows the following process:

$$\lambda(t) = \alpha_J + \beta_J \lambda(t-1) + \gamma_J Q(t-1), \tag{11}$$

where $\lambda(t)$ is the jump intensity at day t, $\alpha_J = (1-\beta_J)\theta_J$ is the constant, θ_J is the long-term jump intensity, β_J is the rate of the exponential decay of the intensity towards θ_J and γ_J is the self-exciting parameter representing the increase in intensity after a jump occurrence.

The equations 9, 10 and 11 represent a standard SVJD model with self-exciting jumps. In order to incorporate the realized variance into the model the following relationship is used:

$$\log\left(RV(t) - J^2(t)Q(t)\right) = h(t) + \varepsilon_{RV}(t). \tag{12}$$

So the model assumes that the logarithm of the realized variance, adjusted for the influence of the jump component, represents unbiased estimate of the underlying log-variance $h(t)$, it is however plagued by certain noise $\varepsilon_{RV}(t) \sim N(0, \sigma_{\mathrm{RV}})$ with volatility σ_{RV}.

The above equations represent the SVJD-RV model. In the full SVJD-RV-Z model there will be one additional equation corresponding to the $Z(t,\Delta)$ variable. Although it is possible to infer jump probabilities directly from $Z(t,\Delta)$, using the standard normal cumulative distribution function, these are generally too high in order to be directly utilized in a SVJD model. Indeed in our dataset the mean value of the $Z(t,\Delta)$ is 1.127 and the mean jump probability 72.92%. This is caused by the existence of a very large number of small jumps, which occur commonly on the ultra-high frequencies, but do not influence the size of the returns on the daily frequency enough in order to be distinguishable from the continuous daily volatility (for further discussion see Fičura and Witzany, 2014).

As we want to focus on the large jumps (visible on the daily frequency), we will utilize the intuitive notion that the large jumps tend to increase the size of $Z(t,\Delta)$ more than the small jumps. So although its values tend to be larger than zero on almost every day, when large jumps happen they should be even larger. This is captured by the following model:

$$Z(t) = \mu_Z + \xi_Z Q(t) + \varepsilon_Z, \tag{13}$$

where μ_Z is the mean value of the $Z(t)$ variable on regular days, containing either no jumps or only small jumps (unobservable on the daily frequency), ξ_Z is the increase in the mean value of the variable on the days when large jumps happen and $\varepsilon_Z \sim N(0, \sigma_Z)$ is the noise of the $Z(t)$ variable with standard deviation not necessarily equal to one (due to the jumps).

So the full model has altogether 12 parameters: $\mu, \alpha, \beta, \gamma, \theta_J, \beta_J, \gamma_J, \mu_J, \sigma_J, \sigma_{RV}, \mu_Z, \xi_Z, \sigma_Z$ and three vectors of latent state variables (**V, J** and **Q**) to be estimated.

The estimation is performed using a MCMC algorithm composed of a Gibbs Sampler, Random-Walk Metropolis-Hastings and Independence-Sampling Metropolis-Hastings. Denoting the vector of all the parameters and latent state variables of the model as $\Theta = (\theta_1, \ldots, \theta_k)$, the MCMC algorithm allows us to sample from the Bayesian joint posterior density $p(\Theta \mid data)$, by constructing a Markov Chain that uses only the information about the conditional densities $p(\theta_j \mid \theta_i, i \neq j, data)$. The chain is then guarantied (as long as certain conditions are met) to asymptotically converge to the density $p(\Theta \mid data)$.

More about the construction of the MCMC algorithm can be found in Witzany (2013). It is based on the results in Jacquier et al. (2007) and Johannes and Polson (2009). For the sake of brevity we report only the final version of the algorithm. It proceeds as follows:

1. Sample initial values $\mu^{(0,)},\lambda^{(0)},\mu_J^{(0)},\sigma_J^{(0)},\alpha^{(0)},\beta^{(0)},\gamma^{(0)},V^{(0)},J^{(0)},Q^{(0)},\sigma_{RV}^{(0)},\mu_Z^{(0)},\xi_Z^{(0)},\sigma_Z^{(0)}$.
2. For $i=1,\dots,T$ sample jump sizes $J_i^{(g)}\propto\varphi\left(J;\mu_J^{(g-1)},\sigma_J^{(g-1)}\right)$ if $Q_i^{(g-1)}=0$ using Gibbs Sampler, or $J_i^{(g)}\propto\varphi\left(r_i;\mu^{(g-1)}+J,\sqrt{V_i^{(g-1)}}\right)\varphi\left(\log\left(RV_i-J^2\right);h_i^{(g-1)},\sigma_{RV}^{(g-1)}\right)\varphi\left(J;\mu_J^{(g-1)},\sigma_J^{(g-1)}\right)$ if $Q_i^{(g-1)}=1$, using Random-Walk Metropolis-Hastings.
3. For $i=1,\dots,T$ sample jump occurrences $Q_i^{(g)}\in\{0,1\}$, $Pr[Q=1]=p_1/(p_0+p_1)$, where: $p_0=\varphi\left(r_i;\mu^{(g-1)},\sqrt{V_i^{(g-1)}}\right)\varphi\left(\log(RV_i);h_i^{(g-1)},\sigma_{RV}^{(g-1)}\right)\varphi\left(Z_i;\mu_Z^{(g-1)},\sigma_Z^{(g-1)}\right)\left(1-\lambda^{(g-1)}\right)$,
$p_1=\varphi\left(r_i;\mu^{(g-1)}+J,\sqrt{V_i^{(g-1)}}\right)\varphi\left(\log\left(RV_i-\left(J_i^{(g)}\right)^2\right);h_i^{(g-1)},\sigma_{RV}^{(g-1)}\right)\varphi\left(Z_i;\mu_Z^{(g-1)}+\xi_Z^{(g-1)},\sigma_Z^{(g-1)}\right)\lambda^{(g-1)}$.
4. Sample new stochastic variances $V_i^{(g)}$ for $i=1,\dots,T$ using Independence-Sampling Metropolis-Hastings with proposal density derived based on Jacquier et al. (1994): $q\left(V_i\mid V_{(-i)},\Theta,r,J,Q\right)=IG\left[V_i;\phi+0.5,(\phi-1)\exp\left(\mu_i+0.5\sigma^2\right)+0.5\left(r_i-\mu-J_iQ_i\right)^2\right]$ where $\phi=\frac{1-2\exp\left(\sigma^2\right)}{1-\exp\left(\sigma^2\right)}$, $\mu_i=\frac{\gamma^2\log\left(EIV_i\right)+\sigma_{RV}^2\left[\alpha(1-\beta)+\beta\left(\log V_{i+1}+\log V_{i-1}\right)\right]}{\gamma^2+\left(1+\beta^2\right)\sigma_{RV}^2}$, $\sigma=\frac{\gamma\sigma_{RV}}{\sqrt{\gamma^2+\left(1+\beta^2\right)\sigma_{RV}^2}}$.
5. Sample new stochastic volatility AR(1) coefficients $\alpha^{(g)},\beta^{(g)},\gamma^{(g)}$ from $h_i=\log\left(V_i^{(g)}\right)$ for $i=1,\dots,T$ using the Bayesian linear regression model (Lynch, 2007): $\beta=\left(X'X\right)^{-1}Xy$, $e=y-X\beta$, where $X=\begin{pmatrix}1\dots1\\ h_1\dots h_{T-1}\end{pmatrix}'$, and sample $\left(\gamma^{(g)}\right)^2\propto IG\left(\frac{n-2}{2},\frac{e'e}{2}\right)$ and $\left(\alpha^{(g)},\beta^{(g)}\right)'\propto\varphi\left[(\alpha,\beta)';\beta,\left(\gamma^{(g)}\right)^2\left(X'X\right)^{-1}\right]$.
6. Sample $\mu^{(g)}$ based on the normally distributed time series $r_i-J_i^{(g)}Q_i^{(g)}$ with variances $V_i^{(g)}$: $p\left(\mu^{(g)}\mid r,J^{(g)},Q^{(g)},V^{(g)}\right)\propto\varphi\left(\mu;\sum_{i=1}^{T}\frac{r_i-J_i^{(g)}Q_i^{(g)}}{V_i^{(g)}}/\sum_{i=1}^{T}\frac{1}{V_i^{(g)}},\sum_{i=1}^{T}\frac{1}{V_i^{(g)}}\right)$.
7. Sample $\theta_J,\beta_J,\gamma_J$ using Random-Walk Metropolis-Hastings with the proposal density $\theta_J^{(g)}=\theta_J^{(g-1)}+N(0,c)$ (for θ_J) and likelihood: $L\left(Q^{(g)}\mid\theta_J,\beta_J,\gamma_J\right)=\prod_{i=1}^{T}\lambda_i^{Q_i}\left(1-\lambda_i\right)^{1-Q_i}$.
8. Sample $\sigma_{RV}^{(g)}$ using the density: $p\left[\left(\sigma_{RV}^{(g)}\right)^2\mid EIV^{(g)},h^{(g)}\right]\propto IG\left[\left(\sigma_{RV}^{(g)}\right)^2;\frac{T}{2},\frac{\sum_{i=1}^{T}\left(\log\left(EIV_i^{(g)}\right)-h_i^{(g)}\right)^2}{2}\right]$.

9. Sample $\mu_Z^{(g)}, \xi_Z^{(g)}, \sigma_Z^{(g)}$ using normally distributed series $Z_i - Q_i^{(g)}\xi_Z^{(g-1)}$, with variance $\sigma_Z^{(g-1)}$ to sample $\mu_Z^{(g)}$, series $Q_i^{(g)}\left(Z_i - \mu_Z^{(g)}\right)$, where $Q_i^{(g)} = 1$, with variance $\sigma_Z^{(g-1)}$ to sample $\xi_Z^{(g)}$, and the centralized series $Z_i - \mu_Z^{(g)} - Q_i^{(g)}\xi_Z^{(g)}$ to sample $\sigma_Z^{(g)}$.
10. Sample $\mu_J^{(g)}, \sigma_J^{(g)}$ based on the normally distributed series $J^{(g)}$ and uninformative priors $p(\mu) \propto 1$ and $p\left(\log\sigma^2\right) \propto 1$, which is equivalent to $p\left(\sigma^2\right) \propto 1/\sigma^2$, i.e. sample:

$$p\left(\mu_J^{(g)} \mid J^{(g)}, \sigma_J^{(g-1)}\right) \propto \varphi\left(\mu_J^{(g)}; \frac{\sum_{i=1}^{T} J_i^{(g)}}{T}, \frac{\sigma_J^{(g-1)}}{\sqrt{T}}\right),\ p\left[\left(\sigma_J^{(g)}\right)^2 \mid J^{(g)}, \mu_J^{(g)}\right] \propto IG\left[\left(\sigma_J^{(g)}\right)^2; \frac{T}{2}, \frac{\sum_{i=1}^{T}\left(J_i^{(g)} - \mu_J^{(g)}\right)^2}{2}\right].$$

2 Application of the SVJD-RV and SVJD-RV-Z models

The models were applied to the time series of the EUR/USD exchange rate in the period between 1.11.1999 and 10.10.2014, containing 3884 trading days and 369533 15-minute returns which were used for the calculation of the power-variation estimators. The intraday data were provided by ForexHistoryDatabase.com and the time corresponds to UTC+2.

Three SVJD models were estimated and will be compared, standard SVJD model using daily returns, SVJD-RV model, using in addition to the daily returns also the realized variance, and the SVJD-RV-Z model utilizing also the Z variable. The MCMC estimation algorithm was implemented in Matlab. The parameters exhibited good convergence. 3000 MCMC iterations for every model were calculated out of which the first 1000 were discarded and the remaining 2000 were used for parameter estimation based on the posterior means. In addition to that Bayesian standard errors were calculated. The parameter estimates are summed up in Table 1.

Tab. 1 – The posterior means and Bayesian standard errors of model parameters

	m	mJ	sigmaJ	alpha	beta	gamma	thetaJ	betaJ	gammaJ	sigmaRV
SVJD	7.9E-05	1.4E-04	0.0075	-0.0473	0.9955	0.0676	0.0491	0.3782	0.0468	
	8.1E-05	0.0016	0.0008	0.0189	0.0018	0.0065	0.0180	0.2473	0.0272	
SVJD-RV	8.8E-05	0.0003	0.0113	-0.0534	0.9948	0.0654	0.0071	0.4205	0.0556	0.4910
	6.1E-05	0.0038	0.0018	0.0194	0.0019	0.0054	0.0028	0.2392	0.0273	0.0092
SVJD-RV-Z	0.0001	0.0003	0.0051	-0.0574	0.9945	0.0740	0.0483	0.6854	0.0158	0.5039
	6.0E-05	0.0005	0.0004	0.0215	0.0021	0.0085	0.0071	0.2860	0.0105	0.0087

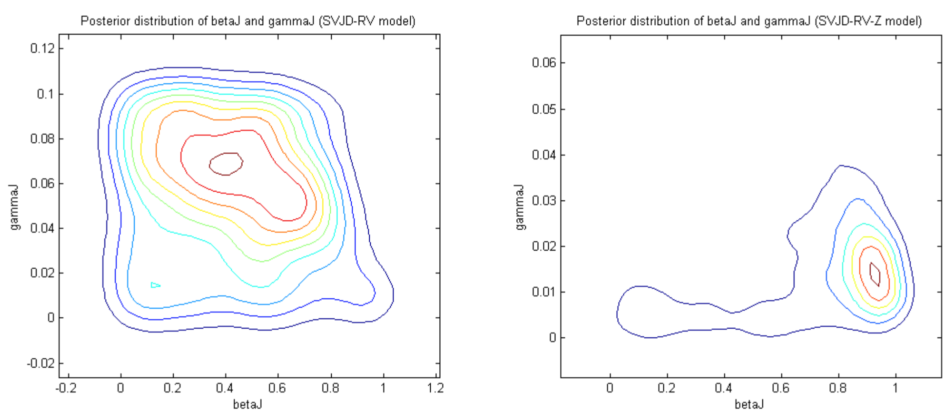
Source: Authorial computation

The upper values in the cells represent the parameter estimates and the bottom values are the Bayesian standard errors. From the results it is apparent that the stochastic variance follows a highly persistent process (beta close to one for all of the models), although a stationary one (beta less than 2 standard deviations away from one).

The long-term jump intensity (thetaJ) is lowest for the SVJD-RV model which has also the largest average mean jump magnitude (sigmaJ). For the other two models the values of thetaJ are similar, but the SVJD model has a larger value of sigmaJ, indicating larger jumps. Considering the jump clustering, the jump intensity is most persistent (highest betaJ) in the case of the SVJD-RV-Z model. This model has also the lowest value of gammaJ.

Further analyses showed that the Bayesian mean estimates may potentially underestimate the extent to which jump clustering occurs as the posterior distributions of betaJ and gammaJ are highly asymmetric. Therefore the Bayesian mode may pose a better estimate. Figure 1 shows the bivariate marginal posterior density of betaJ and gammaJ for two of our models.

Fig. 1 – Posterior densities of betaJ and gammaJ for SVJD-RV and SVJD-RV-Z



Source: Authorial computation

The posterior densities give a much clearer picture about the nature of the jump clustering in the time series. It is apparent that for SVJD-RV-Z the clustering is very strong and persistent as the mode of betaJ is very close to one. For SVJD-RV the clustering is only short lived, manifesting itself in a significantly increased jump probability in the day immediately following a jump occurrence, but not so much in the days afterwards.

The SVJD-RV-Z model has three additional parameters. They are reported in Table 2.

Tab. 2 – Additional parameters of the SVJD-RV-Z model

	mZ	ksiZ	sigmaZ
SVJD-RV-Z	0.9510 0.0254	3.7059 0.1579	1.2544 0.0202

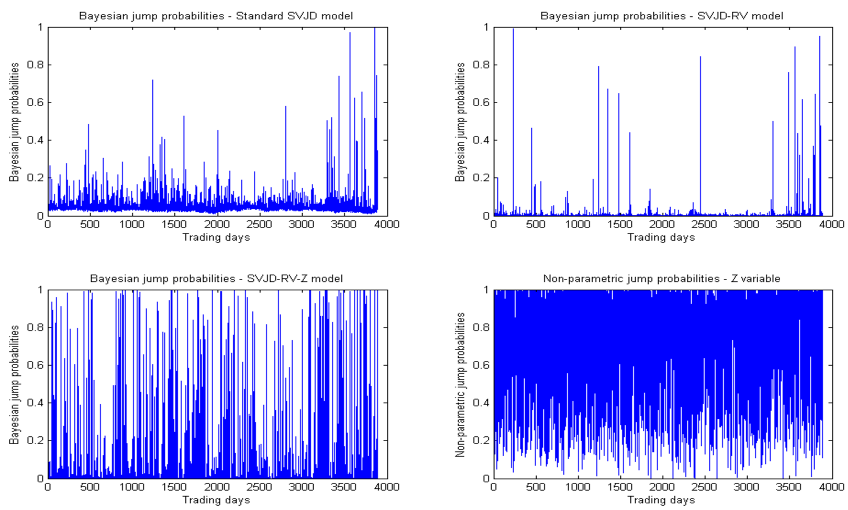
Source: Authorial computation

From the parameters in Table 2 we can see that the average value of the Z variable on the days with no jumps or small jumps, is 0.95, while on the days with large jumps it is about 3.7 higher. The sigmaZ parameter appears to be significantly larger than one (i.e no jump case).

Considering the latent state variables we do not report the stochastic variance estimates in order to save place. Nevertheless from the analyses of the ability of the models to fit the non-parametrically estimated integrated variance, the SVJD-RV-Z model achieved a better fit then the other two models and its estimates were also the least biased.

We further focus on the analysis of the jump component. Figure 2 shows the Bayesian probabilities of jump occurrence based on the models as well as the non-parametric estimates.

Fig. 2 – Bayesian probabilities of jump occurrence based on different models



Source: Authorial computation

It is apparent that all of the SVJD models identify far less jumps in the time series than the non-parametric approach. This is due to the fact that the Z variable finds even the small jumps occurring on the ultra-high frequencies, which are often undistinguishable on the daily frequency. From the SVJD models, the SVJD-RV model tends to identify less jumps than the standard model, but the several ones it found have a relatively large probability of occurrence. The SVJD-RV-Z model does, on the other hand, identify significantly more jumps than the other two models and with greater probabilities of occurrence.

From the charts it is not possible to say which estimates of jumps are the "best" ones. Nevertheless, considering the clustering effects in the jump time series identified by the SVJD-

RV and SVJD-RV-Z models, it seems that there exists some underlying dynamics in the jump component that may play important role in applications such as volatility forecasting and option pricing. So the models pose a promising area for future research.

Conclusion

Methodology was presented of how to utilize the high-frequency power-variation estimators of volatility and jumps in the Bayesian estimation (through MCMC sampling) of SVJD models with self-exciting jumps. The newly developed models utilize the realized variance (SVJD-RV model) and the Z statistics (SVJD-RV-Z) in order to gain more information about the latent states of stochastic volatility, jump occurrences and jump sizes. Based on the results, the models compare favourably to the standard SVJD model as well as to the non-parametric methods of volatility and jump estimation. The main benefits of the models seem to be in their ability to identify large jumps and to model their dynamics. Significant evidence for jump clustering was found, especially by the SVJD-RV-Z model.

The usefulness of the new models will manifest itself mainly in financial applications such as volatility forecasting, VaR estimation and option pricing. For this purpose it would be necessary to construct out-of-cample forecasts, for example through particle filters which is one of the areas of our future research. In addition to that it would be interesting to extend the models to include additional features such as jumps in volatility. Considering the utilization of the power-variation estimators, a constant may be introduced in the modelling of the realized variance in order to tackle the possible bias due to the microstructure noise effects. Additionally other power-variation estimators may be included into the SVJD models in order to utilize their individual benefits or to find out which ones are best suited for applications.

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Contact

Milan Fičura

University of Economics, Prague, Faculty of Finance and Accounting, Department of Banking and Insurance, Winston Churchill Square, 130 67, Prague 3, Czech Republic

milan.ficura@vse.cz

Jiří Witzany

University of Economics, Prague, Faculty of Finance and Accounting, Department of Banking and Insurance, Winston Churchill Square, 130 67, Prague 3, Czech Republic

jiri.witzany@vse.cz