MODELING BILATERAL FLOWS IN ECONOMICS BY MEANS OF EXACT MATHEMATICAL METHODS

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Abstract

This paper suggests a simple deterministic linearbilateral flows / stockmatrix model, which produces estimates of missing relationships inside a group of statistical units, based on matrix equations solved with the help ofthe Moore-Penrose pseudo-inverse matrix. A pseudo-R^2 is used as a measure of model quality. The paper is divided into three parts: the derivation of the model, steps of its calculation and indicators of its quality, anda case study: inward direct investmentstock matrixof2013.The model maybe applied to historical statistics for better understanding of economic cooperation between countries, for early estimations of input-output tables, as well as for correcting incomplete trade, investment and other relationship matrices. The paper is based on official data of the United Nations UNCTAD, including extrapolations of missing values. All calculations were performed with the help of standard software, such as the GNU Octave and Microsoft Excel (OpenOfficeCalc).

Key words:bilateral flows, system of linear equations, foreign trade, foreigndirect investment, input-output

JEL Code:C65, C67, F00

Introduction

Modeling bilateral flows of trade, investment, value added and of other economic variablesstarted with theworks of Nobel Prize laureatesJan Tinbergen (Tinbergen, 1962) and Wassily Leontief(Leontief, 1966), the creators of gravity and input-output methods, laterdevelopedin a variety of ways, for example, with the help ofpanel data (Bellos&Subasat, 2012), structural decomposition method (Owen, Steen-Olsen, Barrett, Wiedmann, &Lenzen, 2014), stochastic calculations (Jansen, 1994) and improved matrix inverses (Casler&Hadlock, 1997)(Gim, 2008). Both models have been frequently used in applied studies, for example, by A. K. Rose (Rose, 2000), as well as by (van der Linden, Oosterhaven, Cuello, Hewings, &Sonis, 2000) and(Zimmermannova, 2009).

This paper describes a simple deterministic linear bilateral flows / stock (accumulated flows) matrix model, based on same principles as Leontief's input-output method (Leontief, 1966), which generates estimates of relationships inside a group of statistical units usingthe (Moore-Penrose) pseudo-inverse matrix(Penrose, 1956)to solvematrix equations (Anton, 2010).The paper is divided into three parts: the description and derivation of the model, steps of itscalculation and indicator of its quality, and a case study:inward direct investment stock matrix of 2013.The paper is based on official data of the United Nations, UNCTAD, missing values were extrapolated.

1 Deterministic linear bilateral flows / stockmodel

The model describes aclosed system[1](Σoutflows = Σinflows, Σoutward stock = Σinward stock) of mutual relationships between *n*abstract unitsor *n-m*units and the rest of group*RoG*in case of incomplete data, under the *assumptionthat total outflowsO_i and inflows $I_i$for all n or n-m and RoG are known*. The relationship matrix of the model in its generalized form is stated in Tab. 1 where for $1 \leq i \leq n$or *n-m* and *RoG,*$O_i$and $I_i$are total outflows and inflows of unit *i*and F_{Oi} and F_{iI} are possible additional factors on the side of both variables (such as exports, demand, importsand taxes in the input-output tables) (Leontief, 1966). In some cases, simplifications are possible: for example,F_{Oi}=0, F_{iI}=0 or F_{Oi}= F_{iI}=0 for international trade and investment and $O_i = I_i$ for input-output tables, as shown in case studies.

Tab. 1:The model's relationship matrix in itsgeneralized form, flows / stock, (n x n)

	Unit 1	**Unit 2**	**...**	**Unit *n* / *RoG***	***Fo***	**Ouflows / outw. stock**
Unit 1	β_{11}	β_{12}	...	β_{1n}	f_{O1}	$\boldsymbol{O_1}$
Unit 2	β_{21}	β_{22}	...	β_{2n}	f_{O2}	$\boldsymbol{O_2}$
...	...	...	...	...	...	**...**
Unit *n* / *RoG*	β_{n1}	β_{n2}	...	β_{nn}	f_{On}	$\boldsymbol{O_n}$
$\boldsymbol{F_I}$	f_{1I}	f_{2I}	...	f_{nI}		$\boldsymbol{\sum f_{iI}}$
Inflows / inw. stock	$\boldsymbol{I_1}$	$\boldsymbol{I_2}$	**...**	$\boldsymbol{I_n}$	$\boldsymbol{\sum f_{Oi}}$	$\boldsymbol{\sum O_i = \sum I_i}$

Source: self-prepared

[1]A system can be defined as "a regularly interacting or interdependent group of items forming a unified whole" (Merriam Webster dictionary).

Under the *assumption that each bilateral flow / outward stock (accumulated outflows) from unit i to unit j, β_{ij} (in matrix form – **B**), is a share of total inflows of unitj*, $I_j$purgedfrom additional factors, formally$\beta_{ij} = \sigma_{ij}(I_j – f_{Ij})$, where$\sigma_{ij}$is the corresponding share, a*bilateral outflows-inflows(outward-inward stock) equation*, i.e. outflows (outward stock) explained by inflows (inward stock) in a closed system,may be formulated as equation (1):

$$O_{i,\ 1\leq i\leq n} = \sum_{j=1}^{n} \beta_{ij} + f_{Oi} = \sum_{j=1}^{n} \sigma_{ij}\left(I_j - f_{jI}\right) + f_{Oi} \qquad (1)$$

Equation (1) can be re-written in a matrix equation form (2) or (3):

$$\begin{bmatrix} O_1 \\ \dots \\ O_n \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1n} \\ \dots & \ddots & \dots \\ \sigma_{n1} & \dots & \sigma_{nn} \end{bmatrix} \cdot \left(\begin{bmatrix} I_1 \\ \dots \\ I_n \end{bmatrix} - \begin{bmatrix} f_{1I} \\ \dots \\ f_{nI} \end{bmatrix} \right) + \begin{bmatrix} f_{On} \\ \dots \\ f_{On} \end{bmatrix} \qquad (2)$$

$$O = \boldsymbol{\sigma} \cdot (I - F_I) + F_O \qquad (3)$$

where$O$, I, $\boldsymbol{\sigma}$, F_O and F_I are (n x 1) - , (n x 1) -, (n x n) -, (n x 1) - and (n x 1) - matrices of outflows (outward stock), inflows (inward stock), shares in inflows (inward stock) and factors on the side of outflows (outward stock) and inflows (inward stock). The *shares matrix*$\boldsymbol{\sigma}$may be derived as indicated in equation (4)(Anton, 2010), and, if the bilateral outflows-inflows(outward-inward stock) equation is reversed, i.e. inflows (inward stock)become explained by outflows (otward stock) in a closed system, the corresponding *shares matrix*$\boldsymbol{\sigma^{-1}}$is derived as indicated in equation (5)(Anton, 2010):

$$O = \boldsymbol{\sigma} \cdot (I - F_I) + F_O \;\; \Longrightarrow \;\; \boldsymbol{\sigma} = (O - F_O) \cdot (I - F_I)^{-1} \qquad (4)$$

$$I = \boldsymbol{\sigma}^{-1} \cdot (O - F_O) + F_I \;\; \Longrightarrow \;\; \boldsymbol{\sigma}^{-1} = (I - F_I) \cdot (O - F_O)^{-1} \qquad (5)$$

Combining equations (3) and (4), an outflows-inflows (outward-inward stock)balance may be derived, as indicated in equation (6):

$$(O - I) = \left[\boldsymbol{\sigma} \cdot (I - F_I) - \boldsymbol{\sigma}^{-1} \cdot (O - F_O)\right] + (F_O - F_I) \qquad (6)$$

Finally, a*general matrix equation expression* of a closed system is achieved, which may be considered the main formal representation of the model, out of which all the other expressions are derived, see equations (7) and (8), where E is anidentity matrix:

$$(O - F_O)(E + \boldsymbol{\sigma}^{-1}) = (I - F_I)(E + \boldsymbol{\sigma}) \qquad (7)$$

$$\frac{O - F_O}{I - F_I} = \frac{E + \boldsymbol{\sigma}}{E + \boldsymbol{\sigma}^{-1}} \qquad (8)$$

2 Calculation of the model and indicator of the model quality

Since (n x 1) - matrices are non-invertible (Anton, 2010), a *right-sidepseudo-inverse matrix*(the closest estimate of an inversematrix) has to be used to determine the shares matrices$\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^{-1}$; in this paper, the *rightMoore-Penrose pseudo-inverse,* $P = M^T \cdot (M \cdot M^T)^{-1}$, (Penrose, 1956), a unique pseudo-inverse matrix fulfilling four criteria[2]:a) general condition: $M \cdot P \cdot M = M$, b) reflexive condition: $P \cdot M \cdot P = P$, c) normalized condition: $(M \cdot P)' = P \cdot M$, and

d) reverse normalized condition: $(P \cdot M)' = M \cdot P$,(Penrose, 1956), where $M = (I - F_I)$ or $M = (O - F_O)$, is applied.P is determinedwith the help of singular value decomposition (SVD).

The model is calculated in *three consecutive steps*: Step 1 –estimation of $(O - F_O)$ and of $(I - F_I)$;Step 2 – computationof corresponding Pmatricesand estimation of model quality; Step 3 – calculation of shares matrix$\boldsymbol{\sigma}$and ofmatrix$\boldsymbol{B}$.

Successin$P$estimation in terms ofidentity $M \cdot P \approx E$, i.e. the variance of residuals $(E - M \cdot P)$,determines*model quality,* since the rest of the model calculationis deterministic. For the purpose of this paper, a specific*pseudo-determination index, pseudo R*, is derived from residuals' coefficient of variance, a ratio between standard deviation, *SD*,of residuals and average of residuals, with a correction of upper limit (acoefficient of variation has no upper bound and can exceed 1), see equation (9) (matrix data treated as cross-sectional data):

$$pseudo\ R = max\left[0,\ \ 1 - {SD_{(E - M \cdot P)}} \Big/ {\overline{(E - M \cdot P)}}\right] \tag{9}$$

Illustration of calculation of the model (a fictional problem):"*Trade matrix between country A, country B and the rest of the world (RoW) forthe year 19XX is givenin Tab. 2, but the data are incomplete. Estimation of*β_{ij}*(***B***)is needed for more profound conclusions."*

Tab. 2: Example,relationship matrix, flows, (3x3)

	Country A	Country B	*RoW*	Ouflows
Country A	n/a	n/a	n/a	**110**
Country B	n/a	n/a	n/a	**55**
RoW	n/a	n/a	n/a	**1135**

[2]The total number of pseudo-inverse matrices is infinite, but there is only one Moore-Penrose pseudo-inverse which ensures a unique solution to the matrix equation.

Inflows	95	55	1150	1300

Source: self-prepared

Step 1: For trade matrices(O – F$_O$) =O and (I – F$_I$) = I, as F$_O$ = F$_I$ = 0, see (10):

$$O - F = \begin{bmatrix} 110 \\ 55 \\ 1135 \end{bmatrix},\; I - FI = \begin{bmatrix} 95 \\ 55 \\ 1150 \end{bmatrix},\; \begin{bmatrix} 110 \\ 55 \\ 1135 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \cdot \begin{bmatrix} 95 \\ 55 \\ 1150 \end{bmatrix} \tag{10}$$

Step 2: The Moore-Penrose pseudo-inverse P is presented in(11):

$$P = [7{,}12 \cdot 10^{-5} \quad 4{,}12 \cdot 10^{-5} \quad 8{,}62 \cdot 10^{-4}] \tag{11}$$

pseudo R is considered to be zero.

*Step3: The estimated matrices**σ** and**B** arestated in (12):*

$$\boldsymbol{\sigma} = \begin{bmatrix} 7{,}83 \cdot 10^{-3} & 4{,}53 \cdot 10^{-3} & 9{,}48 \cdot 10^{-2} \\ 3{,}92 \cdot 10^{-3} & 2{,}27 \cdot 10^{-3} & 4{,}74 \cdot 10^{-2} \\ 8{,}08 \cdot 10^{-2} & 4{,}68\mathrm{E} \cdot 10^{-2} & 9{,}78 \cdot 10^{-1} \end{bmatrix},\quad \boldsymbol{B} = \begin{bmatrix} 0{,}74 & 0{,}25 & 109{,}01 \\ 0{,}37 & 0{,}12 & 54{,}50 \\ 7{,}68 & 2{,}57 & 1124{,}75 \end{bmatrix} \tag{12}$$

The result, ***B***, is interpretable and, most likely, reflects general trend partners among countries A, B and RoW, althougha minimumnumber of unitsled to an extremely low (zero) *pseudo R* and to certain distortions in the abovementioned estimations.

3 Case study

This part of the paper is dedicated toapplication of the model on areal-life example: *inward direct investment stock matrix of 2013* (latest available dataat the time of elaboration of this paper), including extrapolations of missing data (in total 232 countries and dependent territories). The data were taken fromUnctadStat database of the United Nations Conference on Trade and Development(UNCTAD). The reduced investment stock matrix(countries with total outward stock greater than 10 billion USD and inward stock greater than 25 billion inflows) is shown in Tab. 3. The *pseudo R* is ca. 0.5, hence the results are not exact, but they are able to capture the *main patternsof investment* cooperation between countries in the year 2013. All calculations were performed in the GNU Octave and Microsoft Excel (OpenOfficeCalc).

Tab. 3: Direct investment stock matrix, 2013, selected rows and columns, USD million

	Belgium	Brazil	British Virgin Islands	Canada	Chile	China	China, Hong Kong SAR	France
Argentina	764	470	189	372	42	819	1865	1046
Australia	10572	6502	2613	5151	575	11335	25816	14482
Austria	5334	3280	1318	2599	290	5719	13025	7307
Belgium	22609	13905	5587	11016	1229	24241	55210	30972
Brazil	6572	4042	1624	3202	357	7046	16047	9002
British Virgin Islands	11725	7211	2898	5713	637	12572	28633	16063
Canada	16411	10093	4056	7996	892	17596	40076	22482
Cayman Islands	2899	1783	716	1412	158	3108	7078	3971
China	13749	8456	3398	6699	747	14741	33574	18834
China, Hong Kong SAR	30303	18637	7488	14764	1647	32490	73998	41511
China, Taiwan Province of	5510	3388	1362	2684	300	5907	13454	7548
Czech Republic	479	295	118	233	26	514	1170	656
Denmark	5739	3530	1418	2796	312	6153	14014	7862
Finland	3638	2237	899	1773	198	3901	8884	4984
France	36684	22561	9065	17873	1994	39332	89581	50253
Germany	38323	23569	9470	18672	2084	41090	93584	52499
Greece	1039	639	257	506	56	1114	2536	1423
Hungary	888	546	219	432	48	952	2168	1216
India	2685	1651	664	1308	146	2879	6557	3678
Indonesia	360	221	89	175	20	386	879	493
Ireland	11268	6930	2785	5490	613	12082	27517	15436
Israel	1764	1085	436	859	96	1891	4307	2416
Italy	13408	8246	3313	6532	729	14376	32741	18367
Japan	22248	13683	5498	10840	1210	23854	54330	30478
Kazakhstan	653	401	161	318	35	700	1594	894
Korea, Republic of	4908	3019	1213	2391	267	5263	11986	6724
Luxembourg	4069	2503	1006	1983	221	4363	9937	5575
Mexico	3225	1983	797	1571	175	3457	7874	4417
Netherlands	24017	14771	5935	11701	1306	25750	58648	32900
New Zealand	414	254	102	202	22	444	1010	567
Norway	5179	3185	1280	2523	282	5552	12646	7094
Philippines	296	182	73	144	16	317	722	405
Poland	1232	758	304	600	67	1321	3008	1687
Portugal	1835	1128	453	894	100	1967	4481	2514
Russian Federation	11231	6907	2775	5472	611	12041	27425	15385
Saudi Arabia	881	542	218	429	48	944	2151	1206
Singapore	11156	6861	2757	5436	607	11962	27243	15283
South Africa	2146	1320	530	1045	117	2301	5240	2939
Spain	14413	8864	3562	7022	784	15453	35196	19744
Sweden	9769	6008	2414	4760	531	10474	23855	13382
Switzerland	28219	17355	6974	13749	1534	30256	68910	38657
Thailand	1313	808	325	640	71	1408	3207	1799
Turkey	735	452	182	358	40	788	1794	1006
United Arab Emirates	1416	871	350	690	77	1518	3457	1939
United Kingdom	42234	25974	10437	20577	2296	45283	103133	57856
United States	142275	87502	35159	69320	7735	152547	347432	194902
Venezuela (Bolivarian Republic of)	513	316	127	250	28	551	1254	703

	Germany	India	Indonesia	Ireland	Italy	Mexico	Netherlands	Norway
Argentina	648	46	47	128	146	135	402	33
Australia	8978	637	657	1766	2018	1874	5560	458
Austria	4529	321	331	891	1018	946	2805	231
Belgium	19200	1361	1405	3777	4317	4009	11891	980
Brazil	5581	396	408	1098	1255	1165	3456	285
British Virgin Islands	9957	706	729	1959	2239	2079	6167	508
Canada	13937	988	1020	2742	3133	2910	8631	712
Cayman Islands	2462	175	180	484	553	514	1524	126
China	11676	828	854	2297	2625	2438	7231	596
China, Hong Kong SAR	25733	1825	1883	5063	5785	5373	15937	1314
China, Taiwan Province of	4679	332	342	921	1052	977	2898	239
Czech Republic	407	29	30	80	91	85	252	21
Denmark	4874	346	357	959	1096	1018	3018	249
Finland	3089	219	226	608	695	645	1913	158
France	31153	2209	2280	6129	7004	6504	19293	1591
Germany	32545	2308	2382	6403	7317	6795	20156	1662
Greece	882	63	65	174	198	184	546	45
Hungary	754	53	55	148	169	157	467	38
India	2280	162	167	449	513	476	1412	116
Indonesia	306	22	22	60	69	64	189	16
Ireland	9569	679	700	1883	2151	1998	5926	489
Israel	1498	106	110	295	337	313	928	76
Italy	11386	807	833	2240	2560	2377	7052	581
Japan	18894	1340	1383	3717	4248	3945	11701	965
Kazakhstan	554	39	41	109	125	116	343	28
Korea, Republic of	4168	296	305	820	937	870	2581	213
Luxembourg	3456	245	253	680	777	722	2140	176
Mexico	2738	194	200	539	616	572	1696	140
Netherlands	20395	1446	1492	4013	4585	4258	12631	1041
New Zealand	351	25	26	69	79	73	218	18
Norway	4398	312	322	865	989	918	2724	225
Philippines	251	18	18	49	56	52	155	13
Poland	1046	74	77	206	235	218	648	53
Portugal	1558	110	114	307	350	325	965	80
Russian Federation	9537	676	698	1876	2144	1991	5907	487
Saudi Arabia	748	53	55	147	168	156	463	38
Singapore	9474	672	693	1864	2130	1978	5867	484
South Africa	1822	129	133	359	410	380	1129	93
Spain	12240	868	896	2408	2752	2556	7580	625
Sweden	8296	588	607	1632	1865	1732	5138	424
Switzerland	23964	1699	1754	4715	5388	5003	14842	1224
Thailand	1115	79	82	219	251	233	691	57
Turkey	624	44	46	123	140	130	386	32
United Arab Emirates	1202	85	88	237	270	251	745	61
United Kingdom	35866	2543	2625	7056	8063	7488	22212	1831
United States	120823	8568	8841	23771	27163	25226	74828	6169
Venezuela (Bolivarian Republic of)	436	31	32	86	98	91	270	22

	Poland	Russian Federation	Saudi Arabia	Singapore	Spain	Sweden	Switzerland	United Kingdom	United States
Argentina	57	296	39	628	459	128	500	2305	21783
Australia	787	4103	537	8688	6348	1770	6917	31917	301573
Austria	397	2070	271	4383	3202	893	3490	16103	152148
Belgium	1682	8775	1149	18580	13575	3786	14793	68258	644943
Brazil	489	2551	334	5400	3946	1100	4300	19840	187460
British Virgin Islands	872	4551	596	9636	7040	1963	7672	35400	334480
Canada	1221	6370	834	13487	9854	2748	10738	49547	468155
Cayman Islands	216	1125	147	2382	1740	485	1897	8751	82686
China	1023	5336	699	11299	8255	2302	8996	41508	392198
China, Hong Kong SAR	2254	11761	1540	24903	18194	5074	19827	91485	864413
China, Taiwa	410	2138	280	4528	3308	923	3605	16634	157166
Czech Republic	36	186	24	394	288	80	314	1447	13669
Denmark	427	2227	292	4716	3446	961	3755	17326	163710
Finland	271	1412	185	2990	2184	609	2380	10983	103779
France	2729	14238	1865	30147	22026	6142	24003	110751	1046447
Germany	2851	14874	1948	31494	23010	6417	25075	115699	1093206
Greece	77	403	53	854	624	174	680	3136	29628
Hungary	66	345	45	729	533	149	581	2680	25320
India	200	1042	136	2207	1612	450	1757	8107	76599
Indonesia	27	140	18	296	216	60	236	1087	10272
Ireland	838	4373	573	9260	6766	1887	7373	34019	321436
Israel	131	684	90	1449	1059	295	1154	5324	50307
Italy	998	5204	682	11018	8050	2245	8773	40478	382465
Japan	1655	8635	1131	18284	13358	3725	14557	67169	634654
Kazakhstan	49	253	33	536	392	109	427	1970	18615
Korea, Republic of	365	1905	250	4034	2947	822	3212	14818	140015
Luxembourg	303	1579	207	3344	2443	681	2663	12286	116082
Mexico	240	1252	164	2650	1936	540	2110	9735	91984
Netherlands	1787	9321	1221	19737	14420	4021	15714	72507	685097
New Zealand	31	161	21	340	248	69	271	1249	11803
Norway	385	2010	263	4256	3109	867	3388	15634	147723
Philippines	22	115	15	243	177	49	193	892	8432
Poland	92	478	63	1012	740	206	806	3719	35139
Portugal	137	712	93	1508	1102	307	1201	5540	52343
Russian Federation	836	4359	571	9229	6743	1880	7348	33906	320364
Saudi Arabia	66	342	45	724	529	147	576	2659	25122
Singapore	830	4330	567	9168	6698	1868	7300	33681	318241
South Africa	160	833	109	1763	1288	359	1404	6478	61209
Spain	1072	5594	733	11845	8654	2413	9431	43513	411144
Sweden	727	3791	497	8028	5865	1636	6392	29492	278665
Switzerland	2099	10952	1434	23190	16943	4725	18464	85195	804978
Thailand	98	510	67	1079	789	220	859	3965	37463
Turkey	55	285	37	604	441	123	481	2218	20954
United Arab Emirates	105	549	72	1163	850	237	926	4274	40384
United Kingdom	3142	16392	2147	34708	25358	7072	27634	127506	1204761
United States	10585	55220	7232	116922	85425	23823	93093	429537	4058555
Venezuela (Bolivarian Rep. of)	38	199	26	422	308	86	336	1550	14647

Source: self-prepared

Conclusion

As shown in the paper, the described linear bilateral flows / stockmodel is a predominantly deterministic, an easy to use tool for modelling bilateral flows and stock (trade, investment, value added etc.). It may be applied to historical statistics for better understanding of economic cooperation / interaction between countries and industries, for early estimations of input-output tables, as well as for correcting incomplete trade, investment and other relationship matrices.

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