THE DESIGN OF THE ACCEPTANCE SAMPLING PLANS WITH ENHANCEMENT IN ECONOMIC EFFICIENCY

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Abstract

The LTPD sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed by Dodge and Romig for the inspection by attributes. Plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of rejected lots is inspected by attributes) were then proposed and it was shown that these plans are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. This paper recalls some of the properties of the LTPD single sampling plans when the remainder of rejected lots is inspected and proposes new sampling plans for the inspection by variables when another statistic is used in the decision procedure. Considering the situation of a known standard deviation it is shown that the new plans perform well regarding the economic characteristics.

Key words: sampling inspection by variables, inspection cost, LTPDvar package

JEL Code: C44, C80

Introduction

Acceptance sampling is one of the most important quality control tools, used either in vendor-buyer relationships or for management of within-company processes. The aim is to meet desired levels of protection against risk while keeping an eye on economic characteristics of the process. Inference is made based on inspection of a sample of items taken from a lot.

There are many ways of classifying acceptance sampling. One such classification is according to whether an item is inspected by attributes, i.e. just classified as either good or defective (nonconforming) or by variables. Sampling plans for inspection by variables in many cases allow obtaining same level of protection as the corresponding sampling plans for inspection by attributes while using lower sample size. The basic notions of variables sampling plans are addressed in (Jennett and Welch, 1939).

The LTPD sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed by Dodge and Romig for the inspection by attributes. Plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of rejected lots is inspected by attributes) were then proposed and it was shown that these plans are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. LTPD plans for inspection by variables and attributes have been introduced in (Klůfa, 1994), using approximate calculation of the plans. Exact plans, using non-central t distribution in calculation of the operating characteristic, have been reported in (Klůfa, 2010) and implemented in (Kaspříková, 2012). The operating characteristics used for these plans are discussed in (Jennett and Welch, 1939) and (Johnson and Welch, 1940). It has been shown that these plans are in many situations superior to the original attribute sampling plans and similar results have been obtained for the AOQL plans – the analysis is provided in (Kaspříková and Klůfa, 2015) and in (Kaspříková and Klůfa, 2011).

Recent development of acceptance sampling plans includes the work (Aslam et al., 2015) where the EWMA statistic is used for a design of the (p_1, p_2) sampling plans, i.e. sampling plans which satisfy the requirement to control the producer's risk and the consumer's risk. Using the EWMA statistic enables some savings in the cost of inspection as it allows using information on the quality of the previous lots.

With the aim of getting an economically efficient acceptance sampling procedure, the new LTPD plans for the inspection by variables and attributes, which are designed using the EWMA statistics, are proposed in the present paper and a discussion of the economic performance of such plans is provided. The structure of the paper is as follows: the LTPD plans for the inspection by attributes are recalled first, then the LTPD plans for the inspection by variables and attributes using EWMA statistic are introduced and finally the analysis of the economic performance of the new plans is provided.

1 Sampling plans for the inspection by attributes

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes), Dodge and Romig (1998) consider sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\overline{p}; n; c) \tag{1}$$

under the condition

$$L(p_t; n; c) \le \beta, \tag{2}$$

where *L(p, n, c)* is the operating characteristic (the probability of accepting a submitted lot with proportion defective p when using plan *(n, c)* for acceptance sampling), N is the number of items in the lot (the given parameter), $\bar{p}$ is the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, $P_t = 100p_t$ is the lot tolerance per cent defective, denoted LTPD), n is the number of items in the sample $(n<N)$, c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c).

Condition (2) provides a guarantee for the consumer that lots of unsatisfactory quality level, with proportion defective p_t are going to be accepted only with specified probability β (consumer's risk). The value $\beta = 0.1$ is used for the consumer's risk in Dodge and Romig (1998).

2 LTPD plans for inspection by variables and attributes

The new LTPD plans for the inspection by variables and attributes are designed under the following assumptions:

The measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameter μ and known parameter σ^2. For the quality characteristic X there is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L).

The inspection procedure is as follows:

Draw a random sample of n items from the lot and compute sample mean $\bar{x}$ and the statistic T at time t as $T_t = \lambda\bar{x} + (1-\lambda)T_{t-1}$, where λ is a smoothing constant between 0 and 1. The values of the smoothing constant over 0.5 give more weight to the sample in the current lot. Accept the lot if

$$\frac{U - T_t}{\sigma} \ge k, \text{ or } \frac{T_t - L}{\sigma} \ge k. \tag{3}$$

Suppose that c_s^* is the cost of inspection of one item by attributes and c_m^* is the cost of inspection of one item by variables and that the sample is inspected by variables. Then the inspection cost per lot with proportion defective p, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is $n \cdot c_m^*$ with probability $L(p,n,k)$ and $\left[n \cdot c_m^* + (N-n) \cdot c_s^*\right]$ with probability $\left[1 - L(p;n,k)\right]$.

The mean inspection cost per lot of process average quality $\overline{p}$ is therefore

$$C_{ms} = n \cdot c_m^* + (N-n) \cdot c_s^* \cdot \left[1 - L(\overline{p};n,k)\right]. \tag{4}$$

Dividing (4) by c_s^* gives the objective function

$$I_{ms} = n \cdot c_m + (N-n) \cdot \left[1 - L(\overline{p};n,k)\right], \tag{5}$$

where $c_m = c_m^* / c_s^*$ is the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes (this parameter has to be estimated in each real situation, it is usually $c_m > 1$). Note that both the function $I_{ms} = C_{ms} / c_s^*$ and the function C_{ms} have a minimum for the same acceptance plan (n,k). Therefore, we shall look for the acceptance plan (n,k) minimizing (5) instead of (4) under the condition

$$L(p_t;n;k) = \beta \ . \tag{6}$$

Setting the value of c_m to 1 can be used in situations, when both sample and the remainder of rejected lots are inspected by variables. Acceptance sampling by variables can thus be considered just as a special case of acceptance sampling by variables and attributes. Then instead of I_{ms} we may use notation I_m and setting $c_m = 1$ in (5) we obtain

$$I_m = N - (N-n) \cdot L(\overline{p};n,k), \tag{7}$$

i. e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables.

The task to be solved is to determine plan (n,k) minimizing (5) under the condition (6) for given values of input parameters N, c_m, p_t and $\overline{p}$.

The operating characteristic is (see e.g. (Aslam et al., 2015))

$$L(p;n,k) = \Phi\left((u_{1-p} - k)A\right), \tag{8}$$

where

$$A = \sqrt{\frac{n(2-\lambda)}{\lambda}}. \tag{9}$$

The function Φ in (8) is a standard normal distribution function and u_{1-p} is a quantile of order $1-p$ (the unique root of the equation $\Phi(u) = 1-p$).

When the operating characteristic is in the form (8), if $\beta = 0.1$ we get the solution of the equation (6) for k as

$$k = \frac{u_{0.9}}{\sqrt{\frac{(2-\lambda)n}{\lambda}}} + u_{1-p_t}, \tag{10}$$

where $u_{0.9}$ is a quantile of order 0.9 of the standard normal distribution. Inserting formula (10) for k into the I_{ms} function, we obtain a function of one variable n

$$I_{ms}(n) = n \cdot c_m + (N - n) \cdot \alpha(n), \tag{11}$$

where $\alpha(n) = 1 - L(\overline{p}, n, k(n))$ is the producer's risk (the probability of rejecting a lot of process average quality $\overline{p}$). So we search for the sample size n minimizing (11).

3 Calculation and economic characteristics of LTPD plans for inspection by variables and attributes

We will calculate LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes in an example below. The task will be solved using the operating characteristic given by (8). The resulting sampling plan will be evaluated with regard to economic characteristics and compared with the corresponding Dodge-Romig plan in (Dodge and Romig, 1998).

Example. A lot with $N = 1000$ items is considered in acceptance procedure. Lot tolerance proportion defective is given to be p_t=0.01 and consumer's risk $\beta = 0.1$. It is known that average process quality is $\overline{p} = 0.001$. A cost of inspecting an item by variables is 50% higher than the cost of inspecting an item by attributes, so parameter c_m equals 1.5.

Find LTPD acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes, using the operating characteristic given by (8) and the EWMA statistic with smoothing constant 0.9.

The plan can be calculated using a modified version of the code available in LTPDvar package (Kaspříková, 2012) for R software (R Core Team, 2015). The solution based on operating characteristic given by (8), is $n = 20$, $k = 2.58555$.

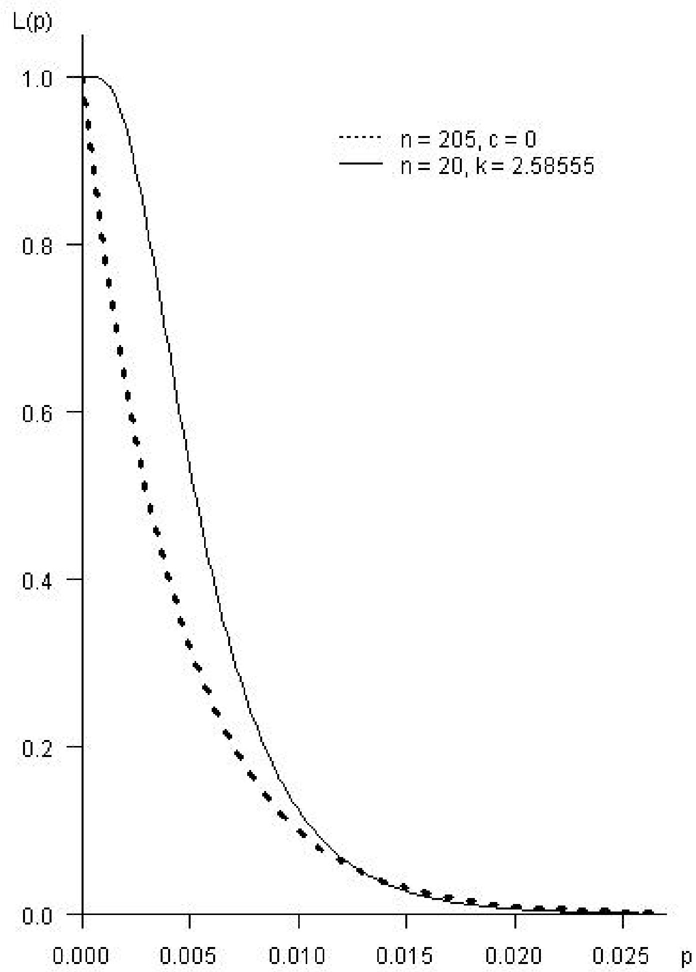
Plan $n = 20$, $k = 2.58555$ gives sufficient guarantees to consumer with regard to requirement (6).

Producer's risk of plan (20, 2.58555) is

$$\alpha = 1 - L(\overline{p}, 20, 2.58555) = 0.006. \tag{12}$$

For the values of input parameters given in our problem, there is plan (205, 0) for acceptance sampling by attributes in (Dodge and Romig, 1998).

Fig. 1: Operating characteristics of sampling plans



Source: the figure has been produced by the author in R software

Besides the fact that the acceptance sampling plan for inspection by variables and attributes has more favourable operating characteristic values (see Figure 1) than the corresponding sampling plan for inspection by attributes for the set of input parameters values considered in the problem solved – plan (20, 2.58555) gives better protection against risk both for the consumer and for the producer – the plan for inspection by variables and attributes is considerably more efficient with regard to economic characteristics for given ratio of cost of inspecting an item by variables to the cost of inspecting the item by attributes.

Let us compare plans (n=20, k= 2.58555) and (n=205, c=0) with regard to the economic efficiency.

For the comparison of the LTPD sampling plans for the inspection by variables (or by variables and attributes) and the corresponding Dodge-Romig LTPD sampling plans for inspection by attributes with regard to economic point of view we will use parameter e, defined as

$$e = \frac{I_{ms}}{I_s} \cdot 100. \tag{13}$$

Expression $(1-e)$ then represents the percentage of savings in mean inspection cost per lot of process average quality when sampling plan for inspection by variables and attributes is used in place of the corresponding plan for inspection by attributes.

Let us denote plan for inspection by variables and attributes as (n_1, k) and the corresponding plan for inspection by attributes as (n_2, c) then it is

$$e = \frac{n_1 \cdot c_m + (N - n_1) \cdot \left[1 - L(\overline{p}, n_1, k)\right]}{N - (N - n_2) \cdot L(\overline{p}, n_2, c)} \cdot 100. \tag{14}$$

Since for $(n_1, k) = (20, 2.58555)$ and $(n_2, c) = (205, 0)$, we get

$$e = 9.8,$$

it can be expected that over 90% savings in inspection cost can be achieved using the LTPD plan for inspection by variables and attributes in place of the corresponding Dodge-Romig plan.

Conclusion

The new LTPD plans for the inspection by variables and attributes minimizing the mean inspection cost per lot of process average quality, which are designed to use the EWMA statistics in the decision procedure, have been proposed and it has been shown that these plans are quite promising with respect to the economic characteristics.

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