

# ESTIMATION OF THE NUMBER OF DESCENDANTS OF PENSIONERS OF THE GIVEN YEAR OF BIRTHS

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## Abstract

The growth of the financial burden of the Czech PAYG pension system is a theme often discussed not only by demographers. A very simple and a very rough measure of the financial stability of the system is the average number of persons in productive age per one person in post-productive age (the reciprocal value of so-called old-age-dependency ratio).

This measure of course does not take into account the differences in the number of children among individual people but also the differences in the average number of children among people by the year of birth. Nevertheless the average number of children of women of younger generations (born in the seventies and later) will be probably lower than the average number of children of women born in the sixties or before.

The paper brings estimated average number of descendants in productive age per one woman in post-productive age of the given year of births. These numbers are compared with the “global” average number of persons in productive age per one person in post-productive age (not distinguishing the year of births of the pensioners). As the upper limit for productive age will be used in the computations the actual retirement age in the Czech Republic (which is permanently increasing), instead of the usually used constant limit of 65 years.

**Key words:** ageing of the population, retirement age, pension system

**JEL Code:** AO, BB

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## Introduction

One of the frequently discussed themes concerning the whole of society is the continuing ageing of the population and its anticipated consequences in the economic, social and political sphere. The population ageing of the Czech population will continue in the next decades not only due to low fertility but also because of expected permanent decrease in mortality (Arltová, Langhamrová & Langhamrová, 2013). The financial burdening of the pension system is usually estimated by the relationship of the number of persons of post-productive

age to the number of persons of productive age. But this ratio does not distinguish and reflect the differences in fertility of separate cohorts of females.

This paper presents a simple estimate of the average number of descendants in productive age per one person in retirement age for separate birth cohorts of females. The upper bound of productive age is considered not to be the usually used age of 65 years, which is (or was until recently) the usual retirement age in many European countries. From the economic point of view an important boundary is the age when there is usually entitlement to an old-age pension (hereafter referred to as retirement age). In the Czech Republic the retirement age is rising steadily since 1996 and according to present legislation (Zákon 155/1995 Sb.) it will continue to do so. Taking into account the actual retirement age in the Czech Republic will change the result values and trends of many characteristics of the financial burden of the pension system. See, e.g. Fiala (2013)

An alternative limit for old age proposed is, for example, that age at which the average (remaining) life expectancy is equal to 15 years (Sanderson & Scherbov, 2010).

## 1 Description of the model

The number of descendants of an individual persons cannot be obtained from the statistical and demographic data available. But it is possible to estimate the size of population of descendants and their sex-and-age distribution for a cohort of women born in a given age.

### 1.1 Distribution of the live born children by the year of births of their mothers

Let's denote  $N_t^{(g)}$  the number of children live born in the year  $t$  to females of the year of births  $g$ . If the number of live births by the year of births of the mother are not available it can be simply estimated by the number of births by the age of the mother

$$N_t^{(g)} = \frac{N_{t,t-g-1} + N_{t,t-g}}{2} \text{ if } g=t-14, t-15, \dots, t-49, N_t^{(t-50)} = \frac{N_{t,49}}{2} + N_{t,50+} \quad (1)$$

where  $N_{t,x}$  is the number of children live born in the year  $t$  to females of the age  $x$ .

The (relative) distribution of children born in the year  $t$  according to the year of birth of their mothers can be defined by

$$n_t^{(g)} = \frac{N_t^{(g)}}{N_t} \quad (2)$$

where  $N_t$  is the total number of live birth in the year  $t$  (but children with unknown age of mother are excluded). Under the assumption that the fertility of females younger than 14 years or older than 51 years is zero we have for each year  $t$

$$\sum_{g=t-50}^{t-14} n_t^{(g)} = 1, \quad n_t^{(g)} = 0 \text{ if } g < t-50 \text{ or } g > t-14. \quad (3)$$

## 1.2 Estimate of the number and sex-and-age structure of the descendants of female cohorts born in the given year

According to the previous distribution of live born children by the year of births of their mothers we can estimate in the end of any year  $t$  the number of living descendants for each female cohort at the age  $x$  (i. e. the cohort of the year of births  $t-x$ ) by the following formula

$$D_{t,x} = \sum_{y=x-50}^{x-14} S_{t,y} \cdot n_{t-y}^{(t-x)}, \quad (4)$$

where  $S_{t,x}$  is the population size at the age  $x$  in the end of the year  $t$ .

Not only persons born in the Czech Republic but also immigrants are taken into account as descendants. The distribution of immigrants according to the year of births of their mothers is supposed to be the same as for the Czech population.

## 2 Data used for computation

For the Czech Republic the number of live birth by the age (at last birthday) of mother were available for each year of the period 1925–1992. The numbers of the children born by mothers in the five-year age group 45–49 years of age have been divided into one-year age groups of mothers. The numbers of live births according to the mothers' year of births have been estimated according to (1). Since 1993 until 2012 the numbers of live births by the mothers' year of births were directly available. The distribution of children born according to their mothers' year of birth was then computed according to (3) for each year of the period 1925–2012. For previous years this distribution has been supposed the same as in 1925, for later years the distribution of 2012 has been used.

The sex-and age structure for the period 1950–2012 are real data from the Czech Statistical Office (CZSO, 2013a). Since 2013 the latest available population projection from the CZSO was used (CZSO, 2013b).

## 3 Development of the number of descendants of persons in older ages

For the purpose of the pension system of PAYG type only the descendants in productive age are important. Due to the fact that an increasing number of young people are continuing to study after completing their elementary education, we shall not consider the lower limit of productive age to be 15 years, but 20. The upper bound of productive age very often used is

65 years. But up to 1995 the retirement age in the Czech Republic, was only 60 years for males and for females was even as low as 53–57 years (according to the number of children reared). Since then there has been a gradual rising of the retirement age, which is to continue even after exceeding the limit of 65 years. This is why the standard upper bound of productive age (65 years) is very misleading for the case of Czech Republic and it is more correct to replace it with the current retirement age. We shall thus consider the upper limit of productive age to be rising – equal to the retirement age at the given moment according to current legislation (See Tab. 1). For the sake of simplicity we assume that every woman brought up two children.

**Tab. 1: Retirement age according to current legislation (as of 31 December)**

Retirement age	1995	2010	2020	2030	2040	2050	2060	2070	2080	2090	2100
Males	60y	62y 2m	63y 8m	65y	66y 6m	68y	69y 4m	70y 10m	72y 2m	73y 8m	75y
Females with two children	55y	59y	61y 8m	65y	66y 6m	68y	69y 4m	70y 10m	72y 2m	73y 8m	75y

y–years, m–months

Source: author’s calculations based on Law No. 155/1995 Coll. (3)

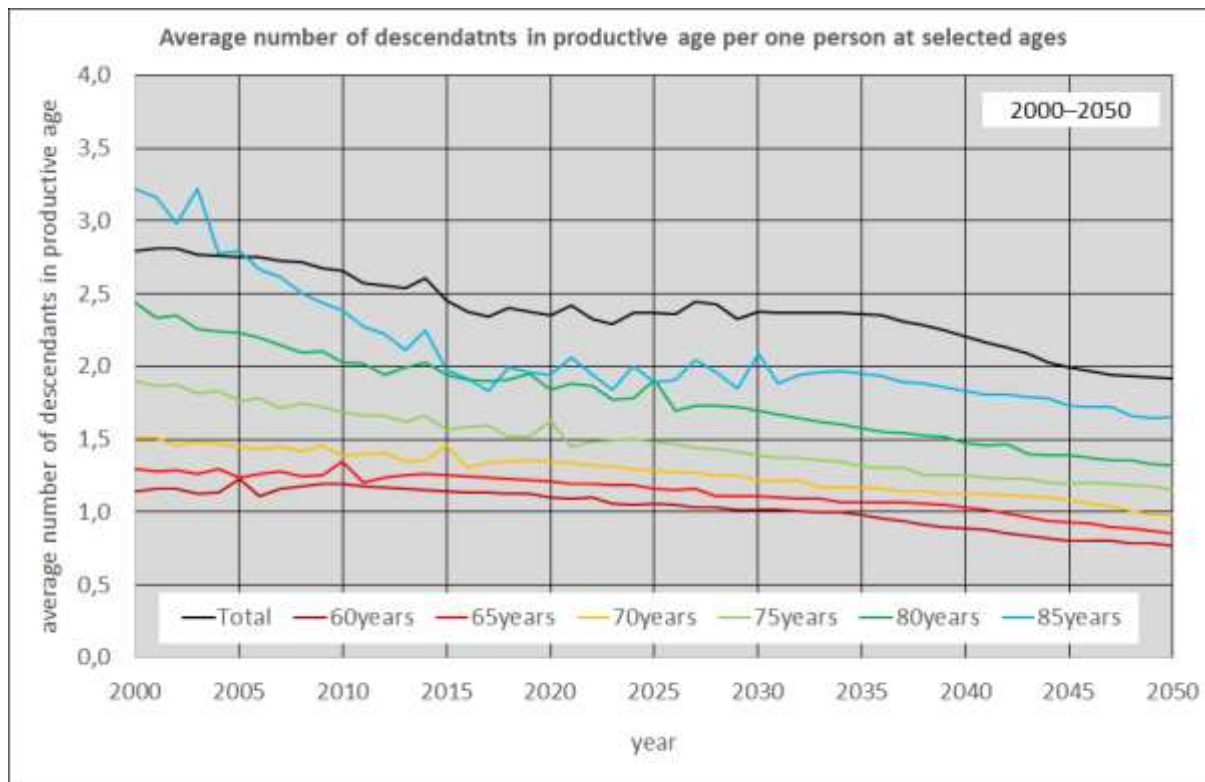
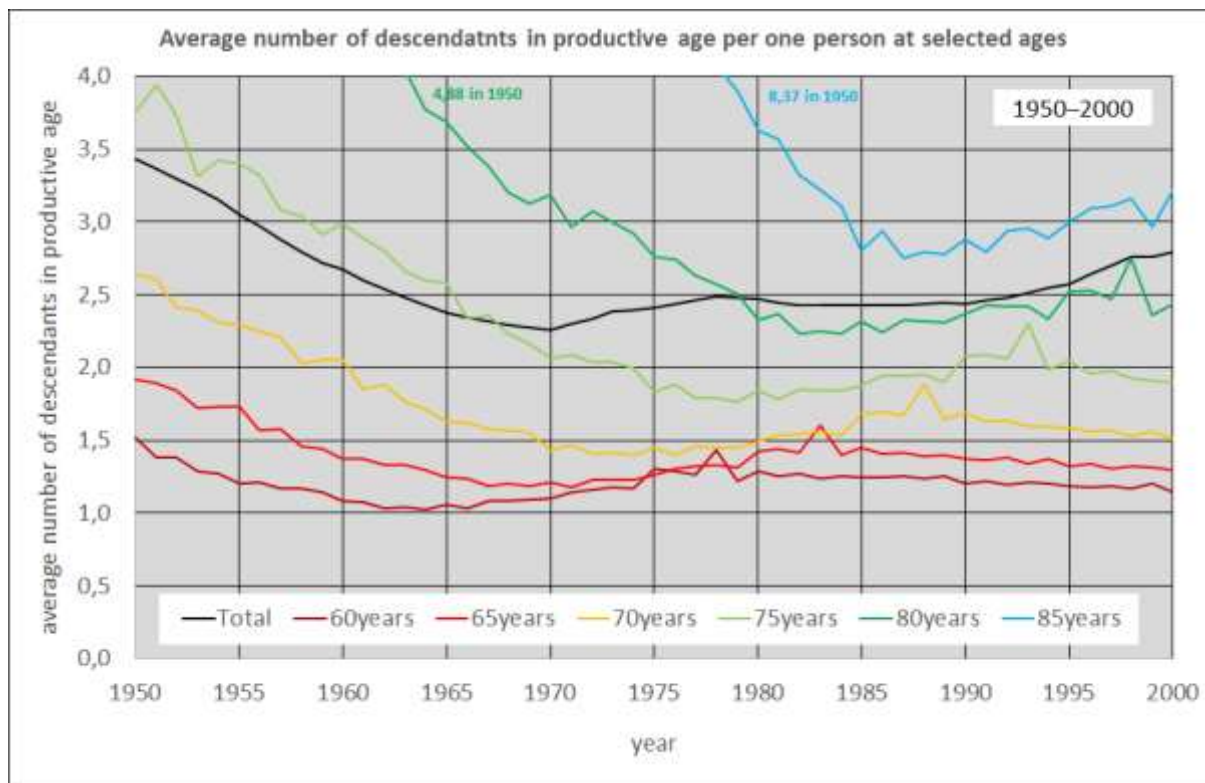
The computation of the number of descendants in productive age  $D_{t,x}^{(a)}$  were carried out according to the formula (4) with appropriate modification of its lower and upper bound, respectively. Not the absolute number of descendants but the average number per one person  $d_{t,x}^{(a)}$  is important for the pension system. The number of descendants has been so compared with the size of the corresponding mothers’ cohort and the size of cohort of males of the same year of births

$$d_{t,x}^{(a)} = \frac{D_{t,x}^{(a)}}{S_{t,x}}. \quad (5)$$

This average number of descendants per one person of the mothers’ cohort depends of course not only on the cohort’s fertility but especially for higher age  $x$  also on the mortality of the mothers’ cohort.

Development of the average number of descendants for selected years of age is shown in the following figure (Fig. 1).

**Fig. 1: Development of average number of descendants in productive age for selected ages**



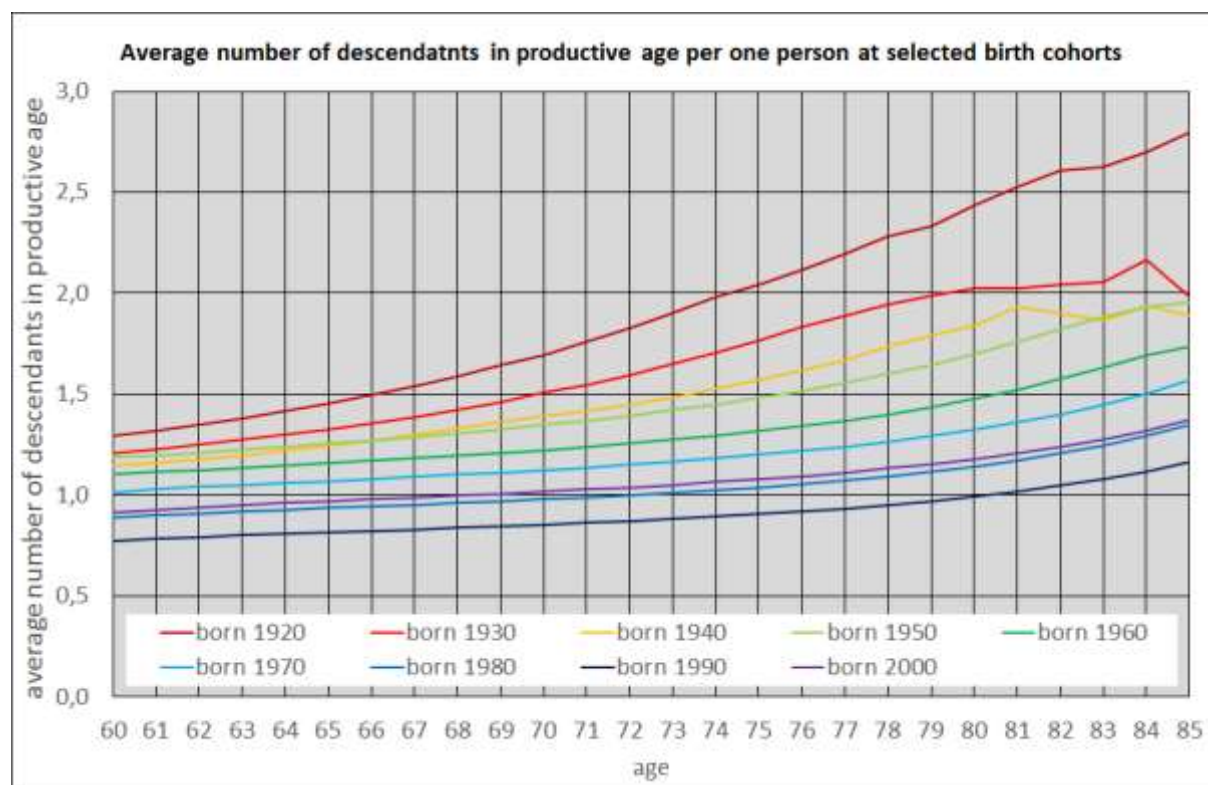
Source: own computations

Relatively rapid drop of the number of descendants in the second half of the previous century is caused not only by the decrease of fertility (from the cohort point of view the cohort fertility was relatively stable) but mainly by the decrease of mortality. The increasing number of people surviving to higher ages followed I the decrease of the average number of descendants per one older person. It can be also seen that the proportion of people of productive age per one person in retirement age reached its local minimum in 1970 and has been slightly increasing since then.

In the first half of this century (more precisely since 1996) the retirement age is permanently rising which will partly stabilize the value of the average number of descendants for some time. Nevertheless due to the continuing decrease in mortality and supposed low fertility the drop of average number of descendants for higher ages will continue.

From the cohort point of view for each cohort the average number of descendants in productive age is slightly rising due to the decreasing size of the cohort caused by mortality (see Fig. 2). Lowest number of descendants is expected for the cohort born in 1990 while the cohort of the year 2000 is expected to have the number of descendants a little bit higher than the cohort of 1980. This can be caused not only by expected increase in fertility but also by the rise of the retirement age.

**Fig. 2: Average number of descendants in productive age for selected births cohorts**



Source: own computations

## Conclusion

The average number of descendants in productive age of mothers' cohorts per one person changes in time and depends on the actual age of the cohort. It is affected not only by the initial number of children of the cohort (i. e. by its complete fertility) but of course also by the mortality (both of the cohort and of their children) and by migration. (Immigrants are also supposed to pay social insurance so they are taken into account in this computations.) Descendants born by women at higher ages may enter the productive age a little bit later than their mother will retire especially in the past years when the retirement age of females was lower than 60 years. On the other hand some females die before their descendants reach retirement age. This will be probably more often case in the future because of higher average age at births and mainly because of higher retirement age. Because of this the ratio of the number of persons in productive age to the number of persons in retirement age can be higher than the average number of descendants per one person.

The appropriate number of descendants is important from the demographic point of view to ensure the financial stability of the pension system. From the economic point of view it is necessary, however, that there should be sufficient suitable jobs for persons of higher age groups, especially those over 60 (otherwise the increase of retirement age has no sense). Especially in some regions it may be a little complicated issue. Due to economic crises unemployment in the Czech Republic has rising during last years. And its regional differentiation is relatively high (Löster & Langhamrová 2012). Especially the increasing long term unemployment in several regions (Löster & Langhamrová, 2011) is a problem not only from economical but mainly from the social point of view. Certain chance of increase of employment follows from the expected increasing level of education of the Czech population (see, e. g. Fiala, Langhamrová & Miskolczi, 2013).

## Acknowledgment

This article was supported by the Internal Grant Agency of University of Economics in Prague No. F4/24/2013 under the title *Mortality and Ageing of the population of the Czech Republic*.

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