MEAN-VARIANCE COEXCEEDANCE NETWORKS: PRELIMINARY RESULTS FOR CEE STOCK MARKETS

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Abstract

We present the preliminary results of an analysis of stock market networks consisting of 125 companies traded in CEE (Poland, Czech Republic, Hungary, Slovenia, Croatia, Latvia, Lithuania, Estonia, and Romania) and German stock markets. Instead of the usual networks based on return correlations among stocks, the networks are constructed using the principle of extreme return coexceedance while the resulting edges are weighted according to the normalized mean-variance return distances. To account for possible asymmetric effects, distinct networks are created separately for extreme positive and negative return coexceedance. The resulting networks are described by their density, measured as average vertex degree and closeness, measured by harmonic centrality. To provide a possible explanation for the time-varying properties of the networks, we employ the Exponential Random Graph Model (ERGM), allowing for inference on the attributes related to the creation of edges. Particularly, we analyse the interconnectedness of companies traded on the same market and within the same industry as explanatory variables for the formation of links within the network.

Key words: extreme coexceedance networks, mean-variance distances, emerging markets, CEE

JEL Code: G15, D85, L14

Introduction

[1]Co-movements between returns on domestic and foreign equities have far reaching consequences for policy makers and investors in their quest for portfolio diversification. In particular, co-movements are of special interest during bearish market conditions, market turmoil or financial crisis, when diversification benefits are most needed, and stability of the financial system is vulnerable. Unfortunately for investors, several of recent studies have shown that during the recent decades we are observing an increase in (international) market co-movements, and thus the diversification benefits have decreased (e.g. Babetskii et al.,

[1] This paper is a work in progress (as of July 2014) and part of an extended paper prepared for publication.

2007; Christiansen and Ranaldo, 2009; Lahrech and Sylwester, 2011; Kenourgios and Samitas 2011; Gjika and Horváth, 2013; Baumöhl and Lyócsa, 2014).

In this paper, we present preliminary results of studying coexceedance networks on a sample of 125 stock of CEE (Poland, Czech Republic, Hungary, Slovenia, Croatia, Latvia, Lithuania, Estonia, and Romania) and German stock markets. Coexceedance networks are a new approach which combines three different strands in the literature: (i) coexceedance approach for measuring contagion among stock markets (Bae et al., 2003; Christiansen and Ranaldo, 2009), (ii) risk-return similarity to measure market co-movement (Eun and Lee, 2010) and (iii) network approach for studying the structure of relationships among equities (e.g. Mantegna, 1999).

Most recent studies studying co-movement between CEE and developed stock markets have shown an increase in co-movement. For example using a sample of data from January 1995 to July 2006, Babetskii et al. (2007) found evidence for the β- and σ-convergence of returns between the Czech, Polish, and Hungarian national markets and the Euro Stoxx Index. Christiansen and Ranaldo (2009) have constructed variables, which measure occurrence of extreme positive/negative returns[2] between new EU member states (2004 and 2007 accession except Malta, Cyprus, and Bulgaria) and old EU member states. Among others they show, that the stock markets of the new EU member states are more exposed to adverse co-movements after their accession. Dependence between Balkan stock markets (Turkey, Romania, Bulgaria, Croatia, and Serbia) and developed markets (UK, Germany, Greece, and the US) have been studied by Kenourgios and Samitas (2011). On a sample of daily stock market prices from January 2000 to February 2009, they found evidence of long-run relationship of Balkan stock markets with those of developed markets (cointegration) but also evidence of short-run relationship (Dynamic Conditional Correlations, DCCs), particularly they show that in general, during the financial crisis (since 26th September 2008 until the end of their sample) the correlations have increased.

The ever popular DCCs were also utilized by Syllignakis and Kouretas (2011), Gjika and Horváth (2013), and Baumöhl and Lyócsa (2014). Using weekly data from October 3, 1997 through February 13, 2009, Syllignakis and Kouretas (2011) found that among six CEE markets, returns on stock markets indices of Czech, Polish, and Hungarian markets had highest conditional correlations with those of US and German stock market indices. Further on, they provide evidence that during the recent financial crisis, the DCCs have increased for

[2] Below 5th percentile or above the 95th percentile on a respective market

all CEE markets. Utilized the asymmetric version of the DCCs using daily observations of CEE-3 stock market indices (PX, BUX, and WIG) and the STOXX-50, from December 20, 2001 to October 31, 2011, Gjika and Horváth (2013) have provided further evidence that market co-movements intensified during the recent financial crisis, but also during times of higher market volatility. Similarly, Baumöhl and Lyócsa (2014) covered a set of 32 emerging and frontier stock markets (among CEE markets Poland, Czech Republic, Hungary, and Romania) and showed that during the period from January to December 2012 and with weekly data frequency co-movements of CEE markets have increased with the MSCI World stock market index (positive linear trend) and that they tend to be higher during periods of higher market volatility.

As far as we are aware, studies covering CEE markets have exclusively focused on co-movements between stock market indices. Such an approach is understandable, as the number of bivariate comparisons makes the interpretations complicated. One of the possible remedies is offered by the network theory, which can be used to study the structure of results (networks) on a global (network) level. Moreover, rather than studying market correlations, co-integration relationships or other model based approaches (e.g. market models as CAPM), we measure extreme market co-movement by utilizing the risk-return distances proposed in Eun and Lee (2010). The structure of the remaining part of the paper is as follows: In Section 1 we present the methodology, Section 2 describes the data used in this study. Section 3 contains the empirical results, while the final section concludes.

1 Methodology

1.1 Coexceedance networks

Dependence between stocks can be represented via networks, where stocks represent vertices and edges occurrence of a particular type of a relationship. Edges which represent a correlation based distance[3] have been used in numerous studies before (e.g. Mantegna, 1999; Lyócsa et al., 2012). A *negative coexceedance network* at time t can be defined as a graph $G(V, E_t)_t$, where V is the set of all stocks and E_t is a set of edges. An edge $E_t(i,j)$ between stock i and j exists, when $r_{it} < r_{it}(5) \wedge r_{jt} < r_{jt}(5)$, where $r_{it}(5)$ and $r_{jt}(5)$ are the 5th percentile of returns for market i and j respectively. The principle of return coexceedance is simiar as in Bae et al., (2003) or Christiansen and Ranaldo (2009). Next, we define a *mean-variance return distance weighted negative coexceedance network* (MV negative coexceedance

[3] $d = (2(1 - \rho))^{0.5}$, with ρ being the correlation coefficient between returns of two stocks.

network in short). We first calculate the normalized mean-variance return distances (rr_{ijt}, $i \neq j$) for each pair of stocks as in Eun – Lee (2010):

$$rd_{ijt} = \left| r_{it} - r_{jt} \right| \tag{1}$$

$$sd_{ijt} = \left| s_{it} - s_{jt} \right| \tag{2}$$

$$w(rd)_{ij} = \left(\sum_{t=1}^{T} rd_{ijt}^2 \Big/ \left(\sum_{t=1}^{T} rd_{ijt}^2 + \sum_{t=1}^{T} sd_{ijt}^2 \right) \right)^{0.5} \tag{3}$$

$$w(sd)_{ij} = \left(\sum_{t=1}^{T} sd_{ijt}^2 \Big/ \left(\sum_{t=1}^{T} rd_{ijt}^2 + \sum_{t=1}^{T} sd_{ijt}^2 \right) \right)^{0.5} \tag{4}$$

$$rr_{ijt} = \left(\left(rd_{ijt} / w(rd)_{ij} \right)^2 + \left(sd_{ijt} / w(sd)_{ij} \right)^2 \right)^{0.5} \tag{5}$$

where r_{it} is the return of i-th stock in weak t and s_{it} is the standard deviation[4] of i-th stock's return in weak t. The normalized mean-variance return distances rr_{ijt} measure the similarity of the first two moments of return distribution. Each edge of networks is labeled according to a corresponding rr_{ijt}. We use these weights to measure how similar are returns between stocks during the occurrence of extreme market co-movements. Positive coexceedance networks are defined in similar manner.

1.2 Networks properties

Two properties of networks are observed: network density and network centrality. Vertex degree $deg(i)$ is measured as number of edges incident to the vertex i. An average vertex degree $ADEG_t$ is a measure of network's density. When our networks will have higher network density, it suggests that extreme mean-variance spillovers are more frequent, which has implications for portfolio managers (rebalancing of their positions) and policy makers (stability of the financial system).

To measure vertex closeness we use harmonic centrality (see Boldi – Vigna, 2013) which is more suited for networks that are not necessarily strongly connected:

$$hc(i) = \sum_{d(i,j)<\infty, i \neq j} \frac{1}{d(i,j)} \tag{6}$$

where $d(i, j)$ is the length of a shortest path from vertex i to j and it is ∞ if vertex j is not reachable from vertex i. Consequently, we calculate network's centrality AHC_t, as an average of each vertex's harmonic centrality.

[4] In this preliminary version of the paper we use weekly data frequency constructed from daily closing prices. As a consequence of non-trading for particular markets or stocks, there are several instances, where no observations for a pair of stocks are observed. In general, these cases are neglible. However, there are many instances with less than 5 observations for a given week. We still used standard deviation, as a proxy for overall variance of returns. When only one observation was available we used the absolute value of return. These issues will be negligible with monthly data frequency.

1.3 Exponential random graph modelling (ERGM)

To describe the nature of the links in the network, as well as the dynamics of their evolution in time, we employ an Exponential Random Graph Model (ERGM). The ERGMs are an extension of a logit model within the context of a network (Wasserman – Pattison, 1996) – given a set of vertices, the model describes the probabilities of creating edges between them, given some exogenous attributes associated with these vertices and possibly some graph topological properties. The exogenous variables for international stock market data may include information such as the country of origin and industry membership for each stock. An example for a topological property is the number of triangles – complete subgraphs on three vertices – the network should possess (e.g. to achieve a high number of triangles, the network has to have a large number of edges, hence the probability of creating an edge is higher).

Formally, an ERGM modelling an empirical graph x can be described by the probability:

$$P(X = x \mid \boldsymbol{\theta}) = \frac{\exp\left(\boldsymbol{\theta}^T s(x)\right)}{c(\boldsymbol{\theta})} \tag{7}$$

where X is an random graph, $\boldsymbol{\theta}$ is the vector of model parameters, $s(x)$ are any characteristics derived from vertex attributes or graph properties and $c(\boldsymbol{\theta})$ is a normalizing constant.

2 Data

Daily closing prices are obtained from Datastream. Our sample starts from 02.01.2006 and end in 31.12.2013. Equities were selected from six CEE and German stock market; Croatia (#18), Czech Republic (#8), Estonia (#8), Hungary (#10), Latvia (#10), Lithuania (#19), Poland (#14), Romania (#4), Slovenia (#4), and Germany (#30 constituents of the DAX index as of December 2013). A complete list is available upon request. In our analysis we used weekly returns (averaged daily continuous returns) and (population) standard deviation of daily returns for a given week (T = 418).

3. Empirical results

3.1 Descriptive evidence

First interesting observation is, that at least within our sample, the weekly coexceedance networks seem to be volatile as during many periods, the number of edges is zero and for others up to 4166 for negative, or 1418 for positive coexceedance network.

Particularly for negative coexceedance networks, 4166 edges correspond to 53.75% of all possible edges, i.e. in week from 6th October 2008 to 10th October 2008, just about one month after the bankruptcy of Lehman Brothers Holdings Inc., more than half of all possible daily extreme coexceedances occurred, which also corresponds to 16.87% of all extreme coexceedances for the whole sample period. This seems like a case for contagion.

In Fig. 1 we have plotted selected coexceedance networks. Most of the time only few extreme coexceedances occur, however, there are several occasions, where equities on markets show higher levels of extreme co-movement, and it turns out, that these co-movements are extreme (see Fig. 1 panels c – f).

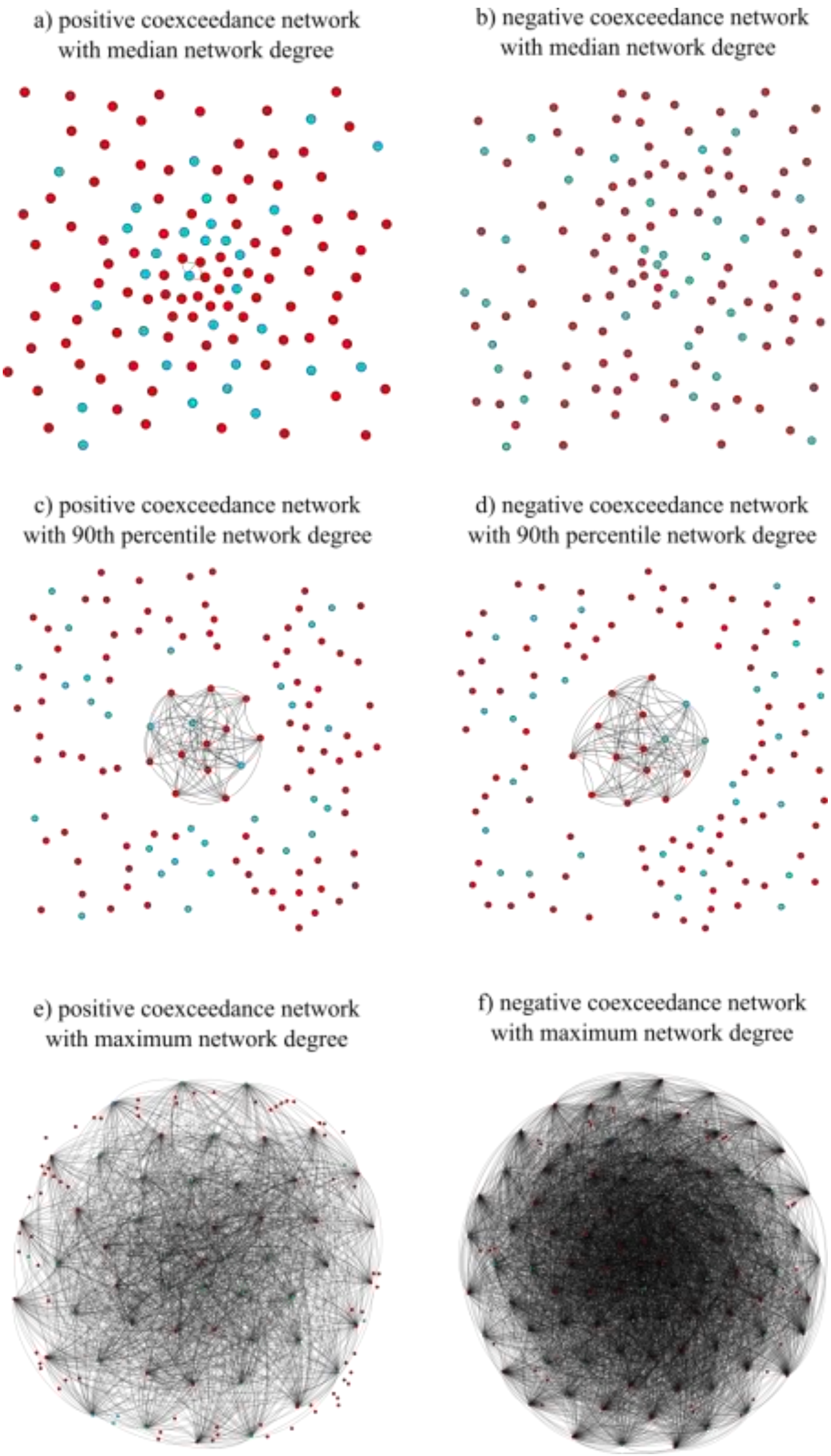
Tab. 1: Descriptive statistics of the number of edges in the coexceedance networks and network's closeness measure

	min	5th	10th	25th	median	75th	90th	95th	max	Mean	sd
Number of edges											
Positive	0.00	0.00	0.00	1.00	5.00	17.00	87.00	171.90	1418	34.02	106.24
Negative	0.00	0.00	0.00	0.00	2.00	13.00	99.60	193.00	4166	59.06	290.99
Harmonic centrality											
Positive	0.0000	0.0000	0.0000	0.0001	0.0007	0.0025	0.0115	0.0223	0.1838	0.0045	0.0138
Negative	0.0000	0.0000	0.0000	0.0000	0.0003	0.0018	0.0132	0.0249	0.5388	0.0077	0.0377
Harmonic centrality - weighted											
Positive	0.0000	0.0000	0.0000	0.0007	0.0098	0.0432	0.2012	0.4506	3.0840	0.0817	0.2494
Negative	0.0000	0.0000	0.0000	0.0000	0.0029	0.0306	0.2668	0.5534	6.6380	0.1480	0.6398
Risk-return distances											
Positive	0.0000	0.0000	0.0000	0.0450	0.0952	0.1491	0.2295	0.2849	2.4830	0.1210	0.1930
Negative	0.0000	0.0000	0.0000	0.0000	0.0804	0.1401	0.2239	0.2892	1.0280	0.0966	0.1096
Kolmogorov-Smirnov test : equality of risk-return distributions											
Test statistics	0.1268	***		p-value	0.0024						

Source: own calculations

Table 1 presence the descriptive evidence for high volatility of extreme coexceedances. It is also evident, that the behavior of extreme co-movement differs according to whether bearish or bullish market conditions are considered. Negative extreme coexceedance takes more extreme values, and more importantly, similarity of returns (risk-return distances) is higher during bearish market conditions. This highlights the importance of portfolio diversification with regard to extreme downside risks.

Fig. 1: Negative and positive coexceedance networks

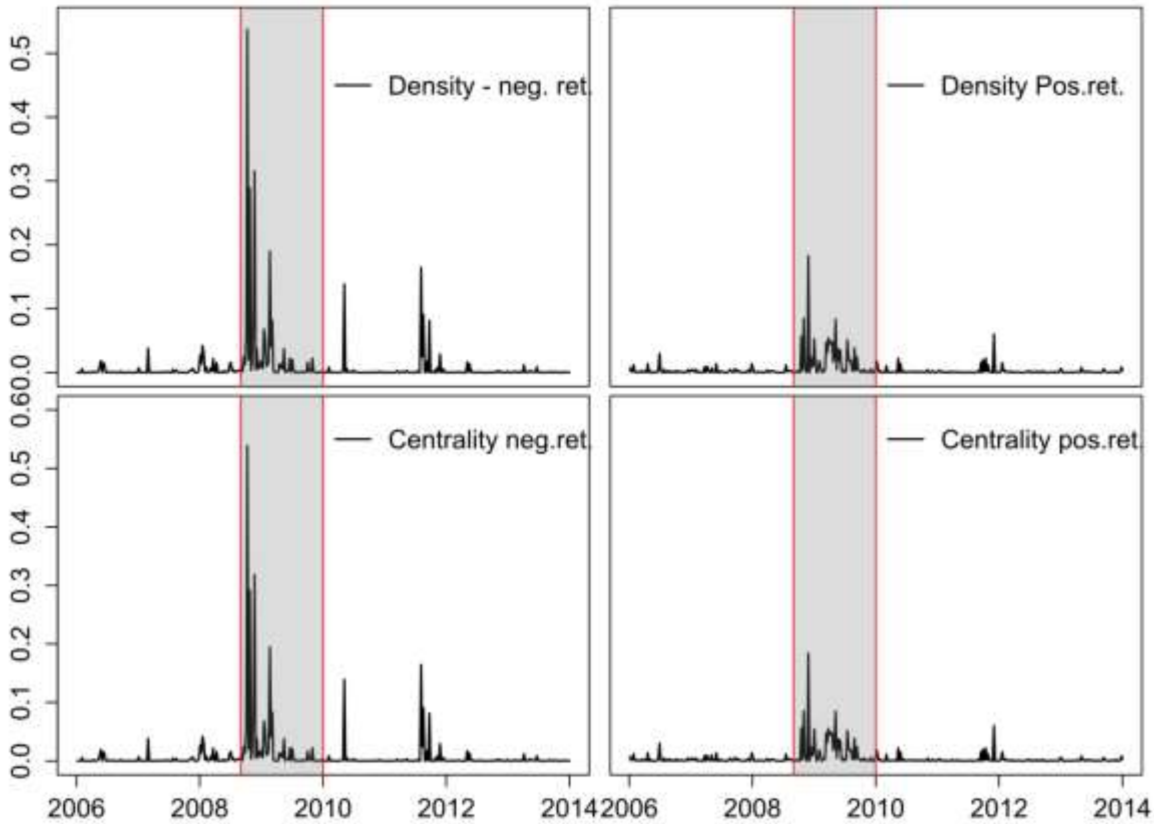


Note: red nodes correspond to CEE equities, while blue to German equities.
Source: in R and Gephi

In Fig. 2 we plotted both network measures for both negative and positive coexceedance networks. There are two interesting observations. The first is, that during the financial crisis dated here as a period from September 2008 to December 2010, the centrality and the density (number of edges) of the network increased, particularly an increase is visible for network's density. This shows how severe the crisis actually was; at some points, compared to tranquil periods, the density of the network increased by tenfold, which makes the task of the portfolio manager very difficult as mean-volatility spillovers across equities are extreme.

Second, we can observe some asymmetry as changes in basic structure of coexceedances are less extreme for bullish markets, i.e. the downside risk faced by investors seems to be greater. Such asymmetric effects are not new, but are usually shown on market or industry index levels.

Fig. 2: Network's density and centrality



Notes: the shaded area corresponds to a period from September 2008 to December 2009, i.e. the period of the financial crisis.
Source: in R

3.2 ERGM modelling

To understand and statistically evaluate the nature of the evolution of network properties described in the previous section, ERGM models have been fitted for both negative and positive coexceedance networks. To make the analysis tractable, two networks describing the overall interconnectedness during pre-crisis and crisis periods have been constructed. As the first step, we created a network consisting of the union of all edges in a given period with edge weights related to the total number of links between the vertices (weights were inversely proportional to the number of links). This allowed for the calculation of a minimum spanning tree (MST), greatly reducing the overall complexity of the network. Such MST networks have been constructed both for pre-crisis and crisis periods.

The ERGM has then been used for modelling of edges and the overall structure of the networks of these two periods. The model itself has used three explanatory attributes – the overall number of edges in a network, and the number of intra-industry, as well as intra-market links. The results are shown in Tab.2.

Tab. 2: ERGM models for ceoxceedence network models

	Positive coexceedance				*Negative coexceedance*			
		Est.	Std. Error			Est.	Std. Error	
Pre-crisis	Edges	-4.3709	0.1144	***	Edges	-4.5405	0.1229	***
	Market	0.5720	0.2326	*	Market	1.5553	0.1870	***
	Industry	0.4709	0.2445	.	Industry	0.1998	0.2539	
		Est.	Std. Error			Est.	Std. Error	
Crisis	Edges	-4.5242	0.1222	***	Edges	-4.568	0.1240	***
	Market	1.4316	0.1924	***	Market	1.5489	0.1871	***
	Industry	0.1817	0.2599		Industry	0.3823	0.2399	

Notes: The coefficient for Industry could not be calculated for dissolution of negative coexceedance networks, as the number of inter-industry links during the crisis period was low. "***" - denotes statistical significance at 0.001, "**" at 0.01, "*" at 0.05 and "**.**" at 0.10.
Source: own calculations

Conclusion

By analyzing the coexceedence networks constructed during and outside of the crisis periods, we show that the crisis manifests itself also in the structure of the network of stock's negative and positive extreme coexceedances.

First, the results show that the crisis period is characterized by substantially higher network densities, which correspond to extreme interconnectedness. This may have a substantial impact on the efficiency of portfolio diversifications strategies, whose benefits rely on low correlation of returns. Second, the results indicate an asymmetry in these effects, which are more pronounced for bearish markets.

Third, the nature of the stock linkages has been explored using an ERGM model. We show that the minimum spanning trees of aggregated extreme coexceedence networks may be explained more by market factors, as the existence of links is significant for vertices from the same country. A similar statement could not be shown for stock from the same industries – thus, the results indicate a dominance of country over industry effects. It further suggests that a potential diversification strategy should exploit market membership over industry membership of a stock even in times of financial crisis.

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