ECONOMIC EFFICIENCY OF THE AOQL PLANS BY VARIABLES WHEN THE REMAINDER OF REJECTED LOTS IS INSPECTED

Jindřich Klůfa

Abstract

In this paper we shall deal with the AOQL single sampling plans when the remainder of rejected lots is inspected. We shall consider two types of the AOQL plans - for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected by attributes). These plans were created by author of this paper and published in Statistical Papers. These new plans we shall compare with the corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view. From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans (saving of the inspection cost is 70% in any cases). Dependence of the saving of the inspection cost on acceptance sampling characteristics is analyzed in this paper.

Key words: Acceptance sampling, inspection by variables, economical aspects

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Introduction

Under the assumption that each inspected item is classified as either good or defective (inspection by attributes – see e.g. (Hald, 1981)), Dodge and Romig consider sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\bar{p}; n, c) \tag{1}$$

under the condition

$$\max_{0<p<1} AOQ(p) = p_L \tag{2}$$

(AOQL single sampling plans), where N is the number of items in the lot (the given parameter), $\bar{p}$ is the process average fraction defective (the given parameter), p_L is the average outgoing quality limit (the given parameter, denoted AOQL), n is the number of items in the sample $(n < N)$, c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c), $L(p)$ is the operating characteristic (the probability of accepting a submitted lot with fraction defective p), $AOQ(p)$ is average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was p). Condition (2) protects the consumer against the acceptance of a bad lot, average outgoing quality is is less or equal to p_L (the chosen value) for each fraction defective p before inspection. The AOQL plans for inspection by attributes are extensively tabulated – see (Dodge and Romig, 1998).

The corresponding AOQL plans for inspection by variables were introduced in (Klůfa, 1997) - the basic notions of variables sampling plans are addressed in (Jennett and Welch, 1939). Calculation of these plans when the non-central t distribution is used for the operating characteristic (see (Johnson and Welch, 1940)) is considerably difficult. This problem was solved in (Klůfa, 2008), exact solution is in (Kaspříková, 2011) – LTPDvar is an add-on package to the R software (see (R Development Core Team, 2008)). Similar problems are solved in (Klůfa, 1994), (Klůfa, 2010), (Chen and Chou, 2001), (Kaspříková and Klůfa, 2011), (Loster and Pavelka, 2013), (Wilrich, 2012) and (Aslam et al. 2013).

1 AOQL plans by variables and attributes

The problem to find AOQL plans *for inspection by variables* has been solved in **(**Klůfa**,** 1997) under the following assumptions:

Measurements of a single quality characteristic *X* are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic *X* is given either an upper specification limit *U* (the item is defective if its measurement exceeds *U*), or a lower specification limit *L* (the item is defective if its measurement is smaller than *L*). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s.

The inspection procedure is as follows: Draw a random sample of *n* items and compute $\bar{x}$ and *s*. Accept the lot if

$$\frac{U-\bar{x}}{s} \geq k, \quad \text{or} \quad \frac{\bar{x}-L}{s} \geq k. \tag{3}$$

We have determine the sample size *n* and the critical value *k*. There are different solutions of this problem. In paper **(**Klůfa**,** 1997) we used for determination *n* and *k* a similar conditions as Dodge and Romig.

Now we shall formulate this problem. Let us consider *AOQL plans for inspection by variables and attributes* – all items from the sample are inspected by variables, but the remainder of rejected lots is inspected only by attributes. Let us denote

c_s^* - the cost of inspection of one item by attributes,

c_m^* - the cost of inspection of one item by variables.

Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is $n \cdot c_m^*$ with probability $L(p;n,k)$, and $n \cdot c_m^* + (N-n) \cdot c_s^*$ with probability $1 - L(p;n,k)$. The mean inspection cost per lot of process average quality is therefore

$$C_{ms} = n \cdot c_m^* + (N-n) \cdot c_s^* \cdot \left[1 - L(\bar{p};n,k)\right] \quad (4)$$

Now we shall look for the acceptance plan (n,k) minimizing the mean inspection cost per lot of process average quality C_{ms} under the condition (2). The condition (2) is the same one as used for protection the consumer Dodge and Romig. Let us introduce a function

$$I_{ms} = n \cdot c_m + (N-n) \cdot \left[1 - L(\bar{p};n,k)\right], \quad (5)$$

where

$$c_m = c_m^* / c_s^*. \quad (6)$$

Since

$$C_{ms} = I_{ms} \cdot c_s^*, \quad (7)$$

both functions C_{ms} and I_{ms} have a minimum for the same acceptance plan (n,k). Therefore, we shall look for the acceptance plan (n,k) minimizing (5) instead of (4) under the condition (2). For these AOQL plans for inspection by variables and attributes *the new parameter* c_m was defined – see (6). This parameter must be estimated in each real situation. Usually is

$$c_m > 1. \quad (8)$$

Putting formally $c_m = 1$ into (5) (I_{ms} in this case is denoted I_m) we obtain

$$I_m = N - (N-n) \cdot L(\bar{p};n,k), \quad (9)$$

i.e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables. Consequently *the AOQL plans for inspection by variables* are a special case of *the AOQL plans by variables and attributes* for $c_m = 1$. From (9) is evident that for the determination AOQL plans by variables it is not necessary to estimate c_m ($c_m = 1$ is not real value of this parameter).

Summary: For the given parameters p_L, N, $\bar{p}$ and c_m we must determine the acceptance plan (n, k) for inspection by variables and attributes, minimizing the function I_{ms} in (5) under the condition (2).

Solution of this problem is in the paper **(**Klůfa**,** 1997), numerical solution is in **(**Klůfa**,** 2008) and **(**Kaspříková**,** 2011).

2 Economic efficiency of the AOQL plans by variables and attributes

For the comparison of the AOQL single sampling plans for inspection by variables and attributes with the corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view we use parameter e defined by relation

$$e = \frac{I_{ms}}{I_s} \cdot 100 \tag{10}$$

According to (7) is

$$e = \frac{I_{ms}}{I_s} \cdot 100 = \frac{I_{ms}.c_s^*}{I_s.c_s^*} \cdot 100 = \frac{C_{ms}}{C_s} \cdot 100,$$

where $C_s = I_s\, c_s^*$ is the mean cost of inspection by attributes (c_s^* is the cost of inspection of one item by attributes). Therefore the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig plan when

$$e < 100.$$

Expression $(100 - e)$ then represents the percentage of savings in inspection cost when sampling plan for inspection by variables and attributes is used instead of the corresponding plan for inspection by attributes.

Economic efficiency measured by parameter e (see formula (10)) is a function of four variables p_L, N, $\bar{p}$ and c_m, i.e.

$$e = e(p_L, N, \bar{p}, c_m). \tag{11}$$

Some values of this function are in Table 1 and Table 2.

From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations **more economical** (saving of the inspection cost is 70% in any cases) than the corresponding Dodge-Romig attribute sampling plans – see also Table 1 and Table 2.

Tab. 1: Values of the parameter e for p_L=0,001

p_L=0,001	$c_m = 2$			$c_m = 3$			$c_m = 4$			$c_m = 5$			$c_m = 6$		
$\bar{p}\backslash N$	500	4000	50000	500	4000	50000	500	4000	50000	500	4000	50000	500	4000	50000
0,000100	34	26	19	46	36	26	57	45	33	67	54	40	77	62	47
0,000200	42	29	19	56	39	27	68	49	34	79	58	41	89	66	48
0,000300	48	31	22	63	42	31	76	51	39	87	60	47	98	68	54
0,000400	53	33	24	69	44	33	82	54	41	94	63	49	105	71	56
0,000500	58	38	27	74	50	36	88	60	45	99	70	53	110	78	60
0,000600	62	43	29	79	55	39	92	66	48	104	76	56	114	84	63
0,000700	66	48	33	83	61	43	96	72	53	108	81	61	118	90	69
0,000800	70	53	38	87	66	49	100	77	58	111	87	66	121	95	74
0,000900	74	58	45	90	72	57	103	82	66	114	91	74	124	99	82
0,001000	77	64	56	93	77	67	106	87	76	117	96	83	126	103	89

Source: Own construction

Tab. 2: Values of the parameter e for p_L=0,0025

p_L=0,0025	$c_m = 2$			$c_m = 3$			$c_m = 4$			$c_m = 5$		$c_m = 6$	
$\bar{p}\backslash N$	500	4000	50000	500	4000	50000	500	4000	50000	4000	50000	4000	50000
0,000250	46	34	28	63	47	40	78	59	51	71	61	82	71
0,000500	54	32	22	73	44	31	89	55	39	66	47	75	55
0,000750	60	43	31	79	58	43	96	71	54	84	65	96	75
0,001000	65	47	28	84	62	39	101	76	48	89	57	101	66
0,001250	69	50	38	88	66	51	105	80	64	92	75	104	86
0,001500	72	53	39	92	69	52	108	82	64	94	75	105	85
0,001750	76	56	42	95	72	56	111	85	68	96	79	107	89
0,002000	79	60	45	98	75	58	114	88	69	99	79	108	88
0,002250	82	68	56	101	84	70	116	97	82	108	93	117	102

0,002500	85	74	67	103	89	80	117	101	90	111	99	119	106

Source: Own construction

For example when p_L = 0,001, N = 4000, $\overline{p}$ = 0,0004 and c_m = 3 is parameter e = 44 (see Table 1), which means that using the AOQL plan for inspection by variables and attributes it can be expected approximately 56 % saving of the inspection cost in comparison with the corresponding Dodge-Romig plan.

Now we shall study dependence of the economic efficiency measured by parameter e on the lot size N. Let p_L, $\overline{p}$, c_m be given parameters. Function (11) for given p_L, $\overline{p}$, c_m is a function of one variable N, i.e.

$$e = e_{p_L, \overline{p}, c_m}(N). \tag{12}$$

From the results of numerical investigations it follows (see also Table 1 and Table 2) that function (12) has decreasing trend in N, which means that *when lot size N increases, then saving of the inspection cost* $(100 - e)$ *increases* (using the AOQL plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

In the second step we shall study dependence of the economic efficiency measured by parameter e on the process average fraction defective $\overline{p}$. Let p_L, N, c_m be given parameters. Function (11) for given p_L, N, c_m is a function of one variable $\overline{p}$, i.e.

$$e = e_{p_L, N, c_m}(\overline{p}). \tag{13}$$

From the results of numerical investigations it follows (see also Table 1 and Table 2) that function (13) has increasing trend in $\overline{p}$, which means that *when the process average fraction defective* $\overline{p}$ *increases, then saving of the inspection cost* $(100 - e)$ *decreases* (using the AOQL plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

Finally we shall study dependence of the economic efficiency measured by parameter e on fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes c_m. Let p_L, N, $\overline{p}$ be given parameters. Function (11) for given p_L, N, $\overline{p}$ is a function of one variable c_m, i.e.

$$e = e_{p_L, N, \overline{p}}(c_m). \tag{14}$$

From the results of numerical investigations it follows (see also Table 1 and Table 2) that function (14) has increasing trend in c_m, which means that *when the fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes* c_m *increases, then saving of the inspection cost* $(100-e)$ *decreases* (using the AOQL plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

Conclusion

From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables and attributes are in many situations **more economical** than the corresponding Dodge-Romig AOQL attribute sampling plans. For the chosen value of average outgoing quality limit p_L this conclusion is valid especially when

1 the number of items in the lot N is large,

2 the process average fraction defective $\bar{p}$ is small,

3 the cost of inspection one item by variables is not much greater than the cost of inspection one item by attributes, i.e. c_m is not large.

Similar conclusions were obtained also for the comparison of the AOQL plans for inspection by variables (special case of the AOQL plans for inspection by variables and attributes) with the Dodge-Romig AOQL plans, but saving of the inspection cost is here less than for the AOQL plans for inspection by variables and attributes. It can be proved that under assumption $c_m > 1$ the AOQL plans for inspection by variables and attributes are always more economical than the corresponding AOQL plans for inspection by variables (for $c_m \le 1$ the AOQL plans for inspection by variables are evidently most economical).

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Contact

Prof. RNDr. Jindřich Klůfa, CSc.
University of Economics in Prague,
Department of Mathematics,
W.Churchill Sq.4, 13067 Prague 3
klufa@vse.cz