THE IMPACTS OF FINANCIAL CRISIS ON INTEREST RATE MODELS

Dana Cíchová Králová

Abstract

The aim of this paper is a validation of popular BGM interest rate model with respect to the aftermath of the last global financial crisis and of following debt crisis in the EU and to answer question whether it is still possible to use this model for managing interest rate risk of portfolio containing instruments linked to interest rate such as bonds or interest rate swaps. Nowadays, various market distortions can be observed. They are caused by market regulations, high liquidity level in the system or increasing role of central banks in markets. Given the fact that the most of the currently used interest rate models (including BGM model) were developed under different market conditions before 2007 is a validation of these models very important. Another objective of this paper is a description of behavior of interest rates models in different periods characterized by varying degree of market regulation and level of liquidity and credit risk. Despite high level of regulations, market distortions and significant increase in liquidity and credit risk, BGM model seems to be robust to changes in the nature of the market that occurred since September 2007 and is suitable for purpose of interest rate risk management.

Key words: interest rate models, violation of assumptions, financial crisis

JEL Code: C51, E47

Introduction

The aim of this paper is a validation of popular Brace-Gatarek-Musiela interest rate model (hereinafter the “BGM model”) with respect to the aftermath of the last global financial crisis (hereinafter the “Financial crisis”) and of the following debt crisis in the EU and to answer the question whether it is still possible to use this model for managing interest rate risk of portfolio containing instruments linked to interest rate such as bonds or interest rate swaps. A characteristic feature of the financial market since beginning the global financial crisis is an increasing regulation of the market, an increase in liquidity and credit risk and a change in the relationship between different financial instruments and market participants. Given the fact that the most of the currently used interest rate models (including the BGM model) were developed under different market conditions before the Financial crisis is the validation of these models very important. Another objective of this paper is a description of behavior of interest rates models in different periods which are characterized by varying degree of market regulation and level of liquidity and credit risk. In this article, I will deal with the period before, during and after the Financial crisis.

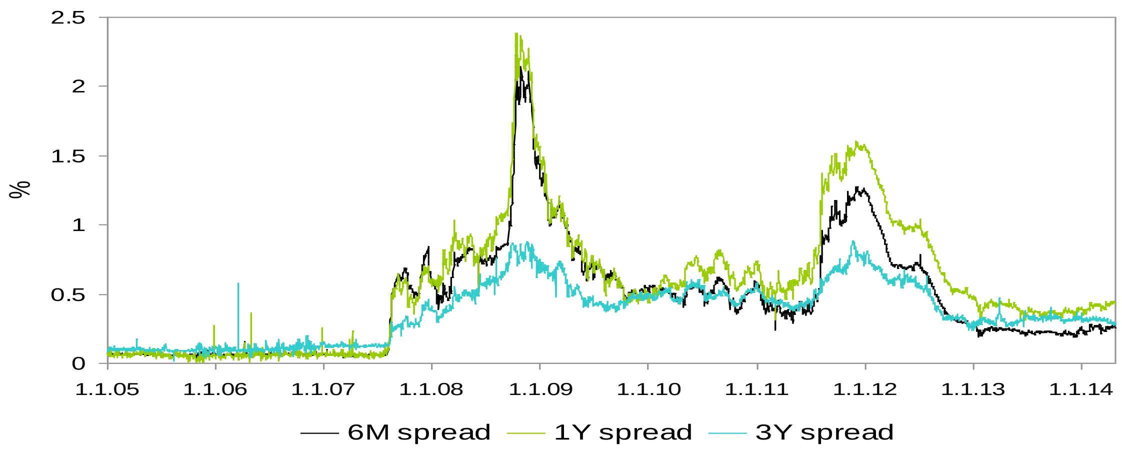
This paper is organized as follows: The first part is a brief description of the development of liquidity and credit risk using basis spread between interbank EURIBOR and EONIA rates. This section also describes an influence of market regulations on interest rates. The second part of the article describes a volatility of interest rates in monitored periods using GARCH model. In the third part of the paper, the BGM model, which is a complex interest rate of whole yield curve, is applied. Simulations of interest rates obtained from the BGM model in each monitored period are compared with the actual development of these rates.

This paper is focused on interest rates in EMU, the source of data used in this paper is Bloomberg and calculations are conducted in MATLAB and in EViews.

1 Change in the character of financial market

The Financial crisis began in the second half of 2007 and caused a shattering of confidence in previously known financial system as a whole. Relations that applied for decades until the beginning of the crisis were suddenly violated and the financial market began to behave quite differently compared to the assumptions under which the popular interest models were created. For example, asset pricing models were constructed under the assumptions that the credit and liquidity risk of the underlying asset affects asset prices only marginally. The increase in liquidity and credit risks can be well documented by the development of market quotes of simple or more complex interest instruments such as swaps, deposits, interest forwards or swaptions, where significant changes in their development before the crisis, i.e. before the half of 2007, during the crisis, i.e. between 2007 and 2011, and after the crisis, i.e. after 2011 can be seen. Approximately since the half of 2007, the primary interest rates in the interbank market show high basis spreads, i.e. the spreads between equally long instruments with different compounding frequency. The same development is shown by the basis spreads reflecting the difference between the EURIBOR interbank interest rate and OIS (Overnight Index Swap). OIS are classic plain vanilla interest swaps with one fixed leg and one floating leg bearing interest in relations to the EONIA rate. Due to the short tenor on EONIA rate, the credit and liquidity risks can be considered as minimal, making the OIS the best approximation of risk-free market rates. Figure 1 shows the development of spreads between 6-month and 12-month EURIBOR rate and equally long OIS. Figure 1 also shows the difference between 3-year plain vanilla interest swap with floating leg linked to 6-month EURIBOR and equally long OIS with floating leg indexed to EONIA. The figure clearly shows that the spreads in all three displayed maturities show a similar tread. Prior to September 2007, these spreads fluctuated within a very narrow band +/- 5 bps. After the September 2007, the spreads began to show an increased volatility and reached levels up to 230 bps at 6-month and 12-month maturities, and 85 bps at 3-year maturity. In the aftermath of the Financial crisis in the second half of 2009, the decrease in all spreads to the levels of approx. 50 to 60 bps with increased volatility of these spreads can be observed. The next significant increase in all spreads can be observed in the course of 2011 and, especially, in the first half of 2012. The debt crisis culminated during this period in EU, as Greece was being saved, and there was a real danger, that the other countries on the periphery of Europe would have to ask for help too. The calming of the markets came after the creation of EU rescue programs and after the announcement of unlimited purchases of affected countries' bonds on secondary market by the European Central Bank (ECB). Since the second half of 2012, the spreads ranged like before the Financial crisis in narrow band of approx. 10 bps wide but at levels about 30 bps higher than before the crisis.

Fig. 1: Spreads between EURIBOR rate/ interest swap and equally long OIS



Source: Bloomberg and own calculation.

Since 2008 the various forms of influencing of natural development of the market have come about. The strongest conventional tool of central banks to influence the interbank rates is setting of Standing facility rates, i.e. Deposit facility rate for overnight deposits with the central bank and Marginal lending facility rate at which the central bank lends. It should be that the deposit facility rate is lower than interbank rate and lending facility rate is higher than interbank rate. Outside the conventionally used tools, the ECB introduced yet another facilities such as fixed-rate refinancing operations with full allotment, substantial extensions of securities accepted as collateral and Long Term Refinancing Operations (LTRO). The ECB is currently considering the introduction of some form of quantitative easing, which further significantly affects market rates at the short and the long ends of the yield curves. The other central banks act like the ECB, such as the Czech National Bank (CNB), which announced the launch of interventions against the CZK by purchasing the EUR on November 2013.

This announcement caused sudden depreciation of CZK to EUR from the level of approx. 25.5 to 27.0. Due to the continuation of the interventions to this day, the CZK is being traded in the band of 27.0 to 27.5 to EUR.

Other important factors affecting the development of market rates are the bans on short selling, implementations of regulations such as EMIR or directives on capital adequacy such as BASEL etc.

2 Volatility on the market

In this section, I focus on the volatility of swap market, which I describe using the GARCH model. The reason for modeling interest rates volatility is my assumption that we can probably also observe a change in the behavior of volatility of individual rates constituting a yield curve together with the increase in liquidity and credit risk since beginning of Financial crisis.

A time varying volatility is a common property of financial time series. ARCH (autoregressive conditional heteroscedasticity) model which was applied by (Engle, 1982) for modeling inflation in the UK, laid the foundation stone of a large class of models which describe conditional volatility. Their importance lies in the fact that they are able to capture the changing market uncertainty.

GARCH (generalized ARCH) model removes some weaknesses of the basic ARCH model. GARCH(m,s) is given by formula (1).

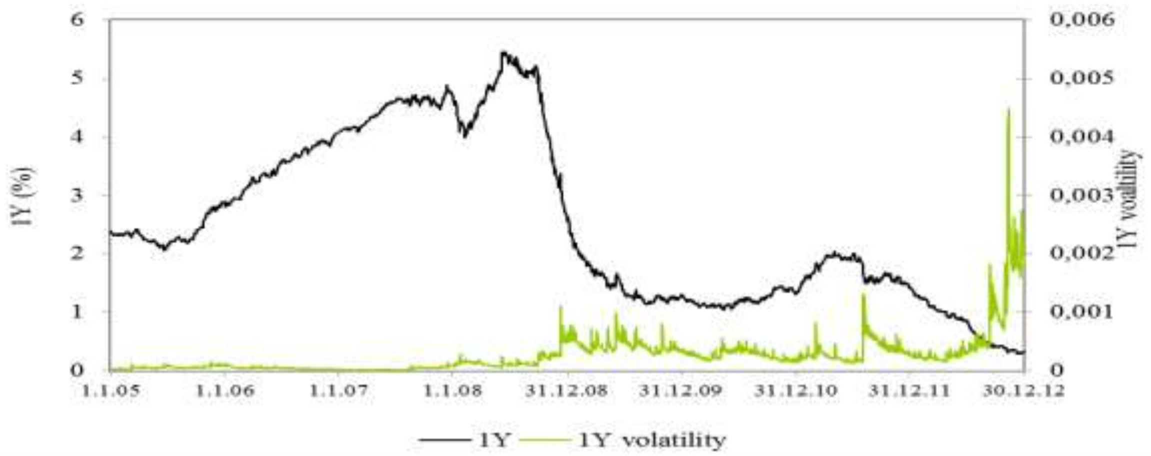
$$y_t = \mu_t + e_t, \qquad e_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i e_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2, \tag{1}$$

where y_t denotes a time series, μ_t denotes conditional mean value and σ_t^2 is conditional variance. Conditional mean value μ_t is usually described by formula for mean value such as linear regression or ARMA process. e_t in formula (1) denotes deviations from the conditional mean value which are uncorrelated identically distributed random variables with zero mean. ε_t in formula (1) are i.i.d. with zero mean and unit variance. Parameters α_i, β_j satisfy constraints (2).

$$\alpha_0 > 0, \qquad \alpha_i \geq 0, \qquad \beta_j \geq 0, \qquad \sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1, \tag{2}$$

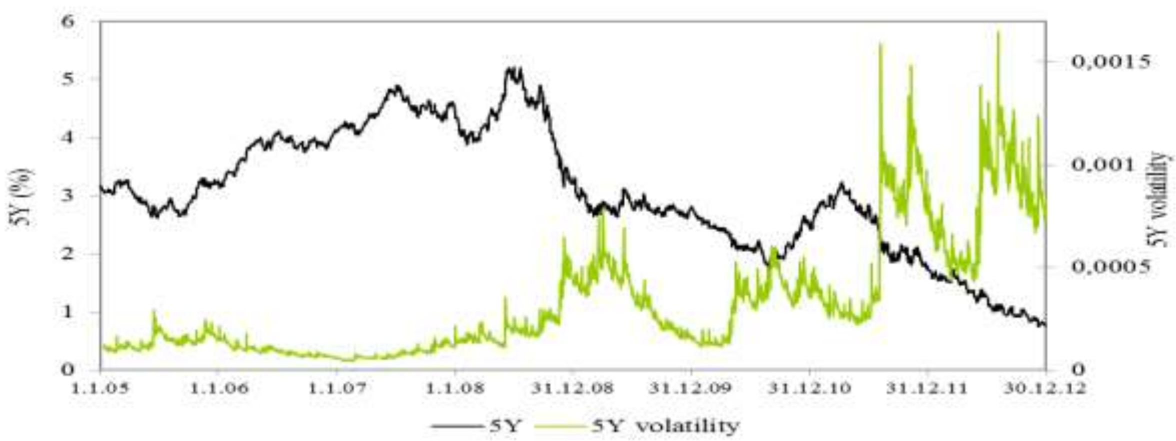
where the last inequality is a sufficient condition for the existence of variance $\text{var}(e_t)$.

Fig. 2: Volatility of 1-year swap rate



Source: Bloomberg and own calculation.

Fig. 3: Volatility of 5-year swap rate



Source: Bloomberg and own calculation.

Figure 2 and 3 show the volatility of 1-year and 5-year swap rate modeled by GARCH(2,2) a GARH(2,1), from which it is obvious that the volatility of interest rates developed differently prior to the beginning of the Financial crisis compared to the period after the crisis. Figure 2 and 3 show, that the volatility of rates was relatively low and ranged in relatively narrow corridor compared to the period after the crisis. During 2008, significant, manifold increase in the volatility compared to the period before 2008 can be observed. It is interesting that although the basis spreads EURIBOR – OIS had been showing the nervousness of the markets even since September 2007, the volatility of these rates reacted more slowly, which can be observed mainly in the 5-year swap, as the value of the volatility had increased from the first half of 2007, but the levels higher than in 2005 were reached in the second half of 2008. High market nervousness in respect to the development of the volatility can be observed in 1-year and 5-year rate after the collapse of Lehman Brothers. It is interesting, that the volatility of interest rates didn't decrease in the aftermath of the Financial crisis, but rather increased, and currently, even after the noticeable calming of markets, reaches its maximum values, which is approx. by more than 100% higher in 1-year rate and approx. by more than 50% higher in 5-year rates compared to the end of 2008. A similar development is shown by the volatility of other maturities on swap curve.

3 Brace-Gatarek-Musiela model (BGM)

In this section, I describe the dynamics of the yield curve using the BGM model that is probably the most complex and the most complicated interest rate model. The BGM model became popular and widely used by quantitative analysts.

The model was created in 1994 by (Miltersen, Sandmann & Sondermann, 1997), then developed in 1995 to a form applicable in practice by (Brace, Gatarek & Musiela, 1997). Its current form belongs to (Jamshidian, 1997), and is based on an abstract formulation of (Musiela & Rutkowski, 1997).

Let us consider a sequence of time points $0 = T_0 < T_1 < \cdots < T_m$, where the distance between the adjacent points is constant. Further, let us denote a forward interest rate in time t for the period $[T_{n-1}, T_n]$ as $L_n(t)$. The BGM forward rate dynamics is driven by process (3)

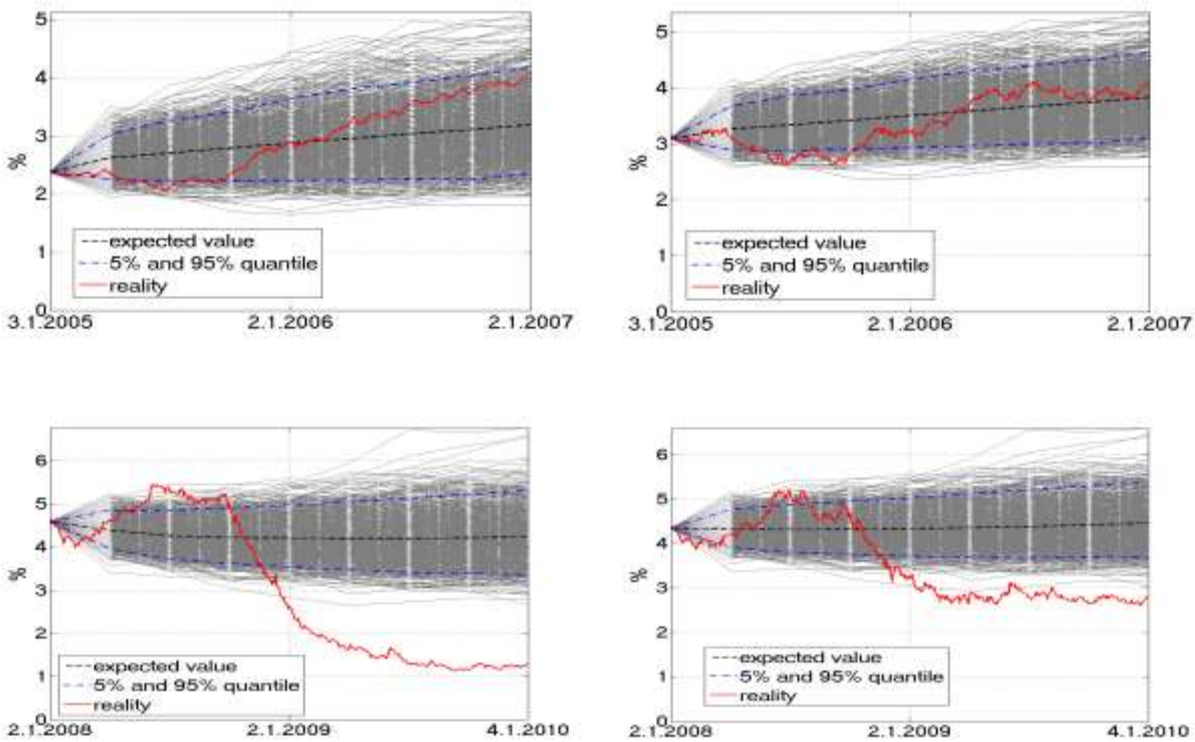
$$dL_n(t) = L_n(t)\gamma_n(t) \cdot dW_n(t), \qquad (3)$$

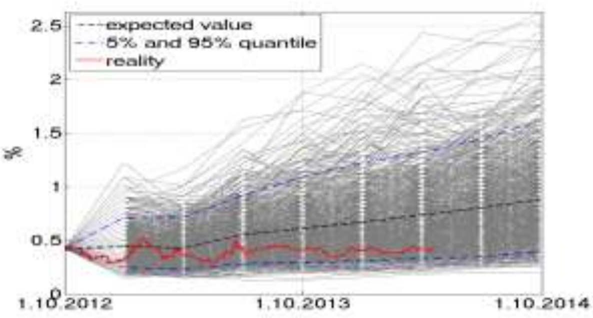
where a deterministic function $\gamma_n(t)$ is the volatility of the appropriate forward rate and $W_n(t)$ denotes a Wiener process under forward measure.

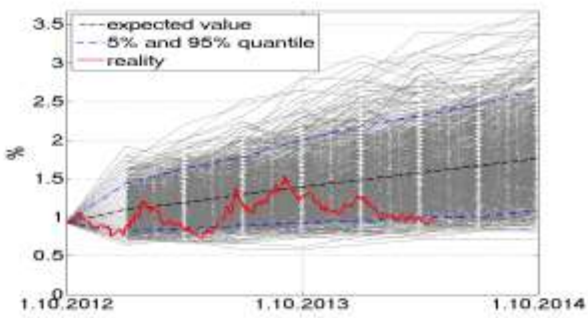
The resulting forward rates follow mean reverting behavior, when the model satisfies several assumptions on volatilities. The assumptions and their derivation can be found in (Brace et al., 1997) and many practical issues of the BGM models can be found in (Gatarek, Bachert & Maksymiuk, 2007).

For modeling of forward rates using the BGM model it is necessary to estimate volatilities of forward rates, which can be usually done first using caps or swaptions volatilities. Many estimation techniques are used for volatility estimates as for example (Rebonato, 2002) or (Brigo, Mercurio & Morini, 2005). In this paper I use a locally single-factor approach, which uses actual swaption volatilities, see (Gatarek et al., 2007) for details. The assumption of this approach is that volatilities are constant in any time interval. Applying the locally single-factor approach, I obtain estimates of forward rates volatilities $\gamma_n(t)$.

Fig. 4: Simulation of 1-year (first column) and 5-year (second column) swap rate







Source: Bloomberg and own calculation.

Further, I use the binomial tree approach to perform simulations. When constructing a binomial tree it is required to determine forward rates for each node, and the conditional and unconditional probabilities of reaching this node. Forward rates for each node were determined using log-normal approximation. Forward rates for time $t = 0$ can be derived from the actual zero coupon yield curve.

Having a binomial tree of all possible forward rates development, I performed simulations of forward rates, which are particular paths of the binomial tree. I perform 1000 simulations for two-years horizon in each monitored period. In the next steps, I convert the forward yield curve simulations to zero coupon yield curve simulations which I further convert to coupon (swap) yield curve simulations. See (Hull, 2006) for details.

Figures 4 shows 2-year-long simulations of 1-year and 5-year swap rates using the BGM model in the period prior to the crisis, i.e. as of 3 January 2005, during the crisis, i.e. as of 2 January 2008, and after the crisis and after the calming of the debt crisis in EU, i.e. as of 1. October 2012. From the first two graphs of Figure 4 it is evident that the actual development of 1-year and 5-year swap rate remains in the interval given by the 5% and 95% percentiles of simulations except for a few days. The second two graphs of Figure 4 shows 2-year-long simulations of swap rates carried out at the beginning of the Financial crisis. When comparing the simulations with the actual development of the rates, a clear failure of the BGM model can be observed, as it didn’t manage to describe the rapid growth of rates in the middle of 2008, as well as a following sharp decrease in interest rates in the second half of 2008. The actual interest rates in all maturities were outside the simulation framework throughout 2009. The third two graphs of Figure 4 shows 2-year-long simulations of swap rates carried out after the end of Financial crisis and the calming of the debt crisis in EU. When comparing the simulations with the actual development of rates, the actual development of all considered rates is in the range given by the 5% and 95% percentile of simulations. From the development of simulations, it is obvious, that the BGM model expected the rising trend of rates of all modeled maturities in October 2012, while the actual development of rates indicates a declining trend. However, this fact does not diminish the quality of the model, because in reality, only one of all possible paths of stochastic process may occur. Another interesting finding is the fact, that in case of the interest rates close to 0, the BGM model recognized asymmetric risk between further limited decline in rates and the possibility of more significant increase in rates, as the difference between 95% percentile of simulations and the expected value is higher than the difference between expected value of simulations and the 5% percentile.

Conclusion

In this article, I explored a few aspects of the use of models for purpose of interest rate risk management of the portfolio of financial assets linked to the interest rate in the periods before, during and after the Financial crisis. In the first part of the paper, I demonstrated an increase in premiums related to credit and liquidity risk observed in the financial market during the Financial crisis and debt crisis in EU compared to the situation before 2007 on the example of the development of basis spreads between EURIBOR or IRS and OIS. Even after the end of the Financial crisis and after calming debt crisis in EU, these spreads remain at high levels and several times higher than before 2007 but lower than during both crisis. In the second part, I applied GARCH model for description of swap rates volatility during all monitored periods and made conclusion that volatility of swap and deposits rates increased significantly compared to the period before the Financial crisis and is currently stabilized at a higher level higher than during the Financial crisis but is slightly lower than during debt crisis. In the third part, I obtained simulations of swap yield curve using the BGM model for each monitored period and made a comparison between the actual development of swap rates and simulations. The conclusion is that while the BGM model completely failed during the Financial crisis it seems to be robust to changes in the nature of the market that occurred since September 2007. However, it is important to note that interventions of central banks and a introduction of various types of market regulations bring to the market redefining relationships that interest rate models cannot explain. On the other hand, the BGM model is, in my opinion, very flexible and suitable for a description of fast changing markets due to the its parameters estimation technique using currently quoted volatility of swaptions. If traders correctly evaluate new information regarding market regulation or change in market character and reflect it in swaptions volatility quotes, parameters of the BGM model will immediately react to new circumstances while parameters estimates using historical data will not.

It should be also noted that this paper does not deal with a no-arbitrage valuation of financial assets which is the main use of interest rate models. From the conclusions of this article, it is not possible to draw conclusions in this area because no-arbitrage valuation is significantly more sensitive to violations of relationship between financial variables than using interest rates model for interest rate risk management of portfolio containing instruments linked to interest rates.

It is clear that the limited size of this paper does not allow to describe and to explore all aspects of the financial markets development during monitored periods in relation to the application of interest rate models. This paper rather opens a discussion and leaves room for further research in this area.

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Contact

Dana Cíchová Králová

The University of Economics in Prague

W. Churchilla 4, 130 67 Prague, Czech Republic

xkrad900@vse.cz