PRODUCT MIX DECISIONS WITH RESPECT TO TOC AND LINEAR PROGRAMMING

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Abstract

Firms that produce various products are confronted with the difficult decision as to the amount of a given product to yield. This decision seriously impacts other aspects of the enterprise that include sales, marketing policy, and the utilization of assets. However, various methods are used to resolve this problem. Among optimization methodologies, the linear programming approach and Theory of Constraints represent the more common. The Theory of Constraints is based on the assumption that every system, such as a firm, contains at least one constraint. Without the latter, the firm could possibly accumulate unlimited profits. The goal of the theory is to maximize throughput of the firm as a whole. Therefore, it is both a holistic and systems approach to the firm. Linear programming is used in many papers to validate the optimality of the solution, therefore, it is often considered to be the most suitable. The Theory of Constraints is a heuristic and therefore, more often used by managers who lack the applied mathematical skills for linear programming. The solution can be transferred to linear programming as well as the Activity Based Costing approach for product mix optimization. This conclusion leads to the basics of this branch of product mix optimization methods as to what costs are relevant in product mix optimization.

This paper focuses on various methods of product mix optimization that enhance the original heuristic solution and provides recommendations for the relevant costs. The linear programming approach is also introduced, a comparison of the methods is performed and assumptions for usage are described.

Key words: product mix optimization, theory of constraints, costing

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Introduction

Through optimization of their product mix, firms derive increased profits. Various definitions of product mix include, “the basic problem consists of both the quantity and the identification of each product to produce. The objective is to maximize profit (or minimize loss) for the organization" (Patterson, 1992). Although it summarizes the problem, it lacks the explicit mention of the conditions wherein this identification occurs. A more suitable definition for this discussion describes the product mix decision as the optimum quantity of each product type in a given period, and recognizes that products compete for a limited number of resources (Hodges & Moore, 1970) to maximize the firm's economic results (Fredendall & Lea, 1997).

Equally, there exist different approaches to determine product mix. The Theory of Constraints (TOC) and Activity Based Costing (ABC) present two views of relevant costs in this decisional process, whereas, linear programming (LP) presents a method. The method is used with inputs from TOC or ABC to determine the optimal product mix. In doing so, it presents an alternative to several heuristics.

This discussion compares TOC, ABC and LP methods of product mix decision-making through the assessment of various approaches to optimization. In addition, it provides recommendations as to usage and modification through the incorporation of varied relevant costs. Different assumptions about relevant costs are made in TOC and ABC that produce various product mixes. This difference in regards to relevant costs is perceived by many authors to be based on the time perspective. This paper argues that it is rather the managerial discretionary power over costs or the inherent nature of the costs that determine relevancy. From this outlook, the research question seeks to determine from which perspective the relevancy of costs should be determined. Subsequently, this is validated through comparison of the aforementioned methods and input particular to relevant costs. In addition, TOC and TOC product mix determination methods are introduced and discussed.

1 The Theory of Constraints and the original TOC heuristic

TOC presents new optics on the firm by means of throughput that measures the central objective of a company that is to generate money now and in the future (Goldratt, 1999). The theory views the firm as a system that possesses at least one constraint. Without this constraint, it stipulates that the enterprise would generate unlimited profit. TOC also emphasizes a global, holistic approach. The distinction between TOC and traditional cost accounting is often presented with the example of the chain analogy. The traditional cost accounting perspective in the terms of TOC represents the "cost world", or the weight of the chain. Therefore, if a company reduces costs in any unit, the weight of the whole chain is reduced. Consequently, the global optimum is reached as the sum of local optima. The TOC approach adopts the "throughput world" represented by the strength of the chain. As such, the overall strength of the chain depends on the weakest link. Under this condition, if the company invests in other than the constrained weakest link, the outcomes are negligible, if not, virtually non-existent. This signifies that the entirety is subordinated to the constraint and that actions justified from the cost world perspective are devastating when subjected to the throughput world (Goldratt, 1999). Many TOC applications and management tools are available. One of these applications is Throughput Accounting (TA). Due to the focus on constraints and as a direct (variable) costing approach, "it particularly supports short- and medium-term production decision-making." (Souren, Ahn, & Schmitz, 2005). For that reason, TOC approach to product mix determination is considered suitable for the short term.

1.1 Throughput

As mentioned previously, throughput is a TOC component and entails two perspectives. These comprise the rate at which a system generates money; alternatively, the sales price minus total variable costs. As the variability of the costs are emphasized, semi-variable costs are not deducted. In the author's opinion, throughput within this context is a restrictive variance of the contribution margin. As all costs alter with a change in output with only a segment variable, throughput deducts a smaller sum of costs. In TOC, there is no allocation of overhead or semi-variable costs to the products.

1.2 Original product mix heuristic

The original product mix heuristic constitute the first two of the TOC five steps process and is restated as (Fredendall & Lea, 1997):

Step 1: Identify the system's constraint(s) – the required load on each resource is calculated when all products are produced. The bottleneck is identified as the resource whose capacity is less than the market demand.

Step 2: Decide how to exploit the system's constraint(s) – Throughput (T) of each product is calculated as the sales price minus the raw material costs; the ratio of the throughput to the processing time on the bottleneck (BN) resource for each product is calculated as T/BN. This reserves the BN capacity to the products in descending order of their T/BN until the BN capacity is exhausted. Free products that do not require processing time on the BN resource are scheduled in descending order of their contribution margin.

Fredendall and Lea (1997) deploy the term contribution margin. However, from the heuristics it is actually throughput with a more stringent approach. Only raw materials are considered as variable costs that coincides with the widely used approach. The original heuristic was criticized in that it does not lead to an optimal solution in specific cases. Souren, Ahn, and Schmitz (2005) conclude that the original TOC heuristic leads to the optimal solution only when several specific conditions are met and not by far, in general.

The circumstances in which the heuristics do not lead to the optimal solution are during the implementation of a new product within an existing product mix as well as the presence of multiple bottlenecks (Aryanezhad & Komijan, 2004). In the case of multiple bottlenecks, most authors orientate the TOC heuristic to the most overloaded bottleneck (Souren, Ahn, & Schmitz, 2005). Another deficit of the heuristic is the case wherein the raw material is not the only variable ("direct") cost (Louderback & Patterson, 1996).

There have been attempts to prove that the original TOC heuristic is able to reach optimum even when multiple bottlenecks are present, but have subsequently been refuted by Balakrishnan (2000). Following which, the authors of the original claim concurred (Finch & Luebbe, 2000).

2 Revised and improved heuristics

Due to the determined inefficiencies of the original heuristic, attempts to overcome the multiple bottleneck problem through heuristic enhancement are proposed. In this section, two of these heuristics are introduced.

2.1 The revised heuristic

Fredendall and Lea (1997) use the multiple bottleneck problem of the original heuristic to develop a revised heuristic. The first step of the heuristic remains untouched, then the dominance check and neighborhood search are implemented.

The dominance check is conducted to select the dominant bottleneck. This is performed by verifying whether the bottleneck with the largest difference between the required and available capacity is the first without sufficient capacity. If not, the first bottleneck without available capacity becomes the new dominant bottleneck. The first bottleneck without capacity is considered to be the "system constraint" or dominant bottleneck. This is also substantiated by Balakrishnan and Cheng (2000). As in the original TOC heuristic, the products are scheduled and the initial solution is obtained.

With the use of a neighborhood search, the authors seek a better solution than that obtained by the initial. The candidate products for interchange are found through consideration of the remaining time of the dominant bottleneck, processing time on the bottleneck of the product, contribution margin (throughput) and processing time to the contribution margin ratio. These products are then interchanged with the product in the initial solution. If there exists a throughput gain, a new solution is then established. The algorithm has 16 steps, some of which, as with the neighborhood search, are complicated.

2.2 Improved heuristic

Aryanezhad and Komijan (2004) criticize the revised algorithm. Using an example, they reveal that the revised algorithm does not reach an optimum solution and propose a new method to cope with the multiple bottleneck problem. They suggest computing the priority sequence of the product for each bottleneck. Once completed, feasible alternatives to reduce and increase processes are determined. This is accomplished through the selection of candidate products and throughput gain computations. An alternative with the highest throughput gain is selected. The authors indicate that the algorithm functions even when the computed throughput gain are negative. This arises as the overall throughput gain occurs in the later stages of the scheduling process when the available time is used to produce the next products in the row. Consequently, the improved algorithm consists of only 8 steps and is therefore more comprehensible and usable than previous.

3 Linear programming

Luebbe and Finch (1992) state that TOC is a philosophy, but LP is a specific optimization technique.

The maximization of throughput is accomplished by means of formulating the objective function. The LP approach does not identify a system constraint or dominant bottleneck, as it considers all constraints. This results by formulating the inequality constraints, in this case, of the resources. The non-negativity constraints to the variables, units of certain products to be produced, have to be included.

TOC can be used for the LP input data when throughput is used in the objective function. Through the work of Balakrishnan and Cheng (2000), LP was proven to reach the optimal solution in the case of a multi-bottleneck problem. LP exploits the system fully; the original

TOC heuristic identifies the system's constraint *a priori* by selecting the dominant bottleneck as being the most overloaded. TOC, however, is considered to have advantages over LP in that it is easier to use, more comprehensive, especially to managers without a mathematical aptitude (Souren, Ahn, & Schmitz, 2005).

4 ABC approach

The ABC approach is based on the causal relationship between products, consumers and their consumption of resources. The relationship is determined by tracing costs on the cost driver that causes or highly correlates with the resource use (Kee & Schmidt, 2000). The approach attempts to resolve the problems with product mix selection, when costs are not proportional to production, but are rather connected with unit, batch and product level activities. ABC is based on the premise that products consume activities and activities consume resources thereby creating costs. In this viewpoint, it is a full costing method. Variable as well as fixed costs are traced to the activities and as well as the products that consumed them.

Kaplan (1992) notes that ABC is not designed to serve to allocate cost to products more accurately. It focuses on the identification of production factors that cause activities to absorb resources and in doing so, incur costs.

Malik and Sullivan (1995) propose an approaches for product mix decision using ABC:

$$\text{Maximize } \sum_{i=1}^{n}\left\{(s_i - m_i - lb_i)x_i - \sum_{j=1}^{r} c_{ji}\left\langle x_i / a_{ji}\right\rangle\right\} \quad (1)$$

$$\text{Subject to } \sum_{i=1}^{n} m_i x_i \le m \quad (2)$$

$$\sum_{i=1}^{n} lb_i x_i \le 1 \quad (3)$$

$$\sum_{i=1}^{n} c_{ji}\left\langle x_i / a_{ji}\right\rangle \le o_j \text{ , } j=1,\dots,r \quad (4)$$

$$0 \le x_i \le u_i \text{ , } i=1,\dots,n \quad (5)$$

Where:

x_i denotes the amount of product *i* for production in a certain time horizon. *n* represents total number of products. s_i is unit selling price of product *i*. m_i denotes unit material cost of product *i*. lb_i is unit labor cost of product *i*. *m* represents cash equivalent of maximum available material resource. *l* is cash equivalent of maximum available labor resource. u_i notes the upper bound on the amount of product *i*. *r* is total number of indirect resources. a_{ji} presents upper bound on the amount of product *i* for production from the amount of resource *j* for c_{ji} costs. o_j denotes cash equivalent of maximum available resource *j*. (x_i/a_{ji}) denotes the smallest integer greater than or equal to x_i/a_{ji}. It is a LP transformation of the product mix problem, in lieu of using TOC as input data, ABC information is substituted. Undeniably, if the input data differ, the outcomes of TOC and of ABC are different.

A different model was constructed to compare ABC and TOC (Kee &Schmidt, 2000). For this purpose, they considered only volume related cost drivers for ABC. Kee and Schmidt (2000) conclude that product mix gained with this model is more profitable than or equal to the TOC model when both product mixes are compared through resources used in production. When both product mixes are evaluated through labor and overhead resources supplied to production, the profitability of the TOC-based product mix is greater or equal to the ABC based product mix.

5 Relevant costs

In consideration of the TOC models, LP and ABC approach, all present a path to determine product mix. TOC and ABC present specific approaches, which could be transformed and solved via LP. Nevertheless, upon close analysis of the TOC and ABC models, if different assumptions on relevant costs are made, both produce the same results. The same premise is valid for LP by using the presented ABC input in LP, or the TOC input, which is throughput. Ultimately, this promotes the same conclusion as determined by Kee and Schmidt (2000) that the TOC and ABC approach to product mix decision are both specific cases of a general model, exclusive of their assumption to consider only volume related cost drivers.

It is possible to decide, which costs will be included in the product mix determination making process. Whether it is only raw material, all variable costs, all direct costs--- it results in the throughput measure or contribution margin used. In essence, both could be used with LP.

As mentioned previously, the research question seeks to determine from which perspective the relevancy of costs should be determined.

Traditionally, TOC labor and overhead are treated as period expenses and are therefore not relevant. ABC leads to tracing labor and overhead to products, and therefore are relevant. TOC treats these costs as committed, hence, tracing back to products is redundant. Kee (1995) presents a prevailing opinion on TOC and ABC. The latter models how product, batch and unit level activities transform into products. Thus, they represent a long term perspective of the product costs. TOC conversely reflects constraints of activity capacities and their consumption in production, and is considered to be a short term perspective. From this argument that frequently appears in the product mix related documentation that draws comparisons between ABC and TOC, it derives that the difference between the approaches is an issue of time. In the short term, labor and overhead costs are fixed whereas, in long term, every cost is variable.

After review of the TOC solution, it may be incorrect in some cases when the assumption on fixed setup and other operating costs is violated. The ABC solution is flawed if the assumption relative to the proportionality of total activity costs is fragmented. It is then essential to agree with Kee and Schmidt (2000) that ABC and TOC are based on different assumptions about the behavior of the costs. Although ABC includes more costs in the decision, TOC is restrictive in the terms of variability assessment and thus understates costs in product mix decision-making. Caspari and Caspari (2004) suggest that in TOC when doubtful as to the categorization of certain costs, they should be categorized as an operating expense. Only after careful examination they could then be transferred to fully variable costs. This view has little consequence to TOC as to whether costs are semi-variable, allocated or fixed. With the TOC approach, costs that are not fully variable are concentrated in the operating expenses category.

The dependency on time, short or long term, derives from the projection that in the short term, there is less opportunity for managers to influence several costs. In the long term, there is more time for managers to influence all costs. “In effect, the more control a firm's management has over labor and overhead resources, the greater the potential for selecting a more profitable product mix and redeploying the excess resources that result from its production” (Kee and Schmidt, 2000). They believe that it is the management’s discretionary power over specific costs that determines which costs are included in the decision. It is plausible, that some over the long term, variable costs become fixed and vice versa. This does not signify that time is not a factor in product mix related decisions. It depends on the period for which the product mix is determined and which costs are treated as relevant. Whether it is the management discretionary power over costs that determines their relevancy or the behavior of the costs as variable or fixed depends on the viewpoint. A manager may have control over costs even in short term as in the case of direct labor. In this case, treatment of labor costs as fixed would produce biased results. Nevertheless, managers' control over specific costs does not always lead to the behavior of the costs as variable. It may be due to the nature of the contract as is often the case with labor, whether the costs will alter with changed volume or not. To gain information on the costs actual behavior is a difficult issue. ABC does provide an insight, but designates more costs as variable, even if based on the assumption of the inclusion of volume-related cost drivers. Likewise, the act of tracing fixed cost to products is misleading. It implies that a change in the volume of production changes costs, but this is valid only for the variable portion. Based on the above, the assumption encompasses that ABC could be used to identify these cost drivers, which incur costs that are contracted on a variable basis and therefore behaves as variable. The costs contracted as fixed are not included in the decision by tracing them to products even with ABC as they would incur nevertheless. By the incorporation of fixed costs into the decision, ABC generally leads to less product diversification through the shift of overhead costs to low volume products (Noreen & Smith, 1995). Setup costs are an example of irrelevant costs when job scheduling is done after product mix determination. Since job scheduling is conducted after the product mix is determined, it has impact on setup cost and should not be included in the decision.

Conclusion

To utilize the resources as efficiently as possible, a company has to determine its product mix correctly. The company, which is able to establish the right combination of products, will gain a competitive advantage. ABC and TOC represent different approaches to determine a firm's product mix. Based on the arguments presented, it is rather the management's discretionary power over costs or the contractual character of the costs, than the timeframe of the decision, which determines the relevancy of the costs. Time still plays a role in determining the time of the decision and period for which the product mix decision is being made. For this purpose the ABC approach is used to trace the costs of a complex production process to products. It appears biased to use this portion of ABC, which traces the fixed costs to product, over which management has, at the time and horizon of product mix decision-making, no discretionary power. The most discussed differences between TOC and ABC is the treatment of direct labor and overhead that require examination with regard to this argument and depends on each firm's specific conditions as to how costs are treated. This decision should not be made a priori, as suggested by various sources cited. The relevant costs incorporated in the decision-making, will therefore differ not only for different firms, but even for different timeframes, product mix planning horizons and managerial discretional power over costs within one corporate entity.

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