GOLD PRICE FORECASTING USING GREY MODEL GM(1,1) AND SELECTED CLASSICAL TIME SERIES MODELS. A COMPARISON OF METHODS.

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Abstract

This article will present three classes of time series models: grey model GM(1,1) for short time series and two classic models – model of Holt's exponential smoothing and ARIMA model. The purpose of this article is to show the properties of these models for forecasting time series with a low number of observations and the possible non-stationarity processes. All presented models are a tools for building forecasts. The differences between the models arise from econometric assumptions. In addition to the adaptive models is not possible the interpretation of the strength of the influence of independing variables. With regard to the quality of the forecasts generated models presented in the article may be comparable. In practice, in certain situation it is important to use the forecasts, which are built on the basis of short past. This may be due to the lack of relevant data. From the point of view of practical importance is the level of accuracy of predictions in relation to expectations. Comparative analysis of forecasting properties will be carried out on the example of the price of gold.

Key words: Grey Systems, GM(1,1) model, Time Series

JEL Code: C1, C2, C32

Introduction

The main goal of this paper is to present some econometric methods aimed at creation of short term forecasts. Additionally assumes the existence of a limited information resulting from the use of very short time series. This article will be considered three groups of models. The first group is a mechanical model of exponential smoothing – model of Holt. The second group consits analitical models for very short tmie series (min two observation), which is based on the theory of grey information systems. The group wil be presented grey GM(1,1) model and its modification rollin GM(1,1) model. The third and last group wil be the classic models of time series – ARMA model. This group represents a model of the time series with a relatively large number of observations. In addition, an important is the issue of the stationarity of the process.

Prediction process is always accompanied by the uncertainity associated with the conduct of the process itself as well as the uncertainity associated with the information available to the econometrician. It can therefore be very intuitive two conclusions. First econometrician never have all the information about the process – working in conditions of limited information (Sroczyńska-Baron, 2013). Secondly it is easy to ask a quastion whether the accuracy of forecast costactewd be on the basis of long time series is significantly higher than be on the basis a short time series or extremally short time series. Of course, despite the fact that the possibility of observing the evolution of the whole process, which is possble only for long time series (Bermingham, D'Agostino, 2014).

The models presented in the paper will be used to build short term forecasts (one-step-ahead forecast) for daily quotations in the gold market. In this article the author does not take the issue of rules of investments in gold because it is not a purpose of work.

1 Simple Holt's exponential smoothing

Holt exponential smoothing is well known and described in the literature in the field of time series. Holt's model belongs to the class of mechanical models which means that it is not required to meet the goals related to estimation process. This is due to the fact that such models do not have explicit analitical form. In a general sense, adaptive models are expressed by the following formula:

$$y_t = \mu + u_t \tag{1}$$

where:

y_t - forecasting variable,

μ - unknownf form of trend,

u_t - error.

The article presents a linear model of Holt. The model assumes linearity of the change in forecasted variable. This applies to changes in both past and the future periods. Holt's model is given by the following equations (Dittmann, 2003):

$$F_{t-1} = \alpha y_{t-1} + (1-\alpha)(F_{t-2} + S_{t-2}) \tag{2}$$

$$S_{t-1} = \beta(F_{t-1} - F_{t-2}) + (1-\beta)S_{t-2} \tag{3}$$

$$y_t^* = F_n + (t-n)S_n \tag{4}$$

where:

F_t - trend at the moment t,

S_t - smoothed value of trend at the moment t,

α , β - smothing parameters. It assumes that: $0 \le \alpha \le 1$, $0 \le \beta \le 1$,

y_t - forecasting variable.

Fit of the model to the time series and the forecast is conditional of the choice of the smoothing parameters. In theory, it is assumed that the fast-changing trends and rapid changes smoothed growth trend correspond to smoothing parameters close or equal to unity. Smoothing parameters are determined through simulation whose aim is to minimize any *ex post* error of forecasts. Prediction equation is given by (Dittmann, 2003):

$$y_t^* = F_n + (t-n)S_n \qquad (5)$$

where:

$t-n$ - forecast horizon, where $t > n$.

Holt model is used for the time series with trend and random fluctuations. The advantages of the model include:

- Simplicity of calculations,
- Ability to conduct a simply simulations,
- The use of low number of observations (min. 12 observations)

The weaknesses of model:

- The possibility of the effect of aging informations,
- Assumed in advance linearity changes of forecasted variable.

2 Grey information systems and GM(1,1) models

The theory of grey system information was established in 1982 in China. The theory was proposed by Julong Deng. The theory allows the description of the system through the prism of the information that is available about him. The theory of grey systems, the informations are shared because of the color assigned to them. Hence, the white information is a complete set of information about the system while the black information means a complete lack of information about the system. Grey information is a part describes the collection system. The description of the system in terms of its informal information is described in the conditions of uncertainity – in conditions of limited information (Barczak, 2013). The theory suggest the possibility of an alternative approach to modelling investigation by trying to accurately describe the reality – the essence of the whitening processes. The main areas of applications of the theory of grey information systems are:

- Grey mathematics,
- Grey econometric models,
- Grey incidence and evalutions,
- Grey models for decisions making,
- Grey game models.

In other words, the theory of grey systems is an alternative to modelling of processes in conditions with incoplete informations (Sroczyńska-Baron, 2013). From the point of view of econometric methods, grey systems theory proposes the use of class GM(1,1)[1] models and GM(1,N)[2]. These models are estimated to be onthe very short informations vectors. For example, the short time series – four realizations of forecasted variable. GM grey class models can in some cases be a good forecasting tool.

2.1 GM(1,1) model

The main equation og GM(1,1) model is given as (Liu & Lin, 2010):

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{6}$$

where:

a - development coefficient,

b - grey action quantity coefficient.

Let $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n)\right)$ be a raw vector of forecasting variable. Assume that $X^{(1)}=\left(x^{(1)}(1),x^{(1)}(2),\ldots,x^{(1)}(n)\right)$ is the result of AGO[3] operator. Hence, the basic form of GM(1,1) model is given as (Liu & Lin, 2010):

$$x^{(0)}(k)+az^{(1)}(k)=b \tag{7}$$

where:

$Z^{(1)}=\left(z^{(1)}(1),z^{(1)}(2),\ldots,z^{(1)}(n)\right)$ - is the series of adjacent averages from variable $X^{(1)}$, which is given as:

$$z^{(1)}(k)=\frac{1}{2}\left(x^{(1)}(k)+x^{(1)}(k-1)\right),\qquad k=1,\ldots,n \tag{8}$$

[1] GM(1,1) – first order Grey Model with one variable.
[2] GM(1,N) – first order Grey Model with N variable.
[3] AGO –Accumulating Generation Operator – cummulative realizations of forecasting variable.

The coefficients of the model (7) can be estimated usung the least squares method:

$$\hat{\mathbf{a}} = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'\mathbf{Y}$$

where:

$$\mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{9}$$

The time response equation is given as (Liu & Lin, 2010):

$$\hat{X}^{(1)}(k+1) = \left(X^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}, \quad k = 1,2,\ldots,n \tag{10}$$

The theoretical values are obtained from (Liu & Lin, 2010):

$$\hat{x}^{(0)}(k+1) = \alpha^{(1)}\hat{x}^{(1)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1-e^{a})\left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} \tag{11}$$

for $k = 1,2,\ldots,n$

Advanteges of GM(1,1) mode (Barczak, 2013):l:

- The possibility of modelling a system under the incomplete informations,
- Applied to the short time series,
- Apply to build a short-term forecasts,
- Easy calculations.

Weaknesses of GM(1,1) model (Barczak, 2013)::

- Model can be used only for positive realizations of forecasting variable,
- The problem of recognition of a classical random component,
- The problem with the conventional approach to the validation process of the model.

The applicability of GM(1,1) model (Barczak, 2013), (Węgrzyn, 2013):

- Short time series,
- Short term forecasts,
- Smoothing time series.

2.2 Rolling GM(1,1) model – RGM(1,1)

Rolling model RGM(1,1) is a modification of the classical model GM(1,1). This model can be used as a moving model for long time series. This allows to smoothing of the time series with the assumed length of the top of the smoothing window. Thus, the basis of the RGM model specification is the choice of the smoothing window width to minimize *ex post* errors of intermadiate forecasts and to minimize the main forecast *ex post* error.

Generally the RGM(1,1) model can be written as:

$$\hat{x}_t^{(0)} = \underset{i=t-k}{\overset{t-1}{GM}}(1,1) \tag{12}$$

where:

$\hat{x}^{(0)}$ - predicted value at the moment i,

k - smoothing parameter – smoothing window width.

RGM(1,1) model is used in the analysis of long time series. Its advantages and disadvantages are the same as the claccic model GM(1,1).

3. Conventional ARMA model

The autoregressive moving-average $ARMA(p,q)$ model is in the form (Tsay, 2010):

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{13}$$

where:

ε_t - is a white noise series,

p, q - is non-negative integers.

Using the back-shift operator, the model can be written as:

$$(1-\phi_1 B - \ldots - \phi_p B^p) y_t = \phi_0 + (1-\theta_1 B - \ldots - \theta_q B^q)\varepsilon_t \tag{14}$$

where:

$B^p = y_{t-p}$ - back-shift operator,

$1-\phi_1 B - \ldots - \phi_p B^p$ - AR polynomial,

$1-\theta_1 B - \ldots - \theta_q B^q$ - MA polynomial.

It is require that there are no common factors between the AR and MA part of the model. It is very important that if the all of the solutions of characteristics equations of ARMA model are less than 1 in absolute value, the the ARMA model is weakly stationary. If AR polynomial have 1 as a characteristic root, the the model becomes autoregressive moving-average model (ARIMA(p,d,q)). In other words this is model of nonstationary process (for example random-walk process). In that case, it is necesary to use differencing.

The one-step-ahead forecast of y_{h+1} can be written as (Tsay, 2010) :

$$\hat{y}_h(1) = \phi_0 + \sum_{i=1}^{p} \phi_i y_{h+1-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{h+1-i} \tag{15}$$

where:

h - is the origin of forecast.

For the l-step-ahead forecast, the formula is (Tsay, 2010):

$$\hat{y}_h(l) = \phi_0 + \sum_{i=1}^{p} \phi_i y_{h+l-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{h+l-i} \tag{16}$$

The forecast error is given as:

$$e_h = y_{h+l} - \hat{y}_h(l) \tag{17}$$

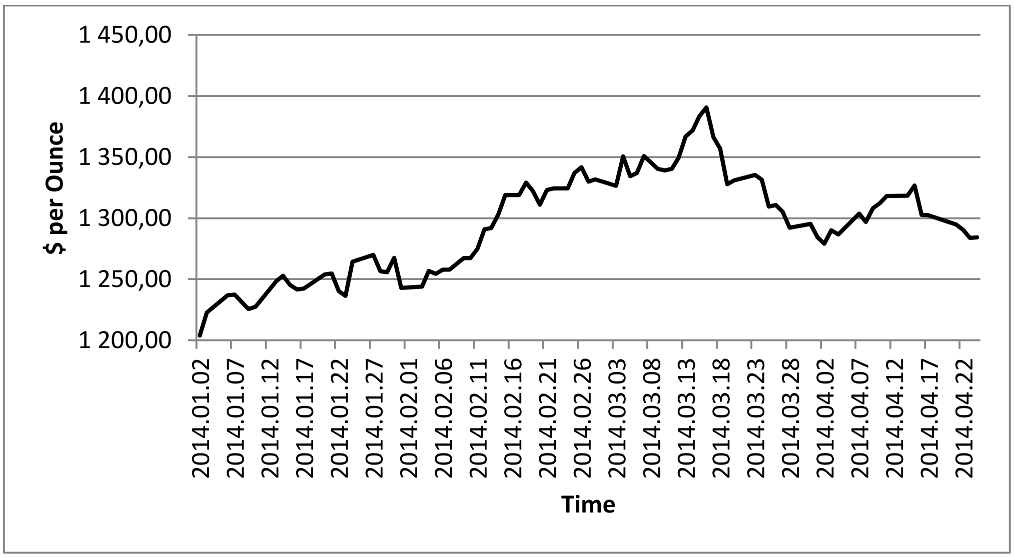
The specification of the ARMA model based on autocorrelation function (ACF) and partial autocorrelation function (PACF). A through analysis of the ARMA model requires a reference to his three representations. It is very important for the ARMA model to recognize the nature of the process due to stationarity. The process should be considered very carefully.

4. Gold price forecasting

Consider the daily gold prices per ounce in the period 2014-01-02 to 2014-04-24 (lenght of time series:T=88). The prices of gold per ounce are expressed in U.S. dollars. One ounce of gold is equal 31,1034768 grams. In practice, an aproximatelly 31,1 gram weight. The unit of gold weight is so-called Troy ounce. The name comming from the name of the French town of Troyes lying on the Seine.

Course of gold prices in the period under consideration shows Figure 1.

Fig. 1: Gold prices from period 2014-01-02 to 2014-04-24



Source: Own work. On the basis of data from www.mennica.com.pl

As shown in Figure 1 gold price ara time series which are relatively strong random fluctuations. The basic descriptive statistics for time series of gold prices is shown in Table 1. You will notice that in the period under consideration the price of gold ranged between 1204 and 1390 $/ounce. From March 20014 recorded a decline in gold prices.

To aasess the accuracy of the forecasts used the following *ex post* errors (Dittmann, 2013):

$$MSE = \frac{1}{m}\sum \varepsilon_t^2 , \tag{18}$$

$$RMSE = \sqrt{MSE} \tag{19}$$

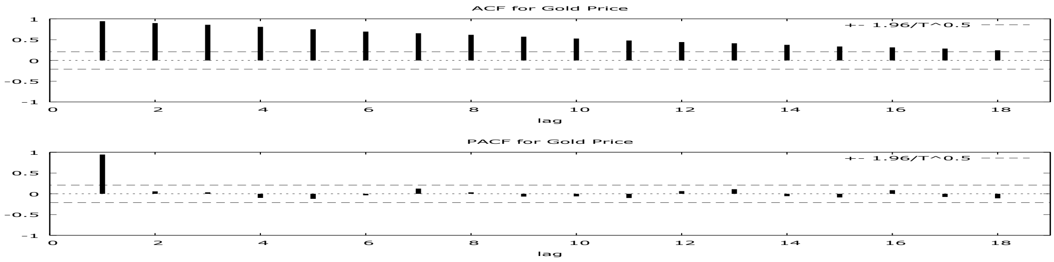
$$MAPE = \frac{1}{m}\sum \left|\frac{\varepsilon_t}{y_t}\right| \cdot 100 \tag{20}$$

where:

m - number of pairs: (actual value, theoretical value).

Issues a specification for the presented models. In the case of Holt's model smoothing parameters α and β determined by minimizing the Mean Square Error (MSE). Model fitted to the entire length of the time series gold prices. For rolling RGM(1,1) model width of the smoothing window is arbitrary equal 4 with one step shift. For GM(1,1) model assumed the length of the time series equal 4. A more complex procedure requires the specification of the ARMA model. The order of ARMA model set be on the basis of analysis of the autocorrelation function - ACF and partial autocorrelation function - PACF (Fig. 2). From the course of PACF shows that the autoregressive process is in order one. After estimating ARM(1,0) model found the nonstationarity process of prices. The delayed was 0,96 and was close to one. In the following, was performed Augmented Dickey-Fuller (ADF) test for the existance of unit root. ADF statistics is equal -1,64082 with asymptotic p-value equal 0,4616. At the 5% level of significance the null hipothesis was accepted. Ultimately proposed ARIMA(1,1,0) integrated model.

Fig. 2: ACF and PACF for prices of gold



Source: Own work.

The main results of the analysis are presented in Table 2. Particularly noteworthy is the ARIMA(1,1,0) model. As the Fig. 2 shows the autocorrelation function ACF reduces ARIMA form to random walk model. There its therefore a high probability that the forecast results obtained using random walk model will be more accurate (Cang & Yu, 2014). From the point of view of grey model GM, the forecast can be considered acceptable because the are built on the basis of short time series, which features such as purely statistical probability distribution, stationarity are not taken into account – incomplete information (Sroczyńska-Baron, 2013). In the case of grey models is essential to the choice of smoothing window in rolling model RGM and for the basic form of the model GM the leght of the time series.

Tab. 2: The results of the forecasts for the period 2014-04-05

Model	Forecast value	RMSE error	MAPE error
Holt smoothing $\alpha = 0{,}97$, $\beta = 0{,}16$	1281,39	11,7584	0,69%
RGM(1,1).Window width equal 4	1279,94	13,9687	0,81%
GM(1,1)Time series T=4	1279,93	1,6482	0,12%
ARIMA(1,1,0)	1284,90	12,3012	0,68%

Source:own work

Conclusion

You can point to the following conclusions: first the ARMA model is not the right model serving predicting gold price or gold returns, second the grey models require more detailed specifications or changes in the period of the time series and third an adaptive models can be treated in terms of the initial assessment of the level of future gold pricess. Grey models can be used to predict the financial indicators (Węgrzyn, 2013).

References

Barczak, B. S. (2013). Zastosowanie teorii szarych systemów do przewidywania przyszłych ofert składanych na aukcjach pierwszej ceny poprzez pryzmat modelu szarego GM(1,1). *Studia Ekonomiczne. Zeszyty Naukowe Wydziałowe Uniwersytetu Ekonomicznego w Katowicach*, *146*, 7-18.

Bermingham, C., D'Agostino, A. (2014). Understanding and forecasting aggregate and disaggregate price dynamics. *Empirical Economics*, *46*(2), 765-788.

Cang, S., & Yu, H. (2014). A combination selection algorithm of forecasting. *European Journal of Operational Research*, *234*(1), 127-139.

Dittmann, P. (2003). *Prognozowanie w przedsiębiorstwie. metody, zastosowania.*. (pp. 63-102). Kraków: Oficyna Ekonomiczna.

Eran, A. (2014). Adaptive learning, endogenous uncertainty, and asymmetric dynamics. *JOURNAL OF ECONOMIC DYNAMICS & CONTROL* , *40*, 355-373.

Liu, S., & Lin, Y. (2010). *Grey systems. theory and applications.*. (pp. 107-123). Berlin, Heidelberg: Springer-Verlag.

Sroczyńska-Baron, A. (2013). The choice of portfolio based on the theory of cooperative games. In O. Deev, V. Ksjurova & J. Krajicek (Eds.), *European Financial Systems 2013. Proceedings of the 10th International Scientific Conference* (pp. 305-311). Brno: Masaryk University.

Sroczyńska-Baron, A. (2013). The analysis of the process of taking over companies based on the theory of games. In T. Löster & T. Pavelka (Eds.), *The 7th International Days of Statistics and Economics, Conference Proceedings* (pp. 1324-1333). Prague: Libuše Macáková, MELANDRIUM. Retrieved from http://msed.vse.cz/msed_2013

Węgrzyn, T. (2013). Stock selection on the warsaw stock exchange financial ratios or profitability ratios. analysis between 2001 and 2011. In T. Löster & T. Pavelka (Eds.), *The 7th International Days of Statistics and Economics, Conference Proceedings* (pp. 1554-1564). Prague: Libuše Macáková, MELANDRIUM. Retrieved from http://msed.vse.cz/msed_2013

Węgrzyn, T. (2013). Stock selection based on financial ratios on the warsaw stock exchange. In O. Deev, V. Ksjurova & J. Krajicek (Eds.), *European Financial Systems 2013. Proceedings of the 10th International Scientific Conference* (pp. 356-361). Brno: Masaryk University.

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