STUCTURAL BREAKS IN VOLATILITY OF STOCK MARKETS

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Abstract

Volatility can be defined as a value of risk of financial instruments over a specified time period. One of the most challenging practical problems is to understand and model a behavior of volatility dynamics through different time periods. In this paper, we deal with an impact of structural breaks on volatility persistence with a help of Iterated Cumulative Sum of Squares (ICSS) algorithm. The aim of this paper is to identify sudden breaks in volatility of financial time series which usually correspond to political or economic events, and measure an impact of these sudden breaks on volatility persistence. When incorporating those breaks into conditional volatility models, long memory property usually significantly disappears. Volatility persistence is estimated using (FI)GARCH family models in two regimes: without sudden breaks and with sudden breaks which are incorporated in volatility models in terms of dummy variables. Empirical analysis is provided on illustrative example using sample data from developed and emerging stock markets. Namely, we consider weekly data of S&P500, WIG20 and PX indexes in the period of 2004 - 2013 years which also includes the stage of recent global financial crisis of 2008-2009 years. Our findings mean that when ignoring an impact of sudden breaks on volatility it leads to overestimation of volatility persistence.

Key words: GARCH, FIGARCH, ICSS algorithm, structural breaks, volatility persistence

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1 Introduction

In recent years, an extensive literature has been developed on studying the volatility of financial markets. One of the most challenging problems in financial econometrics is to understand and model a behavior of volatility through time. Volatility is considered a symptom of highly liquid stock markets. An increase in stock market volatility brings large stock price changes of growths or declines.

Volatility modelling and especially volatility dynamics are important for decision making in financial markets including derivative prices, leverage ratios, credit spreads, and portfolio decisions. In periods of low market volatility it is relatively easy to measure volatility and understand volatility dynamics. In other periods, financial markets are affected by severe disruptions or events like the market crash of 1987, a series of events such as the Russian default or the global financial crisis of 2008-2009 years. During such periods, apparent spikes in volatility and large movements in asset prices complicate estimation of volatility and volatility dynamics. To sum up, these crises dramatically influenced the market volatility and diversification opportunities for investors; see (Seďa, 2012).

Time varying volatility of stock returns has been extensively modelled by the GARCH with high frequency stock data to find high persistence in volatility. The GARCH approach assumes that there is no shift in volatility. However, in emerging markets there may potentially appear sudden breaks in volatility. Therefore, it is important to take account of these shifts when estimating volatility persistence. In this paper, the shifts in volatility are identified by utilizing the Iterated Cumulative Sums of Squares (ICSS) algorithm of (Inclan and Tiao, 1994) and its modification by (Sansó et al., 2004). The conditional volatility models are then estimated by taking account of the volatility shifts.

Sudden changes in volatility of emerging stock markets in Asia and Latin America were examined by (Aggarwal et al., 1999). They concluded that structural breaks are related to local economic and political events. On the other hand, (Hammoudeh and Li, 2008) and (Kang et al., 2009) summarised that the events induced sudden changes in volatility of stock markets have rather global character. Central and Eastern European markets were investigated by (Vyrost et al., 2011) finding an inverted relation between a number of structural breaks and volatility persistence. In addition, the choice of a method of identification of sudden breaks has a direct impact on the magnitude of persistence in volatility of respective time series. When incorporating of sudden breaks in volatility of Central European foreign exchange markets significantly decreased the persistence of volatility, see (Todea and Platon, 2012).

The aim of this paper is to examine an impact of structural breaks on volatility persistence of developed economies represented by US stock market and emerging Central European stock markets represented by Czech Republic and Poland using weekly data over the period of 2004-2013 years. The studies conducted in this paper will be summarized as follows. First, the conditional heteroskedasticity models will be introduced. Second, it will be discussed the ICSS algorithm and its later modification. In Section 3, data sample time series features will be statistically described. Moreover, the main results we achieved will be reported, and Section 4 concludes our analysis.

2 Methodology

The aim of this chapter is to describe methods that will be used for empirical analysis of structural shifts on long memory of volatility of financial time series. In this paper the GARCH, respectively FIGARCH models with and without sudden shifts will be described, and sudden breaks in volatility will be identified with a help of modified ICSS algorithm.

2.1 Conditional Volatility Models

The GARCH model have become a standard tool for modelling conditional variances of the returns from financial time series, see (Bollerslev, 1986). The popularity of these models is caused by their compatibility with major stylized facts for asset returns, the existence of efficient statistical methods for estimating model parameters, and availability of useful volatility forecasts. The stock index returns may have jump risk at the same time which is caused mainly by heteroskedasticity and rapid market fluctuation. In this chapter we will take a look on how to consider heteroskedasticity in the model in order to identify the process of forming appropriate risk premium in such case. Normally, the GARCH(1,1) model taking the form of discrete data and considering heteroskedasticity may takes form:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2, \tag{1}$$

where σ_t^2 is conditional variance estimated here. β_0, β_1, β_2 are constants satisfying the conditions of $\beta_0 > 0$, $\beta_1 + \beta_2 < 1$. For instance, in the GARCH(1,1) model, there is a stringent trade-off between the possibility of having sharp changes in the short term volatility represented by high value of the parameters $\beta_1 + \beta_2$ and the ability to capture the long memory behavior of volatility through high values of $\beta_1 + \beta_2$. Moreover, even with high value of $\beta_1 + \beta_2 < 1$, GARCH models are subject to exponential decline in the autocorrelation.

Another modification of ARCH family models that provides greater flexibility for modelling the conditional variance is the FIGARCH model. According (Baillie et al., 1996) the FIGARCH(1,*d*,1) model a conditional variance can be specified as follows:

$$\sigma_t^2 = \beta_0 \left(1-\beta_2\right)^{-1} + \left[1-\left(1-\beta_2 L\right)^{-1}\left(1-\beta_1 L\right)^{-1}\left(1-L\right)^{d}\right]\varepsilon_t^2, \tag{2}$$

where $0 Ł d Ł 1$ is fractional difference parameter and L means the lag operator. If we use for volatility estimation the FIGARCH model the persistence of shocks to either the conditional variance or the level of long memory is measured by the fractional differencing parameter d that is estimated by quasi-maximum likelihood estimation technique.

Long memory of volatility as estimated by GARCH and FIGARCH models usually tend to be overestimated when sudden changes in volatility regime appear. If the break points in variance of respective time series have been identified for instance with a help of the ICSS algorithm, the GARCH and FIGARCH models may be estimated in two regimes: first without sudden breaks and on the other hand with sudden breaks which are included in volatility models in terms of dummy variables. Therefore, conditional variance in the GARCH(1,1), respectively FIGARCH(1,d,1) models with sudden changes can be defined as follows:

$$\sigma_t^2 = \beta_0 + \beta_1\varepsilon_{t-1}^2 + \beta_2\sigma_{t-1}^2 + d_1D_1 + ... + d_nD_n, \tag{3}$$

$$\sigma_t^2 = \beta_0\left(1-\beta_2\right)^{-1} + \left[1-\left(1-\beta_2L\right)^{-1}\left(1-\beta_1L\right)^{-1}\left(1-L\right)^d\right]\varepsilon_t^2 + d_1D_1 + ... + d_nD_n, \tag{4}$$

where $D_1 + ... + D_n$ denote dummy variables that can be equal the value of 1 when sudden shifts in volatility appears, elsewhere take a value of 0.

2.2 ICSS Algorithm and its Modification

In this paper the Iterative Cumulative Sum of Squares or the ICSS algorithm originally proposed by (Inclan and Tiao, 1994) and its modification will be considered. The aim of this algorithm is to detect break points in variance The ICSS algorithm assumes stationary variance process over an initial period, until certain events generate a break point, then the variance returns to stacionarity again until the next appearance of sudden break and so on. This algorithm determines the number and position of sudden shifts in volatility with a help of cumulative sums of squares.

Let assume that ε_t is an independent time series with zero mean and variance s_j^2. The unconditional variance for each particular period is given by s_j^2 for $j=0,1,...,N_T$, where N_T means the total number of structural breaks in variance assuming T observations, and assuming that $1<K_1<...<K_{N_T}<T$ is the set of break point. To summarize, the unconditional variance s_j^2 over N_T intervals can be written as given below:

$$\sigma_t^2 = \begin{cases} \sigma_0^2, & 1<t<K_1 \\ \sigma_1^2, & K_1<t<K_2 \\ \cdot & \\ \sigma_{Nt}^2, & K_{Nt}<t<T. \end{cases} \tag{5}$$

A cumulative sum of squares is calculated in order to determine the total number of sudden breaks in volatility and time point when variance breaks appears. Therefore, the cumulative sum of square residuals from the first observation to the K-th time point can be defined as referred below:

$$S_K = \sum_{t=1}^{K} \varepsilon_t^2, \tag{6}$$

where K=1,…,T. In addition, test statistic D_K is defined as follows:

$$D_K = \left(\frac{S_K}{S_T}\right) - \frac{K}{T}, \tag{7}$$

where $D_0=D_T = 0$, and S_T denotes the sum of squared residuals of the whole sample period. In case of no change in unconditional variance, the D_K statistic will oscillate around zero. Structural changes in unconditional variance are estimated applying the critical values obtained from the distribution of D_K assuming the null hypothesis of constant variance. If the maximum absolute value of D_K is greater than the critical value, the null hypothesis of homogeneity in variance is rejected.

Original ICSS algorithm as proposed by (Inclán and Tiao, 1994) may have several size distortions; see (Sansó et al., 2004). Specifically, it suffers important size distortions for leptokurtic innovations. In addition, the size distortions are more extreme for heteroskedastic conditional variance processes. Their findings invalidate in practice the use of the ICSS test for financial time series. Therefore, (Sansó et al., 2004) proposed two test statistics:

$$k_1 = \sup_k \left| T^{-1/2} B_k \right|, \tag{8}$$

where

$$B_k = \frac{S_k - \frac{k}{T} S_T}{\sqrt{\left(\eta_4 - \sigma^4\right)}}, \tag{9}$$

and $\hat{\eta}_4 = T^{-1}\sum_{i=1}^{T} e_t^4$, $\hat{\sigma}^4 = T^{-1}S_T$ for $k \in \{1,2,...,T\}$. The calculation of statistic k_2 is defined as follows:

$$k_2 = \sup_k \left| T^{-1/2} G_k \right|, \tag{10}$$

assuming that

$$G_k = w_4^{1/2}\left(S_K - \frac{k}{T} S_T \right). \tag{11}$$

Using the modified ICSS algorithm denotes that we are calculating test statistics for various sample sizes. Thus, the critical values for each test statistic might be obtained from a response surface as provided by (Sansó et al., 2004) as long they provide better results when using small samples. In this paper, it will be used modified version of the ICSS algorithm.

3. Empirical Findings

The goal of this chapter is to present and demonstrate empirical findings using the methods discussed and described in Section 2. In first step we identify sudden breaks in volatility. Then the GARCH(1,1) and FIGARCH(1,*d*,1) models are estimated to quantify volatility persistence. As a next step of our analysis the GARCH and FIGARCH models are reestimated incorporating dummies representing break points as identified by modified ICSS algorithm.

3.1 Data Sample

Empirical analysis is performed on a data set of weekly closing rates of S&P500, WIG20 and PX indexes in the period of 2004 - 2013 years which also includes period of recent global financial crisis of 2008-2009 years. It includes total of 519 weekly observations. This period was chosen purposely, to investigate changes of respective markets volatility during time with a special emphasis on the behavior in the time during and after the global financial crisis of 2008-2009 years. The returns r_t at time t are defined as the logarithm of respective index p, that is, $r_t = \log\left(p_t - p_{t-1}\right)$. Tab. 1 shows selected descriptive statistics and the results of the Jarque-Bera test for all stock returns. Symbol * denotes a rejection of relevant null hypothesis at the 5% significance level.

Tab. 1: Descriptive statistics of S&P500, WIG20 and PX indexes

Index	*Mean*	*St. deviation*	*Skewness*	*Kurtosis*	*J-B test*	*Q(12)*

S&P500	0.0005	0.0285	-0.2979	7.9065	433.615	31.744*
WIG20	0.0007	0.0366	-0.4405	4.6194	60.3226	20.808
PX	0.0008	0.0353	-0.3254	10.2685	1137.518	14.341

Source: own calculations

The means of all sample returns are relatively small, while the standard deviations are significantly higher. Based on the values of skewness, increased values of kurtosis and *J-B* test of normality, the weekly return series show mostly leptokurtic distribution which has a higher peak and heavy tail, instead of normal distribution. The Lj.-Box statistics $Q(12)$ for the squared return series are high which indicate rejection of the null hypothesis of no correlation. The sample statistics of analysed time series data indicate that it is desirable to consider heteroscedasticity and jump risk when estimating the volatility of all investigated markets.

3.2 Identification of Sudden Breaks in Volatility

The ICSS algorithm calculates standard deviations between change points to identify the number of sudden changes. Tab. 2 indicates the time periods of sudden changes in volatility as identified by modified ICSS algorithm. Looking at Tab. 2, all series returns had similar time points of sudden changes in volatility which are correlated rather with global economic events like global financial crisis or subprime mortgage crisis. However, local events are also important, especially in case of U.S. economy.

Tab. 2: Sudden breaks in volatility as detected by modified ICSS algorithm

Index	St. Deviation	Time Period	Economic Event
PX	3,356	1/2004 – 9/2008	Entry to European Union in 2004, economic growth
	9,754	9/2008 – 7/2009	Global financial crisis
	4,824	7/2009 – 12/2013	Debt crisis, economic stagnation
WIG20	2,681	1/2004 – 5/2006	Entry to European Union in 2004, economic growth
	4,616	5/2006 – 10/2008	Global correction in equity markets due to fears of inflationary pressures
	5,836	10/2008 – 9/2009	Global financial crisis
	4,462	9/2009 – 12/2013	Debt crisis, economic stagnation
S&P500	2,136	1/2004 – 10/2008	Economic growth
	5,112	10/2008 – 4/2009	Global financial crisis
	2,637	4/2009 – 8/2011	Debt crisis, economic stagnation
	4,730	8/2011 - 12/20013	U.S. credit rating decrease, fears of a possible return of the global economy into recession, investors to move away from more risky assets to safer.

Source: own calculations

It can be seen that more sudden changes appeared in Polish and U.S. stock markets. In fact, all three markets experienced a similar change due to volatility spillovers across international markets. Volatility in one financial market is transmitted to other markets. In other words, a single event can affect different sectors simultaneously, and cause similar volatility breaks. Based on the values of standard deviations, we can conclude that the highest risk value was observed on the Czech market, which is followed by Polish market. Conversely the lowest value of the volatility has been noticed on U.S. stock market.

3.3 Conditional Volatility Models Estimation

In previous subchapter there were identified sudden changes in volatility. Next logical step of our analysis is to incorporate these sudden shifts into conditional volatility models in terms of dummy variables, and investigate potential impact of those shifts on volatility persistence. In Tab. 3 and Tab. 4 there are shown estimations of the GARCH(1,1) and FIGARCH(1,*d*,1) models in two regimes: with and without dummy variables. According Tab. 3, the GARCH(1,1) models without dummies exhibit highly significant parameters b_1 and b_2, and in addition, the sums of $\beta_1 + \beta_2$ are always close to one. It means that sudden shifts have a persistence impact on the volatility of all investigated time series. Volatility persistence reached higher level in case of US stock market. On the other hand, when including sudden shifts in terms of dummies, the persistence of conditional volatility is reduced in all series significantly. PX index shows decline in volatility persistence of 0,179, while WIG20 index shows even larger decrease in volatility persistence with 0,273. Persistence decline is largest in case of S&P500 and reached the level of 0,325. The results therefore clearly indicate that standard GARCH(1,1) model tends to overestimate volatility long memory by ignoring sudden shifts in conditional volatility.

Tab. 3 GARCH(1,1) model parameters with and without dummy variables for sudden breaks in volatility

GARCH(1,1) model without dummy variables						
Index	b_1	b_2	*volatility persistence*		*Q(12)*	*AIC*
PX	0,326*	0,584*	0,910		8,274	-4,241
WIG20	0,128*	0,816*	0,944		7,265	-3,924
S&P500	0,244*	0,726*	0,970		13,794	-4,739
GARCH(1,1) model with dummy variables						
Index	b_1	b_2	*volatility persistence*	*persistence decline*	*Q(12)*	*AIC*

PX	0,248*	0,483*	0,731	0,179	9,712	-4,032
WIG20	0,096*	0,575*	0,671	0,273	9,268	-3,769
S&P500	0,156*	0,489*	0,645	0,325	14,256	-4,521

Source: own calculations

According Tab. 4 we found that FIGARCH(1,*d*,1) models without dummy variables detected that fractional difference parameter *d* significantly from those cases when *d*=0 or *d*=1. It seems that volatility of all the series reveals long memory patterns. However, if one includes sudden shifts into the FIGARCH(1,*d*,1), the values of estimated parameters *d* significantly declined. Moreover, it becomes statistically insignificant at 5% level, and the volatility persistence disappears. Thus, it appears that ignoring sudden changes in conditional variances spuriously generates the presence of long memory in volatility. As a result, the long memory property in the volatility of all stock markets is often exaggerated by sudden changes corresponding especially to global financial events.

Tab. 4 FIGARCH(1,*d*,1) model parameters with and without dummy variables for sudden breaks in volatility

FIGARCH(1,*d*,1) model without dummy variables					
Index	b_1	b_2	*d*	*Q(12)*	*AIC*
PX	0,257*	0,483*	0,451*	8,420	-4,174
WIG20	0,108*	0,548*	0,354*	7,504	-3,832
S&P500	0,152*	0,479*	0,236*	14,143	-4,672
FIGARCH(1,*d*,1) model with dummy variables					
Index	b_1	b_2	*d*	*Q(12)*	*AIC*
PX	0,417*	0,379*	0,096	10,547	-3,947
WIG20	0,231*	0,736*	0,138	15,412	-3,412
S&P500	0,184*	0,590*	0,035	17,583	-4,417

Source: own calculations

4 Conclusions

This study examines the impact of structural breaks as estimated by modified ICSS algorithm on volatility persistence, or the long memory property on developed (USA) and emerging stock markets (Czech Republic and Poland). According our empirical estimations we can conclude that identification of sudden shocks in all investigated markets is rather caused by global political or economic event like the global financial crisis of 2008-2009 years although also local events may have significant impact. When including sudden breaks into GARCH and FIGARCH models in terms of dummy variables, the long memory of volatility significantly declines. This result suggests that ignoring the effect of sudden changes overestimates volatility persistence. Our findings denote that major economic and political events usually lead to overestimation of the persistence in volatility. While a certain reasonable level of volatility is certainly a desirable and natural on stock markets, as it reflects an impact of new information on markets starting from some level a volatility can be consider harmful. In that case, stock price already on financial markets usually do not reflect real prices of the underlying asset. Excessive level of volatility can lead for instance to inefficient resource allocation, and upward pressure on interest rates due to an excessive uncertainty in the equity markets. In addition, it may bring about potential errors by risk managers for instance when interpreting Value-at-Risk or in option pricing.

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