SIGNALING MODEL OF LABOUR DEMAND

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Abstract

This paper deals with the issue of labour demand in the specific situation of signaling behaviour. The concept of Signaling is known since its introduction by the Nobelist Michael Spence in 1973. However, its applications were usually drawn from the viewpoint of prospective employees. Our point of view in this paper is to look at the problem from the employers' side. From that perspective, demand for labour force is inevitably blurred by the randomness of workers' productivity. The employer doesn't know if newly hired employee really possesses the productivity he or she had signalled by an educational level. Only after couple of "working interactions" genuine productivity is revealed.

This leads to a game-like situation, because all agents of the labour market can repeatedly change their behaviour on the market, employees the preferred level of education and employers the wage scheme that is offered to employees accordingly to their education level. In the paper, I focus on the optimization problem of employers and I employ the dynamic approach. One of the most important variables in this decision is reliability of the signal, therefore my model explains how the results differ if the signal reliability changes.

Key words: Signaling, labour demand, job market, education

JEL Code: D82, D83, I21

Introduction

In the presented paper I deal with some of the problems in the labour market under uncertainty. This uncertainty is caused by asymmetry of information between the employer and prospects, i.e. possible employees. It is only the prospects having precise information about their productivity, whereas for the employer the hiring decision possesses risk. The point of departure is the theory of signaling behaviour, which was introduced to economics by Michael Spence in 1973 (Spence, 1973). My research question is whether signaling has an impact to the labour market and if it does in which direction it will influence it.

The theory of signaling says that the most obvious signal about one's productivity is education. There are some fundamental features of the theory, such as negative relationship between the innate and unobservable productivity of a worker and his/her cost of education and imperfect reliability of the signal. This is why the whole model has to be presented in a dynamic environment. It is because there can be several rounds of a hiring game when employers set up their wage offers and only after a period of time when the real productivity of a worker is revealed, they can assess, if their expectations were confirmed or not. If not, then enter the labour market anew, this time with different expectations expressed in different wage schemes offered.

We can say that the company's aim is to minimize the difference between signalled and real productivity. That's why a component of the model presented below is the production function of a firm including the loss function from the difference in productivities.

1. Basic characteristics of the model

If company's aim is to maximize the profit, it also has to minimize the difference between signalled and real productivity. Only if real productivity equals the signalled one, the employer reaches its optimal state that means it doesn't need to change the wage schemes for next rounds of hiring. We mark the productivity of an employee in time t as P_t and in time t signalled future productivity as $S_t[P_{t+1}]$. The simplest loss function that is to minimize is then following difference

$$L = S_t[P_{t+1}] - P_{t+1} \tag{1}$$

Simply put, employee's education provides information about his/her productivity, however this assumption can stay unfulfilled and the employer than suffers a loss as a difference between these two values.

The signalled productivity can be identified with expectations. Since if there prevail the principles of signaling behaviour in the labour market, employers expect productivity that is signalled. There emerges uncertainty and a certain level of probability that the expectations won't be met.

We have to recall that employers presume the employee to deliver signalled productivity and thus they form their wage schemes accordingly. They also know that in time $t+1$ they will know about employee's productivity with certainty. However they don't want to change their wage schemes (for it is costly), so they suppose

$$w_{t+1} = w_t = S_t[P_{t+1}] \tag{2}$$

Let's construct the loss function expressing the loss a company faces if real and expected productivity don't match. This is consequence of an information asymmetry in the labour market. We assume that the decision of the employer of the amount of labour under the previous conditions takes place at time t, when the employer has available only the signaling information on the expected productivity at time $t+1$. The actual productivity at time $t+1$ is unknown and therefore employer's decision is based on known values, i.e. the difference between current worker productivity and in the past signalled level of current productivity. This is the formulation of conditional expectations of the employer as a loss function shifted back one period. We get a modified expression (1):

$$L_{t-1} = S_{t-1}[P_t] - P_t \tag{3}$$

The aim of the employer for two consecutive follow-up periods is to reduce losses. By reducing the losses the conditional expectation of the employer will be adjusted, and therefore the employer also reduces cost that would otherwise be spent. For employers would be therefore desirable to get $L_{t-1} > L_t$. When using expressions in equations (1) and (3) we get inequality

$$(S_{t-1}[P_t] - P_t) > (S_t[P_{t+1}] - P_{t+1}) \tag{4}$$

which we can modify into relation between changes in real productivities and expected productivities

$$(P_{t+1} - P_t) > (S_t[P_{t+1}] - S_{t-1}[P_t]) \tag{5}$$

This means that in order for the employer to be better-off the growth of the real productivity (left side) must be larger than growth of the signalled one (right side).

Now we suppose there is a stable state on the market and employers don't realize any losses. Than the wages will not change until there's growth in productivity. We can reformulate (5) into an equation and employ (2) to get

$$w_t = w_{t-1} + P_{t+1} - P_t \tag{6}$$

that is current wage depending on past wage and the changes in productivity. We also know that the productivity growth is only an expectation.

2. Inter-temporal Loss function

In the foregoing, we have assumed only two periods. Now we extend the time horizon to n periods and form the inter-temporal loss function based on expression (3). The function is simply a sum of losses in all periods and reads

$$L = \sum_{t=0}^{n} (S_t[P_{t+1}] - P_{t+1}) \tag{7}$$

All the losses are counted the same and there is no depreciation of future values. On the other hand there are changes in the values of losses over time and we see that in terms of the topic presented, it is education that affects the company's loss from expected productivity. Education serves as a signal of productivity and if there might be problem with the difference between real and expected, thus signalled, productivity, isn't that education to blame?

Let's assume we are able to assign the level of education to an invariable. According to the statistical data there are several groups classified into educational levels. The universal scale of ISCED (The International Standard Classification of Education) levels of education is not used by the Czech statistical office, however there is another five-grade level used. We assign to each level a value from 1 to 5 and calculate a weighted average, where the weights of each level are relative numbers of the specific educational groups in the population. This way we can, based on statistical data, form an educational index for the Czech Republic and track its development in time. In the following table there are values assigned to each educational level:

Tab. 1: Index of education levels

Educational group	Elementary/ unfinished	Secondary without graduation	Secondary with graduation	Higher and bachelor's degree	University
Level index	1	2	3	4	5

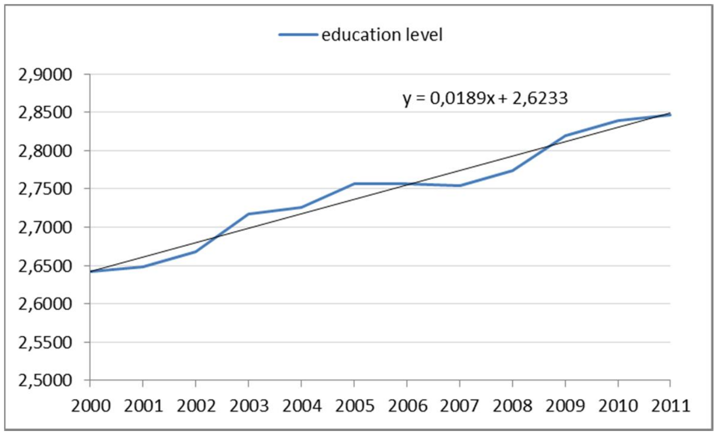
Now we can count the weighted average for last twelve years and we get following results:

Tab. 2: Index of education 2000 - 2011

2000	2,6425
2001	2,6481
2002	2,6685
2003	2,7177
2004	2,7261
2005	2,7564
2006	2,7569
2007	2,7548
2008	2,7739
2009	2,8194
2010	2,8393
2011	2,8468

To depict the development of the general educational level in the Czech Republic we can show it in a chart and we can also compute the linear regression.

Fig. 1: Index of education 2000 – 2011



Now we can ask about the influence the level of education has on the loss magnitude in certain period. The impact is dual. On one side the higher the educational level the lower the signal reliability. The season for that is that with higher levels of education the character of education changes. Each level describes not only growth from previous level but also defines different types of education. This changes cause a decrease in verification chances. It is much easier to verify that education signals productivity correctly at the lower levels than at the higher ones. The imprecision of a signal rises with the educational level.

On the other hand, from very similar reasons we can presume quite countervailing effect. For the difference between signalled and real productivity can be smaller with increasing educational levels. That's because education becomes more universal in comparison to vocational and secondary education with higher education levels. Thanks to this universality the difference between signalled and real productivity can be reduced.

If we had to decide which of these effects would prevail, it is the former, connected with the imperfection of signals. We can assume this signal unreliability grows faster with education level than the decreasing effect of universality. When we include the educational invariable (σ) to into the loss function (7) we get the final expression for company's inter-temporal loss under the signaling behaviour in the labour market:

$$L = \sum_{t=0}^{n} \frac{1}{\sigma_t} \left(S_t^{\sigma_t}[P_{t+1}] - P_{t+1} \right) \tag{8}$$

We can see there the effect of the invariable σ both on the decrease of the loss value (expression $1/\sigma_t$) and the increase in this value (exponent σ_t).

3. Production function

Keeping the prerequisites valid we can now express the company's production function in the situation of possible loss caused by signaling behaviour. The revenue side is given by productivity and amount of labour, whereas the cost side by the loss from signaling. Simple production function under these conditions reads

$$Y = \sum_{t=0}^{n} (P_t N_t^{\alpha}) \sum_{t=0}^{n} (S_{t-1}[P_t] - P_t) \tag{9}$$

which we reformulate to the form that we can maximize

$$\max \sum_{t=0}^{n} P_t \left(N_t^{\alpha} - \frac{S_{t-1}[P_t] - P_t}{P_t} \right) \tag{10}$$

This expression nevertheless doesn't include the educational invariable σ introduced in (8). We have to shift the invariable value one period back and then we can express the final production function in the maximization form

$$\max \sum_{t=0}^{n} P_t \left(N_t^{\alpha} - \frac{S_{t-1}^{\sigma_{t-1}}[P_t] - P_t}{\sigma_{t-1} P_t} \right) \tag{11}$$

Now we can proceed to the model of labour demand under the signaling conditions. The control variable is N – the amount of labour, the state variables are P and σ, productivity and education respectively. In general we can say we want to maximize company's production function where variables are labour, productivity and level of education. We have also to include the discount factor β.

$$\max_{\{N_t\}_{t=0}^{n}} \sum_{t=0}^{n} \beta^t Y(N_t, P_t, \sigma_{t-1}) \tag{12}$$

The state equation then will be

$$P_{t+1} = F(P_t, N_t) \tag{13}$$

this means the fact that future productivity depends on current productivity and current amount of labour. Our task is to find relation between values of productivity and labour amount in the same period, so the amount of labour is a function of productivity and education (in the past). This will be the simplified labour demand function of an employer. In general it reads:

$$N_t = g(P_t, \sigma_{t-1}) \tag{14}$$

We further proceed in according to the principles of dynamic optimization and we form the value function and so called Bellman equation. The value function for the initial values of state variables is

$$v(P_0) = \max_{\{N_t\}_{t=0}^{n}} \sum_{t=0}^{n} \beta^t Y(N_t, P_t, \sigma_{t-1}) \tag{15}$$

and the respective Bellman equation then

$$v(P_0) = \max_{\{N_t\}_{t=0}^{n}} \{Y(N_0, P_0, \sigma_{-1}) + \beta v(N_1, P_1, \sigma_0)\} \tag{16}$$

This equation determines how we have to choose the control variable in order to maximize the current value of production function and at the same time discounted value of the production function in subsequent periods depending on the development of state variables.

We adjust the Bellman equation and with help of expression (14) to express it as function of state variables only.

$$v(P_t) = \max_{P}\{Y(g(P_t, \sigma_{t-1}), P_t, \sigma_{t-1}) + \beta v(g(P_{t+1}, \sigma_t), P_{t+1}, \sigma_t)\} \tag{17}$$

The necessary optimization condition is derivation of this equation equal to zero, which is gives the Euler equation, generally set as

$$v(P_t) = \max_{P}\{Y(g(P_t, \sigma_{t-1}), P_t, \sigma_{t-1}) + \beta v(g(P_{t+1}, \sigma_t), P_{t+1}, \sigma_t)\} \tag{18}$$

Now we can use again the state equation saying that current productivity is function of past productivity and past amount of employment and then adjust the Euler equation to

$$\frac{\partial Y(g(P_t, \sigma_{t-1}), P_t, \sigma_{t-1})}{\partial N_t} = -\beta \frac{\partial Y(g(P_{t+1}, \sigma_t), P_{t+1}, \sigma_t)}{\partial N_t} \tag{19}$$

It is then clear that the Euler equation shows the relation between current and past production depending on changes of labour amount in presence. This relation is being negative, for simultaneous increase in productivity an in labour must be counterbalanced by the countervailing motion in the future, e.g. growth of production by decrease in labour productivity. This growth can be achieved only by productivity increments following the decrease in labour quantity or by lowering of the loss caused by the difference between signalled and real productivity. As has been already set above, this lowering is also one of the main efforts of the employer.

Now we can substitute the production function (11) into the Euler equation and derive, so we obtain

$$\alpha \frac{P_t}{N_t^{1-\alpha}} = -\beta \left\{ \frac{\partial F(P_t, N_t)}{\partial N_t} \frac{-1}{\sigma_t P_{t+1}^2} \right\} \tag{20}$$

By modifications of this expression we can try to obtain a specific example of the general function expression (14). For simplicity, let's substitute the derivation of state equation (first component in the brackets) by θ and then we can modify, so we eventually get

$$\frac{N_t^{1-\alpha}}{P_t} = \frac{1}{\theta} \frac{\alpha}{\beta} \sigma_t P_{t+1}^2 \tag{21}$$

And after further modifications in order to single the current amount of labour out as the control variable we get the final expression for the dynamic labour demand under signaling

$$N_t = \left(\frac{1}{\theta} \frac{\alpha}{\beta} \sigma_t P_t P_{t+1}^2 \right)^{\frac{1}{1-\alpha}} \tag{22}$$

where θ is derivation of the state equation (13). Current amount of labour then depends on current educational level, current productivity and future productivity. Since the future productivity is unknown, we have to outcome from the expectations and current signals and the demand equation changes into

$$N_t = \left(\frac{1}{\theta} \frac{\alpha}{\beta} \sigma_t P_t S_t [P_{t+1}]^2 \right)^{\frac{1}{1-\alpha}} \tag{23}$$

Conclusion

The labour demand under the condition of signaling behaviour of prospects in the labour market changes its character from a couple of reasons. The main reason is that there exists ambiguity in the level of productivity the prospective employees possess. This ambiguity causes a situation of uncertainty and need for repeated games of signaling and hiring. This repetition leads to specific dynamics of the demand evolution over time. Therefore the model is presented in dynamic environment and that's how the essential difference between expectations and reality can be overcome over time.

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