CAPM AND OPTIMAL PORTFOLIO

Petra Gavlaková – Elena Gregová

Abstract

One of the most important factors affecting investment decisions is cost of capital. Many economists have therefore examined the possible ways of calculating the cost of capital. There still doesn't exist consensus among experts but in developed markets the Capital Asset Pricing Model (CAPM) is commonly used for valuation of assets and for creating the optimal portfolio. It is the mathematical model that explains a relationship between risk and return of particular securities. This model was created parallel and independent during the period 1964 – 1966 by authors W. F. Sharpe, J. Lintner and J. Mossin. However foundations of this model were developed by H. Markowitz and J. Tobin in the early 50's of 20th century. The basic idea is an assumption that expected yield (or loss) of an investment is proportional to the risk (coefficient Beta) of particular investment. CAPM therefore clarifies the relationship between expected return and Beta; and allows the investor to create optimal portfolio that eliminates the risk of investment. This paper deals with the Capital Asset Pricing Model, describes its advantages and also drawbacks.

Key words: CAPM, risk, portfolio, capital

JEL Code: G11, G12

Introduction

The capital market investment is an interesting possibility how to maximize the investor`s capital. It is always necessary to create some strategies to eliminate the risk of high loss. In order to manage investment strategies we can use the Portfolio Theory. It is always desired to create such an optimal portfolio that would balance the risk and yield.

The paper deals with creating optimal portfolio using the Capital Asset Pricing Model - the mathematical model most commonly used for this purpose. It describes the relation between risk and return of assets. It was parallel created by W. F. Sharpe, J. Lintner and J. Mossin during the years 1964 – 1966. All of them were inspired by model developed by H. Markowitz and J. Tobin in the early 50's of 20th century.

CAPM brings a new point of view to the assets valuation. Sharpe`s model is based on the idea of balance between risk (β) and expected yield and loss of investment. This method allows investor to create such a portfolio that is acting consistently with the market.

1 Specific and Market Risk

People invest because they expect some return. The more risky is the investment, the higher return investor demands to compensate him for accepting the risk. However, not all risks are equal. If an investor holds a diversified portfolio of equity investments then he will require a return to compensate only for those risks which contribute to the risk associated with the portfolio as a whole. Risks which are unique or specific to a particular equity investment are diversified away by holding the portfolio, and investor will therefore not require any additional return as a reward for bearing such risks (Ogier, Rugman & Spicer, 2004).

Risk reduction and portfolio size is a well known and much-studied topic in financial markets since the establishment of Modern Portfolio Theory (MPT) in the seminal work of Markowitz in 1952. As a later extension to Markowitz by Evans and Archer in 1968 and Elton and Gruber in 1977, it was shown that the variation in total returns on portfolios of stocks can be split into two components, systematic and non-systematic risk. The former component explains the underlying variation in market returns while the latter is wholly attributable to the variation in returns of a portfolio's specific assets, hence often referred to as specific risk (Baum & Baum, 2006).

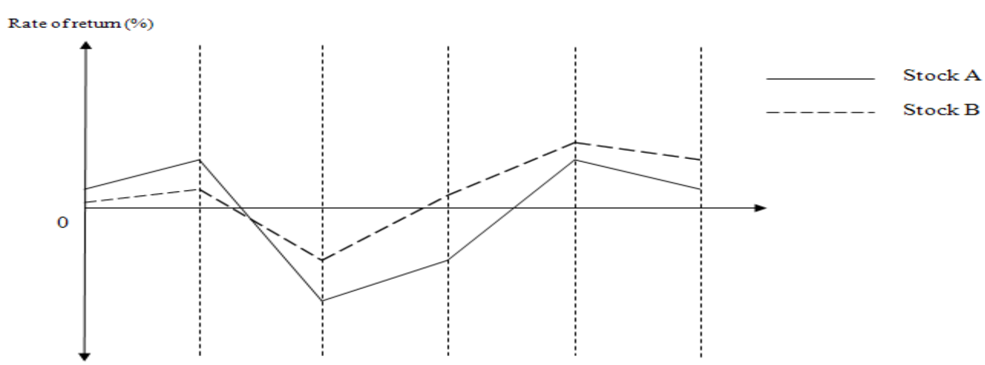
Specific risk – it is often called also unsystematic, variable, diversifiable or unique risk (for example a rival company goes out of business, labor force strike in the factory, construction of a new plant is more expensive than expected etc.). The specific risk consists of four parts: managerial, operational, financial and advance risk.

- *Managerial risk* – means the possibility that managers of company won't be competent and will lead the firm to insolvency. Such a risk often occurs in the new companies that may have a problem to succeed in financial markets.
- *Operational risk* – risk that firm won't be able to produce enough revenue to cover the fixed costs of its activities. It relates to active side of the firm's balance sheet.
- *Financial risk* – relates to passive side of the firm's balance sheet and it is the risk that firm won't be able to cover the fixed costs such as fixed interest payments.

- *Advance risk* – depends on investor's requirements for assets of the company in bankruptcy. Generally it means the order in which the investors' requirements will be satisfied.

Market risk – the risk of the loss of the portfolio value caused by price changes of assets in financial markets. It is sometimes called systematic risk and it can't be diversified. Whereas specific risk results from concrete situation in particular company, market risk is influenced by macroeconomic events (for example Growth in gross domestic product – GDP is faster than expected, interest rates rise, the local currency appreciates, the rate of inflation falls etc.).
Well-diversified equity portfolio can eliminate specific risks. The basic rule of proper portfolio creation is the rule of opposite movement of returns (negatively correlated returns) or at least independent movement (non-correlated returns). The implications of portfolio diversification for risk and return were first explored by Harry Markowitz in 1952. He pointed out that individuals can reduce the variability of the returns on the investments they make simply by investing in a number of different companies or investments. When considering investments in only two stocks it is possible to put bounds on the extent to which risk is reduced (Markowitz, H., 1952).
For example, we have stocks A and B. Stock A has a standard deviation 35% and stock B 55%. If the returns on these stocks were perfectly correlated, i.e. if they always moved in step with one another (Picture 1) then the standard deviation of a 50-50% portfolio of the two stocks would simply be the average of their individual deviations, in this case 45%.

Fig. 1 Perfectly positively correlated stocks

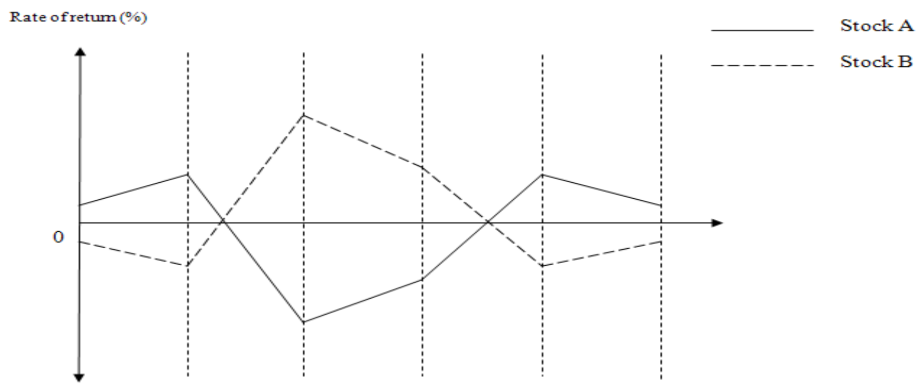


Source: (Cisko & Klieštik, 2009)

If, on the other hand, the two stocks were perfectly negatively correlated, i.e. when the returns on A go up the returns on B go down by the same amount, and vice versa (Picture 2) then the standard deviation of the 50-50% portfolio would be zero. In such circumstances, an investment in A would effectively provide some insurance against an investment in B, giving the investor absolute certainty of returns.

In real life the returns on most companies are positively (but not perfectly) correlated. In other words, the returns on investments and companies vary because of a range of uncertain factors, some of which are specific to the particular company or investment, and some of which affect all companies together (most often changes in the economic environment).

Fig. 2 Perfectly negatively correlated stocks



Source: (Cisko & Klieštik, 2009)

As more and more stocks are added to a portfolio, the contribution to variability of returns of the factors which affect only an individual company or investment declines, and the importance of the correlation between variations in returns begins to dominate. Essentially this happens because, if a portfolio contains a sufficient number of diverse stocks, then on average in any period the lower returns on investments which perform adversely due to specific factors will be offset by higher returns on those which perform well due to favorable specific factors (Ogier, Rugman & Spicer, 2004). At the limit, once a sufficient number of stocks has been added to a portfolio, all the variability in returns is associated with those factors which influence all of the stocks systematically – the factors which cause returns on individual stocks to be correlated. Hence, in a well-diversified portfolio the risks to which the investor remains exposed are purely those associated with the market as a whole; risks specific to a particular stock or investment are not relevant.

2 Optimal Portfolio Theory

The goal of portfolio selection is to find an optimal allocation of wealth across a number of assets. First papers dealing with theory of portfolio were introduced at the end of 19th century.

In 1952 Harry Markowitz published an article that is considered to be a beginning of new approach to investing based on a modern portfolio theory. He proposed a model in which an investor selects a portfolio at time t-1 that produces a stochastic return at t. The model assumes:

- investors are risk averse,
- all investors invest during the same time horizon,
- investment decision making is based on expected risk and yield,
- there exists a perfect capital market,
- choosing among portfolios, investors care only about the mean and variance of their one-period investment return.

As a result, investors choose "mean-variance-efficient" portfolios, in the sense that the portfolios minimize the variance of portfolio return, given expected return, and maximize expected return, given variance (Fama & French, 2004). The portfolio model provides an algebraic condition on asset weights in mean-variance-efficient portfolios. Markowitz model can be described: when investor wants to minimize an overall portfolio risk he must combine assets that are negatively correlated or that are not correlated.

William Sharpe in 1964 and John Lintner in 1965 added two key assumptions to the Markowitz model to identify a portfolio that must be mean-variance-efficient. The first assumption is complete agreement: given market clearing asset prices at t-1, investors agree on the joint distribution of asset returns from t-1 to t. The second assumption is that there is borrowing and lending at a riskfree rate, which is the same for all investors and does not depend on the amount borrowed or lent.

The key result of individual investor's portfolio theory is the intention to invest into some combination of risky assets and risk-free asset.

3 Capital Asset Pricing Model

CAPM assumes that the cost of equity for any investment will increase only with the extent of systematic risk to which the investment exposes the equity investor. CAPM gives the formula for the cost of equity as:

$$K_e = R_f + \beta_e \cdot EMRP \qquad (1)$$

Where:

K_e – Cost of equity

R_f – Risk-free rate

B_e – Equity beta of investment
EMRP – Equity market risk premium

CAPM derives the cost of equity by adding to the risk-free rate a premium for risk which is a product of the equity market risk premium (a measure of the reward required by investors for investing in a risky equity) and the investment's beta (a measure of the relative systematic risk of the particular equity investment). So the CAPM suggests that the cost of equity will vary between different investments only to the extent that investments exhibit differing degrees of systematic risk. There are two fundamental relationships: the capital market line and the security market line. These two models are the building blocks for deriving the CAPM.

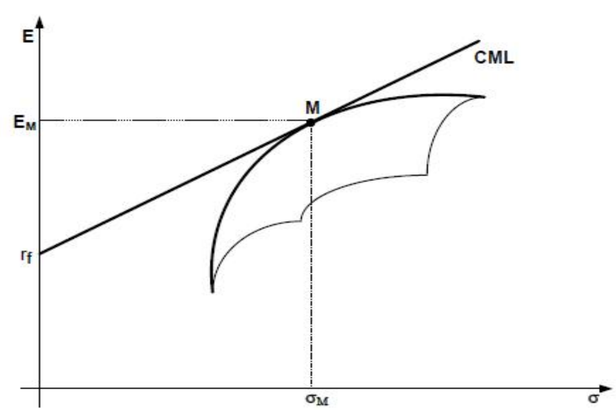
3.1 Capital Market Line

The capital market line (CML) specifies the return an individual investor expects to receive on a portfolio. It is a linear relationship between risk and return on efficient portfolios that can be written as:

$$E(R_p) = R_f + \sigma_P \left[\frac{E(R_m) - R_f}{\sigma_m} \right] \qquad (2)$$

Where:
R_p - Portfolio return
R_f - Risk-free asset return
R_m - Market portfolio return
σ_p - Standard deviation of portfolio returns
σ_m - Standard deviation of market portfolio returns

Fig. 3 Capital Market Line



Source: (Cisko & Klieštik, 2013)

The expected return on a portfolio can be thought of as a sum of the return for delaying consumption and a premium for bearing risk inherent in the portfolio. The CML is valid only for efficient portfolios and expresses investors' behaviour regarding the market portfolio and their own investment portfolios.

3.2 Security Market Line

The security market line (SML) expresses the return an individual investor can expect in terms of a risk-free rate and the relative risk of a security or portfolio. The SML with respect to security *i* can be written as:

$$E(R_i) = R_f + \beta_i \{E(R_m) - R_f\} \qquad (3)$$

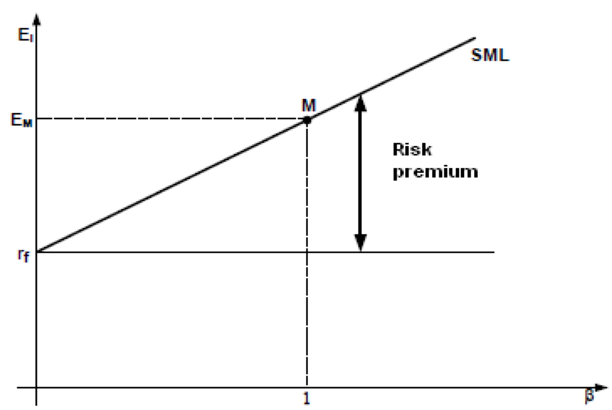
Where:

$$\beta_i = \frac{\sigma_i \cdot r_{im}}{\sigma_m} = \frac{\mathrm{cov}(R_i, R_m)}{\sigma_m^2} \qquad (4)$$

R_m is the correlation between security return, R_i and market portfolio return. The β_i can be interpreted as the amount of non-diversifiable risk inherent in the security relative to the risk of the market portfolio.

The SML is applicable to portfolios as well. Therefore, SML can be used in portfolio analysis to test whether securities are fairly priced, or not.

Fig. 4 Security Market Line



Source: (Cisko & Klieštik, 2013)

It is necessary to distinguish between CML and SML. Even if their geometric interpretations are very similar, CML describes the relation between expected yield and overall risk of portfolio consisting of risky and riskless assets. SML describes the relation of any individual asset and market by its covariance with market. It means, if covariance of asset and market rises, the expected yield of this particular asset rises too. Investor should find out how sensitive is the yield of intended investment relative to the yield of market portfolio.

3.3 Beta Coefficient

Beta of portfolio is the weighted average of individual betas of assets that are forming the portfolio.

$$\beta_p = \sum_{i=1}^{n} X_i \cdot \beta_i = X_{1\cdot} \cdot \beta_1 + X_2 \cdot \beta_2 + \ldots + X_n \cdot \beta_n \qquad (5)$$

Beta coefficient β_i measures the sensitivity of asset yield on the changes of market yield rate.

- if $\beta_i > 1$, assets are classified as aggressive. Rate of return of i-asset rises faster than market portfolio rate of return,
- if $\beta_i < 1$, assets are classified as defensive. Yields fluctuates less than the market,
- if $\beta_i = 1$, assets are neutral. Rate of return of i-asset behaves identically as market portfolio rate of return.

Values less than 0,5 and higher than 2 are rare and unsustainable.

Conclusion

The Capital Asset Pricing Model is the best-known model used to determine the expected rate of return desirable for a variable income investment. This model builds conceptually on the relationship between risk and return taking into account:

- investors hold well-diversified portfolios,
- investors want to maximize their economic utility (more return is preferable to less),
- investors are risk-averse,
- investors cannot influence prices,
- investors can lend and borrow at the risk-free rate,
- there are no transaction costs or taxes,
- all of the necessary information is free and easily accessible by all participants at the same time,
- the traded securities are divisible into small parcels.

The underlying assumptions of investment theory, and specifically of the CAPM, are that investors are rational, that they aim to maximize economic utility, and that they are risk-averse (Porras, 2011).

CAPM has some drawbacks for which has been criticized and even denied by some economists. For example:

- it is created on the basis of historical values but working with expected yield,
- is single-factor, which is the major limitation; doesn`t include dividends, liquidity of assets, taxation, company size, book value of firm etc.,
- assumes the same interest rate for deposits and loans.

The standard CAPM is set in the contingent-states world; it is assumed that investors at date 0 know the contingent returns on each asset at date 1 and their probabilities. This is why they know the variance of return of each asset and all the covariances between them. The assumption seems hopelessly unrealistic. Yet the model does combine practicality with a rigorous theoretical basis, and this helps explain its great prominence in finance. If one is willing to assume that the model holds at least approximately then the assumptions required for its derivation can be put aside. It can be applied in the real world, because the three ingredients required to estimate the expected return on an asset – the risk-free rate, the market risk premium ant the asset´s eta – can all be estimated (Armitage, 2005). With some adjustments it is possible to have modified model solving problems mentioned before.

Acknowledgment (Project)

The article is an output of scientific project VEGA 1/0357/11 Klieštik, T. and col.: Research on the possibility of applying fuzzy-stochastic approach and Corporate Matrics as tools of quantification and diversification of business risk.

References

[1] Alexander, C. (2008). *Value at risk models*. Chichester: John Wiley&Sons, Inc.

[2] Armitage, S. (2005). *The cost of capital: Intermediate theory*. New York: Cambridge University Press.

[3] Baum, A. (2006). *Managing specific risk in property portfolios*. Unpublished manuscript, Department of Real Estate and Planning, University of Reading Business School, Reading.

[4] Cisko, Š., & Klieštik, T. (2009). *Financial management of company I*. (1st ed.). Žilina, Slovakia: EDIS Publishers.

[5] Cisko, Š., & Klieštik, T. (2013). *Financial management of company II*. (1st ed.). Žilina, Slovakia: EDIS Publishers.

[6] Copeland, T. E., Koller, T., & Murrin, J. (2000).*Valuation: Measuring and managing the value of companies*. (3rd ed.). New York: John Wiley&Sons, Inc.

[7] Damodaran, A. (2004). *Damodaran on valuation*. (2nd ed.). New York: John Wiley&Sons, Inc.

[8] Fama, E. F., & French, K. R. (2004). *The capital asset pricing model: Theory and evidence*. CRSP Working Paper; Tuck Business School Working Paper, Retrieved from http://ssrn.com/abstract=440920 or http://dx.doi. org/10.2139/ssrn.440920.

[9] Froot, K. A., & Stein, J. C. (1998). Risk management, capital budgeting, and capital structure policy for financial institutions: An integrated approach.*Journal of Financial Economics*, *47*(1), 55 - 82.

[10] Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*(1), 77 - 91.

[11] Ogier, T., Rugman, J., & Spicer, L. (2004). *The real cost of capital*. Great Britain: Pearson Education Limited.

[12] Porras, E. R. (2011). *The cost of capital*. Hampshire, England: Palgrave Macmillan.

Contact

Petra Gavlaková, Ing.
University of Žilina in Žilina, Faculty of Operation and Economics of Transport and Communications, Department of Economics, Univerzitná 1, 010 26 Žilina
petra.gavlakova@fpedas.uniza.sk

Elena Gregová, doc. Mgr., PhD.
University of Žilina in Žilina, Faculty of Operation and Economics of Transport and Communications, Department of Economics, Univerzitná 1, 010 26 Žilina
elena.gregova@fpedas.uniza.sk