CHANGES IN THE DEVELOPMENT OF THE NORMAL LENGTH OF LIFE AND LIFE EXPECTANCY IN THE CZECH POPULATION

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Abstract

The article deals with the analysis of the development of the mortality of the population in the Czech Republic from 1950. In the past the life expectancy was gradually increasing. This means that there was a gradual improvement in mortality. On the basis of projections of future development we may assume that there will be an improvement in mortality. It will be increasingly important to have the best idea about mortality in individual matters. In our study we are focussing on the analysis of the development of the life expectancy at an exact age x. The life expectancy is not the sole characteristic of length of life. The second indicator is the normal length of life, which is also a very frequently used indicator. For the equalisation of specific mortality rates we use what is so far the most-used Gompertz-Makeham model. An important part of the article will be the comparison of the development of the life expectancy and normal length of life for the whole period. In a further part we will carry out a comparison of various methods of estimating the normal length of life. We will publish the results separately for males and for females.

Key words: mortality, normal length of life, life expectancy

JEL Code: J11, J19

Introduction

For the investigation of mortality is the most commonly used an indicator known as life expectancy. Given that the positive development of mortality has great impact on the reforms of social and pension system, it is good to follow other indicators. It is important to mention for example normal length of life or probable length of life. In this study, will be presented two of them – normal length of life and life expectancy. The subject of research is the development of mentioned indicators in the Czech Republic. The next part will be about comparing of two ways of estimate of the normal length of life. The aim is to determine how

much these two methods differ from each other and whether there are changes during the reference period.

1   Methodology

Recently, the length of human life is extended. One reason for this development is the increasing level of medical care. An important role also plays higher interest in a healthy lifestyle. Lengthening of human life means that people live longer. On the other hand, there is a question, in what medical condition a person spends the last years of his life.

Given this development, it is increasingly important to have the most accurate picture of the future development of human life and the development of mortality. The most frequently used indicator to capture the length of human life is the life expectancy (as was mentioned before). It indicates how long somebody lives in average (assume: prevailing patterns of mortality at the time of the construction of mortality tables were to stay the same throughout his – her life). But we must not forget on the other indicators such as normal or probable length of life (e.g. Fiala, 2005). The probable length of life is defined as the age at which lived just half of births (in this case we are talking about the probable length of life of a person at age 0). The normal length of life then gives the age at which most people die. In other words, it is the modus age of deaths. In this study will be presented two methods used for estimating it. One will be based on the age at which most people die and the second method, which is derived from the Gompertz-Makeham function (e.g. Fiala, 2005).

Life expectancy can be found in mortality tables. The own calculation follows several steps. At first, shall be calculated specific mortality rates, the probability of death and survival. Another part of the calculation relates to tabular population. At first is selected, the default number of live births in tabular population and it continues by calculation of the number of survivors, deaths, number of years lived and a number of years of residual life. In the last step will be obtain the life expectancy (e.g. Fiala, 2005).

If we examine mortality in individual ages, we find out that the mortality trend of oldest persons is different from the younger age groups. Therefore, it is necessary to model their mortality. For smoothing to empirical values of specific mortality rates is possible to use any of already existing models. In this study the Gompertz-Makeham model will be presented. This assumes that with increasing age the intensity of mortality is constantly growing (this assumption is not met at the highest ages) (e.g. Gompertz, 1825 or Makeham, 1860). For this reason, the Gompertz-Makeham function is used to compensate for the age range of 60-85 years old (e.g. Boleslawski and Tabeau, 2001 or Thatcher et al., 1998):

$$\mu_x = a + bc^x,$$ (1)

where x is age, μ_x is the intensity of mortality, a, b and c are parameters, which would be estimated by ordinary least squares (OLS) (Burcin et al., 2010 or Gavrilov and Gavrilova, 2011).

The second indicator characterizing the human lifespan is the normal length of life. As has been said, this is the age at which the most persons die (i.e. the age at which the density number of deaths is the highest) (Canudas-Romo, 2008). The density values can be easily estimated as (e.g. Koschin, 2005):

$$\delta(x) = \mu_x.l(x),$$ (2)

where $l(x)$ is the number of living at exact age x, $\mu(x)$ is the intensity of mortality.

To find the maximum of density is sufficient to solve the following equation:

$$\delta'(\hat{x}) = 0.$$ (3)

Maximum is interesting mainly in higher ages (i.e., we are not interested in maximum in childhood). At first we calculate the density of number of deaths by the formula (2) and then we put its derivative equal to 0.

The intensity of mortality can be also modeled by using the Gompertz-Makeham function (1). If we proceed from the requirement that the normal length of life is defined as the age at which the density of deaths is the highest. Its calculation is performed according to formula (2). With various modifications finally we arrive at formula for calculating the normal length of life (e.g. Koschin, 2000):

$$\hat{x} = \frac{\ln[\frac{1}{2b}.(\ln c - 2a + \sqrt{(\ln c - 4a).\ln c})]}{\ln c},$$ (4)

where a, b, c are parameters of the Gompertz-Makeham model.

The second method of calculating the normal length is easier. We estimate it as the age at which the most persons from tabular population die, finally it is important to increase its value by 0.5:

$$\hat{x} = \max(d_x),$$ (5)

where d_x is the number of deaths in tabular population.

As with the previous method of estimation we are not interested in the maximum in childhood. Therefore in the calculation must exclude age 0.

2   The results

In this study, will be followed the development of mortality of males and females in the Czech Republic since 1950 (until 2011). The evaluation will be done by life expectancy and normal length of life. For the normal length of life will be also compared two ways to estimate.

For own calculation was used Solver in MS Excel and DeRaS.

Fig. 1 – Lifespans – males in the Czech Republic
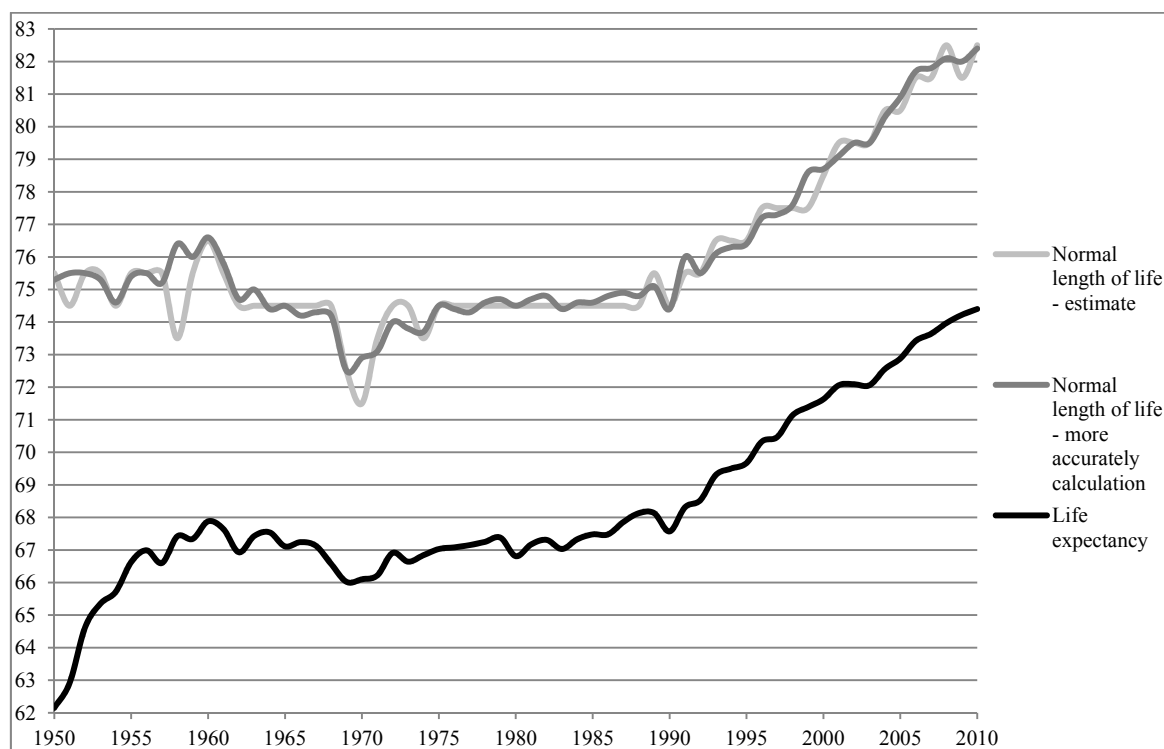
Source: own calculation, data: Eurostat

The first figure shows, that during the whole period there were increases in longevity of males in the Czech Republic. The life expectancy has grown more than the normal length of life. Low values of life expectancy at the beginning of the reference period are mainly due to high infant mortality. For the normal length of life is evident that during the period under review increased. The increase was not as pronounced. In contrast to the life expectancy there are no

major fluctuations. From the development is also clear that, according to both methods of estimation of normal length of life we will get similar results.

Fig. 2 – Lifespans – females in the Czech Republic
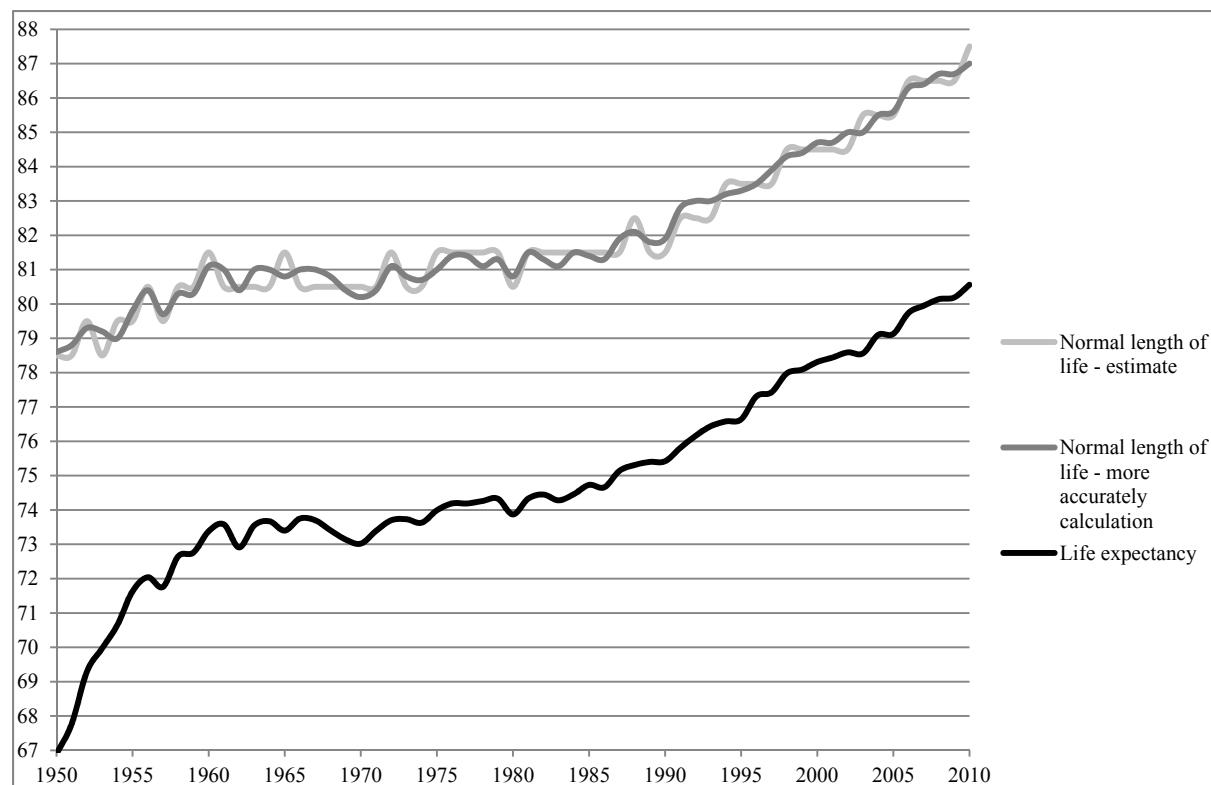
Source: own calculation, data: Eurostat

The second graph shows a similar situation as the first one. During the monitored period there was lengthening of both indicators. Life expectancy recorded significant growth mainly at the beginning of the period. This fluctuation we can't see in the normal length of life because in its estimate we did not take into account the number of deaths at age 0.

It is possible to say that the lengthening of the human life can have an impact in many areas. It would have a great impact on the structure of an employment. Because there will be a reduction in the number of people in working age, it is important for people (in general) to work longer. On the other hand, it is good to know that the solution of unemployment for people in retirement age (especially the long term one) is very difficult (e.g. Löster and Langhamrová, 2011).

In future research of mentioned indicators would be interesting to apply cluster analysis. The research could be focused on clustering regions in the Czech Republic according to the values

of life expectancy and normal length of life. For the actual analysis would be used one of the existing methods of cluster analysis (Řezanková et al., 2011).

Conclusion

From obtained outputs it is obvious that during the period under review occurred how the extension of life expectancy for Czech males as for females. The life expectancy at birth recorded a larger increase than the normal length of life. An important observation is that the normal length of life doesn't show clear trend. Its values fluctuate much more than the values of the life expectancy. Similarly, we can say that during the period there were some significant approximation values of life expectancy at birth and normal life expectancy (an exception is the beginning of the period). The difference between the life expectancy and normal length of life is approximately 7 years.

If we compare the values of lifespans for males and females we reach the conclusion, that females live longer (it is consistent with the original assumption). Lower values of lifespans for males are caused mainly by the so-called "male excess mortality". The difference is approximately 6 years for life expectancy, the normal length of life is the beginning of the period difference 3.5 years and at the end of the reporting period is 6 years.
If we compare the two methods used for estimating normal length of life, we find out that the results are not very different. We can say that the estimate of normal length of life by age at which the most persons die, it is relatively accurate estimate. So we can say that the calculation of normal life expectancy by estimating the age at which the most persons die, provide relatively accurate values of normal length of human life.

Acknowledgment

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