SCENARIO ANALYSIS OF CASH-FLOWS IN THE CZECH LIFE INSURANCE MARKET

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Abstract

The aim of this paper is to introduce a complex econometric model of cash-flows of life insurers operating in the Czech market, which is also suitable in a prognosis framework. Namely, various economic-actuarial links among given insurance variables, e.g. the acquisition and administrative expenses, the technical reserves or the insurance premiums, are captured by means of a dynamic econometric system of linear simultaneous equations. The considered model estimated via the three-stage least squares method (3SLS) is fully statistically verified. Therefore, it offers several useful interpretations and applications. In particular, one can deal with analysis of anticipations of possible future developments from the actuarial point of view. Generally, results for particular generated scenarios (deterministic or random) can be taken into account by life insurance companies, e.g. in their internal calculations or for financial planning purposes, or by an insurance regulator, e.g. for stress testing presented by Solvency II. Moreover, randomly generated scenarios are capable of delivering different empirical probabilities corresponding to any prediction horizon.

Key words: econometric system of simultaneous equations, life insurance, scenario analysis, Solvency II.

JEL Code: C30, C32, C39.

Introduction

In the present paper, a complex econometric model of cash-flows for the Czech life insurance market is introduced. In particular, a dynamic econometric system of linear simultaneous equations is applied to capture different links among observed insurance variables. Generally, a statistically verified econometric model concerning this phenomenon is useful from both the economic and the actuarial point of view. The issues relative to the discussed topic, i.e. to econometric modelling of cash-flows in life insurance, are handled in various academically or practically oriented works (Feilmeier & Junker, 1982; Schüler & Hüls, 1988; Cipra, 1998; Baranoff, 2007; Hendrych, 2011).

1 Econometric model of the Czech life insurance market

For simplicity, let us consider only relationships arising from the annual technical report of life insurance (i.e. the compulsory part of the annual balance sheet). Links among these economic data can be investigated through econometric modelling based on actuarial theory (Cipra, 2010). On the other hand, one could possibly extend the introduced model into a more general form including other insurance or economic variables.

In particular, assume the following life insurance variables: CS_t - the indemnity (including claims and surrenders of policies) in year t (in thousands of CZK), DV_t - the contribution to the technical reserves in year t (in thousands of CZK), EAA_t - the acquisition and administrative expenses in year t (in thousands of CZK), EX_t - the number of policies terminated in year t (in pieces), N_t - the number of the new life policies (the newcomers) in year t (in pieces), P_t - the insurance premium written in year t (in thousands of CZK), $PORT_t$ - the insurance portfolio (the number of active insurance policies) in year t (in pieces) defined by the identity $PORT_t = PORT_{t-1} + N_t - EX_t$, $PROF_t$ - the investment income in life insurance in year t (in thousands of CZK), $PROFS_t$ - the profit share in life insurance in year t (in thousands of CZK), V_t - the technical reserves in year t (in thousands of CZK) and R_t - the life insurance result in year t (in thousands of CZK), t=1,…,15 (t=1 refers to the year 1997, T=15 to the year 2011). The yearly based data have been subtracted from the summary annual reports of the Czech Insurance Association (ČAP), i.e. the association of commercial insurance companies with 98% share in the total premium written in the Czech Republic.[1]

Proceed to the formulation of the dynamic econometric system of linear simultaneous equations. It describes relationships among the particular life insurance market variables. Such a system allows capturing more than one dependent variable simultaneously, and thus one can model an analyzed economic phenomenon in a more sophisticated way.

Suppose the following system (for t=1,…,T):

$$
\begin{aligned}
CS_t &= \beta_{11} + \beta_{21}EX_t + \beta_{31}N_t + \beta_{41}PORT_t + \beta_{61}PROF_{t-1} + \phi_{71}V_{t-1} + \varepsilon_t^{CS}, \\
DV_t &= \beta_{12} + \beta_{52}PROF_t + \gamma_{12}CS_t + \gamma_{32}EAA_t + \gamma_{42}P_t + \gamma_{52}PROFS_t + \varepsilon_t^{DV}, \\
EAA_t &= \beta_{13} + \beta_{33}N_t + \beta_{43}PORT_t + \gamma_{13}CS_t + \gamma_{43}P_t + \phi_{73}V_{t-1} + \varepsilon_t^{EAA}, \\
P_t &= \beta_{14} + \beta_{24}EX_t + \beta_{34}N_t + \beta_{54}PROF_t + \phi_{74}V_{t-1} + \varepsilon_t^{P}, \\
PROFS_t &= \beta_{15} + \beta_{25}EX_t + \beta_{35}N_t + \beta_{45}PORT_t + \beta_{55}PROF_t + \beta_{65}PROF_{t-1} + \varepsilon_t^{PROFS}, \\
R_t &= \beta_{16} + \gamma_{96}TR_t - \gamma_{86}TC_t + \varepsilon_t^{R}, \\
V_t &= DV_t + \beta_{17} + \phi_{77}V_{t-1} + \varepsilon_t^{V}, \\
TC_t &= CS_t + DV_t + EAA_t + PROFS_t, \\
TR_t &= P_t + PROF_t.
\end{aligned}
\tag{1}
$$

[1] http://www.cap.cz/default_en.aspx, accessed 2 May 2013.

The considered dynamic system (1) originally includes seven stochastic equations (the equations with stochastic residuals). The stochastic equation for the life insurance result R_t in year t is expressed as the difference between a part of the total revenues TR_t in year t and a part of the total costs TC_t in year t in the presence of the intercept and the residual term. Parts (apparently minority) of the total revenues and the total costs can be transferred out of the technical account of life insurers. The last stochastic equation describing the evolution of the technical reserves V_t in year t respects the fact that the lagged technical reserves can be also adjusted, e.g. by the technical interest rate.

In the suggested model, the intercepts and variables EX_t, N_t, $PORT_t$, $PROF_t$ (and thus $PROF_{t-1}$) are assumed to be strictly exogenous (uncorrelated with all residuals components), i.e. these variables enter the system from outside. This choice seems to be reasonable due to the apparent external character of such variables. The lagged variable V_{t-1} is supposed to be predetermined (uncorrelated with current and future values of residual components), i.e. it is fully determined by the given system in time t-1. Overall, the model (1) includes 9 endogenous and 7 exogenous (6 strictly and 1 predetermined) variables.

To clarify, the model (1) follows a general structural form of a dynamic system of linear simultaneous equations (Greene, 2003). In particular, several assumptions are commonly considered: ***(A1)*** $\mathrm{E}(\boldsymbol{\varepsilon}_t)=\mathbf{0}$, $\mathrm{cov}(\boldsymbol{\varepsilon}_t)=\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^{\mathrm{T}})=\boldsymbol{\Sigma}$ for a given positive definite matrix and $\mathrm{cov}(\boldsymbol{\varepsilon}_s,\boldsymbol{\varepsilon}_t)=\mathrm{E}(\boldsymbol{\varepsilon}_s\boldsymbol{\varepsilon}_t^{\mathrm{T}})=\mathbf{0}$, $s\neq t$, ***(A2)*** $\mathrm{E}(\boldsymbol{x}_t\boldsymbol{\varepsilon}_t^{\mathrm{T}})=\mathrm{E}(\boldsymbol{x}_t)\mathrm{E}(\boldsymbol{\varepsilon}_t^{\mathrm{T}})=\mathbf{0}$, ***(A3)*** $\mathrm{E}(\boldsymbol{x}_t\boldsymbol{x}_t^{\mathrm{T}})=\boldsymbol{Q}$, $\boldsymbol{Q}$ is a finite matrix, ***(A4)*** the matrix $\boldsymbol{\Gamma}$ containing real parameters concerning (exclusively) endogenous variables is an invertible matrix with elements -1 on its diagonal. Here, $\boldsymbol{\varepsilon}_t$ stands for the (stochastic) vector of structural residuals and $\boldsymbol{x}_t$ denotes the vector of strictly exogenous variables with the first element normalized to 1 (everything for all given times t).

Note that the indices of the parameters β, γ and ϕ in (1) refer to the corresponding components of the parameter matrices $\mathbf{B}$ (for exogenous variables), $\boldsymbol{\Gamma}$ (for endogenous variables) and $\boldsymbol{\Phi}_1$ (for predetermined variables with the time lag 1), respectively (Greene, 2003; Lütkepohl, 2005). Moreover, each equation in the system (1) satisfies the necessary condition of identification (Dhrymes, 1994), i.e. the number of the variables on the right-hand side of each equation is less or equal to the number of exogenous variables in the system.

To estimate the unknown parameters of the previous stochastic simultaneous econometric equations, the three-stage least squares method (3SLS) is considered. This full information estimation technique as a special case of the more universal full information generalized method of moments exploits all information available in the system (Hall, 2005). Hence, it offers a set of suitable properties (under general assumptions). Namely, such estimates are consistent, asymptotically normally distributed and asymptotically efficient (Dhrymes, 1994). In Table 1, the 3SLS estimates of the parameters in the model (1) are shown (together with their estimated standard errors).

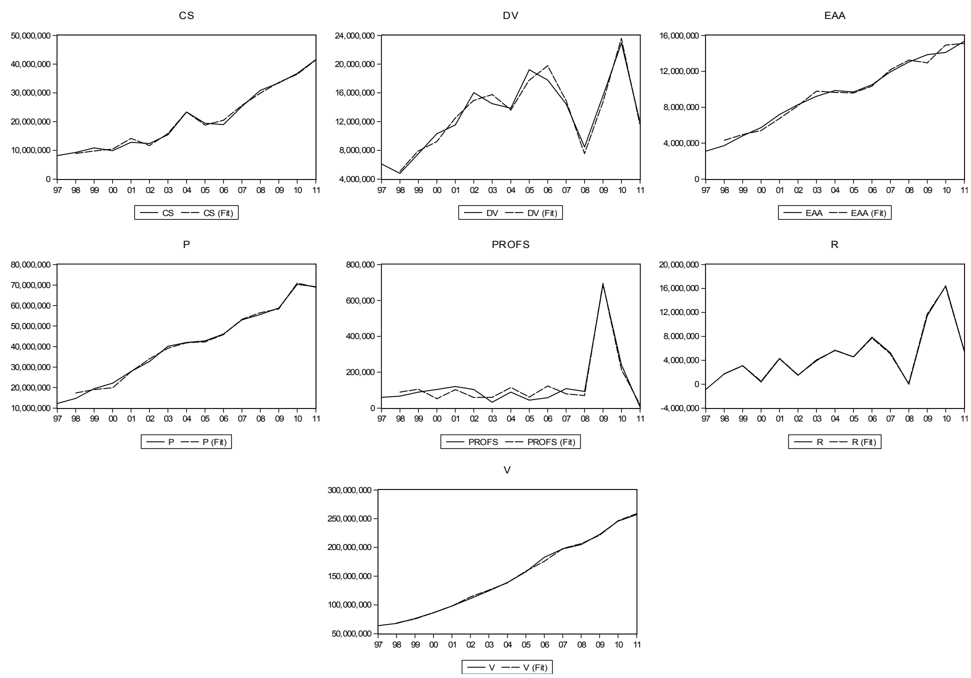
Tab. 1: The 3SLS estimates of the model (1).

Par.	*Estimate*	*Std. Error*	*Par.*	*Estimate*	*Std. Error*	*Par.*	*Estimate*	*Std. Error*
β_{11}	3.98E+07	3.14E+06	β_{13}	-9.59E+05	9.81E+05	β_{15}	-2.75E+05	7.40E+04
β_{21}	-7.87607	0.74371	β_{33}	1.47455	0.53419	β_{25}	0.20359	0.01709
β_{31}	4.20584	0.79968	β_{43}	0.19327	0.15804	β_{35}	-0.21267	0.03536
β_{41}	-7.15314	0.51759	γ_{13}	0.01527	0.04408	β_{45}	0.05979	0.00945
β_{61}	-0.79075	0.06806	γ_{43}	0.13803	0.04018	β_{55}	0.01016	0.00157
ϕ_{71}	0.28853	0.00984	ϕ_{73}	0.01243	0.01356	β_{65}	-0.00894	0.00141
β_{12}	1.42E+06	3.38E+06	β_{14}	-1.16E+07	1.42E+06	β_{16}	7.34E+04	1.11E+05
β_{52}	0.57694	0.17263	β_{24}	-2.87155	0.54592	γ_{96}	0.97818	0.01200
γ_{12}	-0.98678	0.06805	β_{34}	15.01998	1.36312	γ_{86}	0.98723	0.01468
γ_{32}	0.42954	2.25319	β_{54}	0.55060	0.05953	β_{17}	-6.84E+05	1.55E+06
γ_{42}	0.59608	0.45387	ϕ_{74}	0.26516	0.00535	ϕ_{77}	1.00701	0.01008
γ_{52}	-6.84361	5.38862						

Source: Author (by EViews 7.0).

One can see that the estimated model fits the data suitably (see Figure 1). Moreover, it demonstrates its adequacy in the sense of statistical validity (see the assumptions above). For instance, the sample correlation matrix of the estimated residuals delivers several relatively high correlations; see the assumption ***(A1)***. The Portmanteau test does not detect autocorrelations of the residuals (Lütkepohl, 2005). Furthermore, the Joint Jarque-Bera statistics cannot reject multivariate normality of the residuals. Finally, the Hausman test and the Sargan tests (Greene, 2003) cannot reject the proper model specification, i.e. the proper selection of exogenous and endogenous variables; compare with the assumption ***(A2)***. Point out that the parameters β_{12}, γ_{32}, γ_{42}, γ_{52}, β_{13}, β_{43}, γ_{13}, ϕ_{73}, β_{16} and β_{17} are not significant (using 5% significance level asymptotically).

Fig. 1: The observed endogenous variables with their fitted counterparts.



Source: Author.

2 Scenario analysis

From the practical point of view, anticipations of possible future developments of the considered (endogenous) insurance variables can be really useful, e.g. in the framework of manager accounting and financial planning. In the considered context, one can apply two different approaches to scenario analysis: (i) analysis of deterministic scenarios simply based on expert (optimistic or pessimistic) values of strictly exogenous variables and (ii) analysis of random scenarios generated via corresponding econometric models.

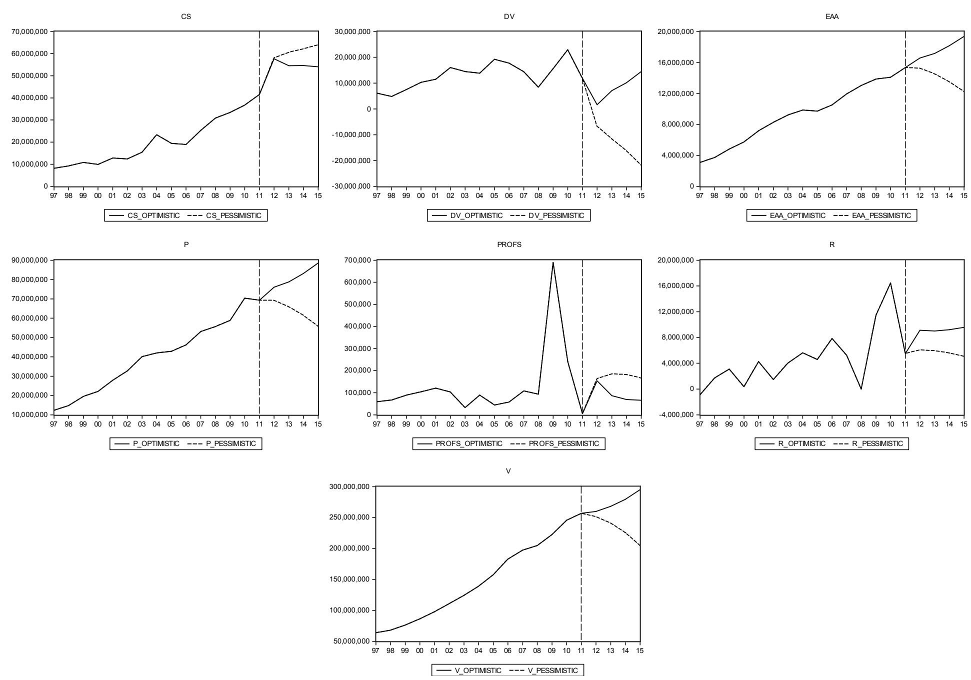
All computational efforts are handled by the econometric software EViews (version 7.0) and by authors' programme procedures.

2.1 Deterministic scenarios

To illustrate the main idea of deterministic scenario analysis, two different scenarios are supposed. The first one is rather pessimistic: the number of terminated policies EX_t increases by 10% each year, the number of new insurance policies N_t decreases by 10% each year and the investment income $PROF_t$ retains its first sample quartile value, t=16,…,19, i.e. from 2012 to 2015. The second one follows optimistic expectations in the Czech life insurance market: the number of terminated policies EX_t decreases by 10% each year, the number of new insurance policies N_t increases by 10% each year and the investment income $PROF_t$ retains its third sample quartile value.

The results of this analysis are summarized in Figure 2. One can see that the differences between two proposed scenarios are truly significant. On the other hand, they respect rational anticipations, e.g. for the pessimistic scenario the technical reserves decrease, the insurance premium volume decreases, the indemnity increases or the acquisition and administrative expenses decrease mainly due to the smaller insurance portfolio and low investment returns.

Fig. 2: The results of optimistic/pessimistic scenario prognoses.



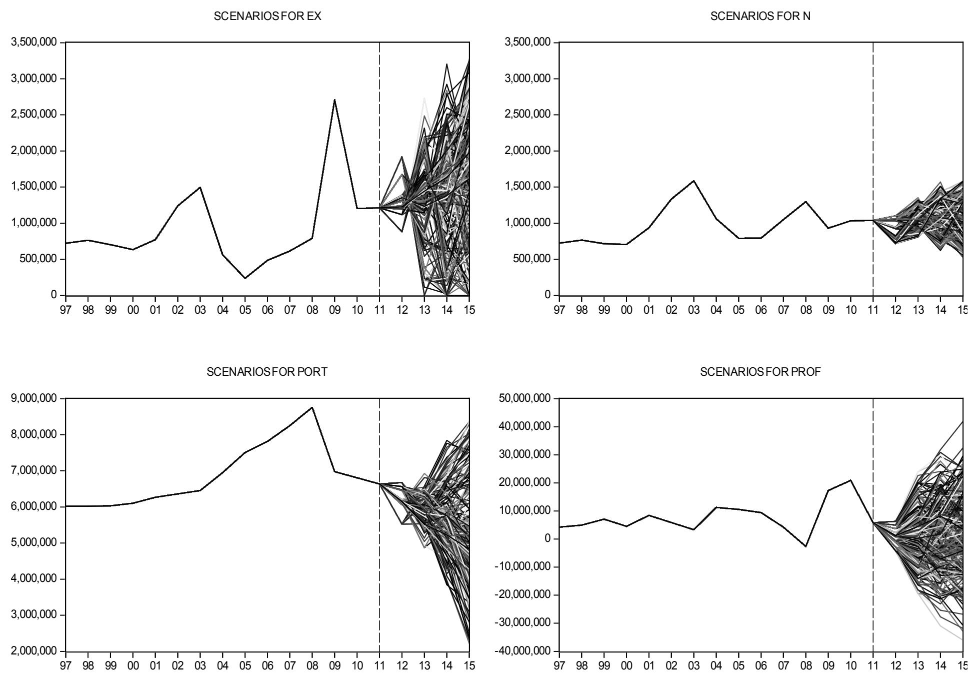
Source: Author.

2.2 Randomly generated scenarios

Analysis of randomly generated scenarios might be more sophisticated in the sense that it possibly uses more realistic scenario predictors of strictly exogenous variables, which respect evolution of the data generating process, e.g. by statistical time series models as ARIMA and GARCH (or their more complex multivariate counterparts). In particular, given original exogenous data can be fitted by such a model. Moreover, future anticipated values of strictly exogenous variables are obtained via stochastic predicting when predictions follow the estimated model with impact of randomly generated innovations.

For example, in the case of the econometric system (1), the exogenous variables EX_t, N_t and $PROF_t$ are captured by univariate linear ARMA models with normal residuals (the high values of the initially nonnegative integer variables EX_t and N_t allow using normal approximations with adjustments to integers). All models have been fully statistically verified and offer suitable fits. Finally, one obtains 100000 different random scenarios for all strictly exogenous variables (see Figure 3).

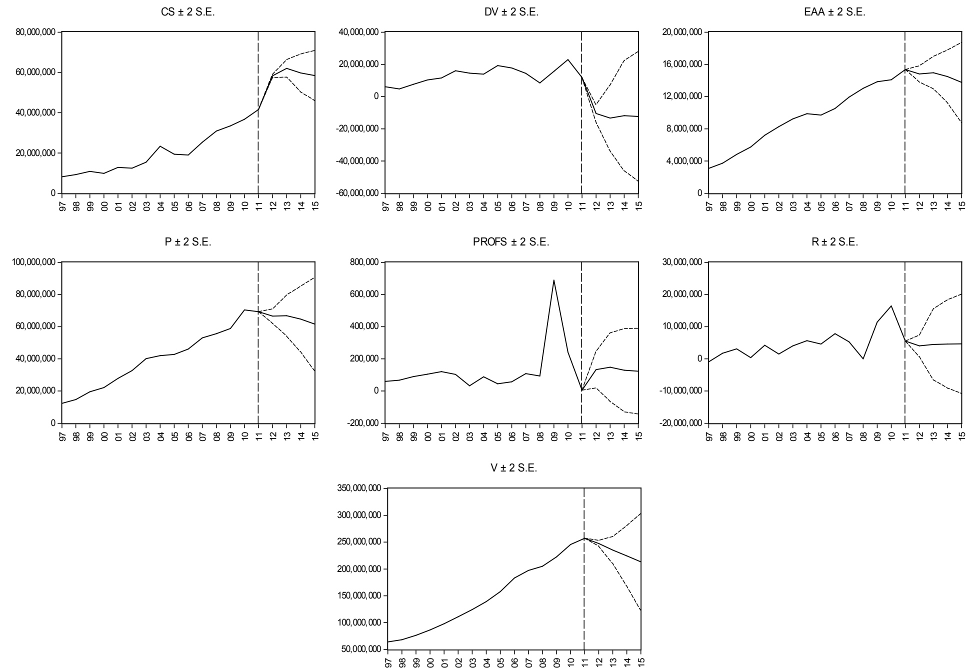
Fig. 3: The generated scenarios (100000) of exogenous variables.



Source: Author.

The obtained scenario projections have been used to predict all endogenous variables of the estimated econometric system (1). Figure 4 graphically presents the achieved results. From the economic point of view, the results offer useful interpretations. For instance, in the case of scenarios for the life insurance result R_t, one can conclude that the sum of predicted returns over the prediction horizon is negative in 18.75% of scenarios, minimally one negative return over the four year prediction horizon is in 38.84% of scenarios, only negative returns are in 0% of generated scenarios, new maximum is achieved in 8.77% and new minimum in 34.46% of scenarios.

Fig. 4: The mean and ± 2 std. error bounds of 100000 generated scenario predictions.



Source: Author.

Conclusion

The contribution dealt with the complex econometric model of cash-flows for the Czech life insurance market. In particular, the dynamic econometric system of linear simultaneous equations was considered to capture economic-actuarial relationships within the annually published summary technical reports of the Czech life insurers. Considered scenario analysis offered effective predictions of possible future developments which are generally useful from both the economic and the actuarial point of view. Two approaches were considered: the deterministic and the stochastic one. The first one could capture different expectations, e.g. optimistic and pessimistic scenarios. The obtained results respected natural anticipations, e.g. in the sense of the increasing/decreasing technical reserves, the corresponding changes in the insurance premium, etc. The second one included a broader simulation framework, and thus enabled to test future eventualities in details, e.g. to deliver various empirical probabilities of specific phenomena such as the probabilities of newly achieved maximum and minimum over the given prediction horizon (e.g. for the life insurance result R_t, these calculated probabilities were 8.77% and 34.46% over the given four year horizon 2012 - 2015, respectively).

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