THE MOTION EQUATIONS IN THE THEORY OF PRICING

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Abstract

The article briefly discusses the need to find causal mechanisms in the areas of electromagnetic, gravitational, and nuclear strong force interactions to explain phenomena observed in socio-physical systems. Linear and non-linear kinematic equations of motion are derived for economically variable socio-physical systems. The methodology of non-relativistic theoretical mechanics is used to derive these deterministic kinematic equations.

A deterministic linear equations of motion of the second order are derived to describe the degressive and progressive development of the instantaneous relative depreciation of a commodity over time in a convergent sequence of models of market structures with perfect competition. The same approach is used to derive a linear motion equations of the second order to describe the degressive and progressive development of instantaneous relative price of a commodity over time in a convergent sequence of models of market structures with perfect competition. To model the progressive/degressive development of the immediate relative depreciation of a commodity over time, a deterministic non-linear motion equation of the second order is derived.

Key words: Depreciation, differential equation, econophysics, equation of motion

JEL Code: A190 , C650

Introduction

The systematic use of methods of theoretical and mathematical physics in basic and applied research in the development of the states of economic systems can be relatively reliably traced to the end of the first half of the 19th century and especially for the Cambridge and Lausanne Schools of Economics (Zeithamer, 2012a).

Economic phenomena and processes at that time were described and analyzed using analogies between the evolution of physical systems and the evolution of economic systems. Biographical research has shown that one of the reasons for the successful application of theoretical physics in economics is that many economists had initially studied physical and mathematical sciences, or fields related to the physical or mathematical sciences (Zeithamer, 2012a).

The gradual spread of methods taken from theoretical and mathematical physics to economics during the 19th and especially the 20th century eventually led to the beginning of basic theoretical research, which in the 21st century consists of the systematically targeted application of theoretical, mathematical and statistical physics to economics (Mantegna & Stanley, 2000; McCauley, 2004; Chakrabarti, Chakraborti & Chaterjee, 2006; Drozen 2008; Voit, 2010; Zeithamer, 2012b). A part of this basic theoretical research in economics has become known as econophysics. Great confidence in the power of mathematical statistics, which is justified in measuring techniques (James, 2010; Štroner & Pospisil, 2011), is somewhat weakened by works which, based on detailed knowledge of statistical methods and their use in physics itself, come to the conclusion that it is not possible using statistics alone to determine the causal mechanisms that explain observed phenomena in economics (Chakrabarti, Chakraborti & Chaterjee, 2006; Roehner, 2002, 2007).

We also encounter this state of knowledge in the interdisciplinary field of Sun-Earth relations on the physical and biophysical level. Other related scientific fields include the physics of the heliomagnetosphere and magnetosphere of planets in the Solar System, meteorology of the Earth's atmosphere and the atmosphere of planets in the Solar System.

Finding causal mechanisms which explain observed socio-physical phenomena on a level of gravitational, electromagnetic or nuclear interactions is a very difficult, long and costly task. The same applies to the education of experts in commodity price theory, thoroughly based on the knowledge of basic physical force interactions. The theoretical constructions presented in this work are intended to facilitate solving both tasks mentioned in future modern commodity price theory. Specifically, there are linear and non-linear elementary kinematic equations, which do not explain the phenomena observed in the socio-physical system with interactions of force, but merely describe the developing state of the socio-physical system. These kinematic equations lead to quantifiable mechanisms which explain observed development in the state of the socio-physical system using analytical dynamics, i.e. force interactions. The analytical dynamics of socio-physical systems is not the subject of this work, however, it is one of the subjects of basic and applied economic and physical research being conducted by the author.

1 Linear motion equation of commodity state without inflexion

The late nineteenth century is a period in which there was a synthesis of the partial knowledge of economic laws formulated by the previous generation of economists, and increased attempts to describe these laws using the language of physics and mathematics, comprising both the mathematical and physical meaning of the differential of variable quantities; the foundations of "econophysics in the broader sense" were laid and expanded (Zeithamer, 2012a).

The first half of the twentieth century witnessed a deepening integration of economics, mathematics, and physics. At the Czech School of Economics during this time, no reliable sources have yet been found indicating such an interdisciplinary approach or related original work. At the end of the twentieth century however, we do find an economist at the Czech School of Economics whose work represents econophysics in the broader sense. This economist is František Drozen (born 30. 5. 1949), who was inspired by the work of German engineer August Wöhler (22. 6. 1819 – 21.3.1914). František Drozen constructed an analogy between the process of fatigue crack growth in axles and the process of price reduction for goods. This approach to modeling the process of falling prices for goods can be found in its final form in several of Drozen's works (Drozen, 2003, 2008).

In this work it is assumed, as in Drozen's works (Drozen, 2003, 2008), that the market value of a commodity is quantified only by the market price $\boldsymbol{n}$ of the commodity.

We now make the generalizing assumption that the instantaneous acceleration of reduction of the market value is directly proportional to the instantaneous rate of reduction of the market value (Zeithamer 2010). Then the deterministic differential equation of price which expresses this model is

$$\frac{d^2\boldsymbol{n}}{dt^2}(t) = -A\frac{d\boldsymbol{n}}{dt}(t), \qquad (1)$$

where $A > 0$ is the proportionality constant, and a negative sign is used to indicate that $\boldsymbol{n}$, the market value of commodity, i.e., a price, is decreasing and the acceleration of reduction of the market value decreases over time. The initial conditions now are that over time $t = 0$ the market value is $\boldsymbol{n}(0) = \boldsymbol{n}_0$ and $\frac{d\boldsymbol{n}}{dt}(0) = \boldsymbol{r}_0 < 0$. A more detailed approach to modeling the process of falling prices with acceleration can be found in the following works (Zeithamer 2010, 2011a, 2011b, 2012b).

2 Linear motion equation of commodity relative depreciation

In a convergent sequence of market structures with perfect competition, the instantaneous commodity relative depreciation RD is defined by the magnitudes of instantaneous commodity relative depreciation according to the relationship (Drozen, 2008; Zeithamer, 2011b)

$$RD(t) = \frac{w(t) - w(t_0)}{w(t_0)}, \tag{2}$$

where $w(t_0) = w_0$ is the magnitude of instantaneous commodity depreciation at the initial time t_0 and $w(t)$ is the magnitude of instantaneous commodity depreciation at time $t(t \geq t_0)$. In addition to instantaneous commodity relative depreciation RD, the instantaneous commodity relative price RP is also defined under the condition of perfect competition by the magnitudes $RP(t)$ at time t according to the relationship (Drozen, 2008; Zeithamer, 2011b)

$$RP(t) = \frac{p(t_0) - p(t)}{p(t_0)}, \tag{3}$$

where $p(t_0) = p_0$ is the magnitude of instantaneous commodity price p at the initial time t_0 of monitoring the instantaneous commodity price on a select model market and $p(t)$ is the magnitude of instantaneous commodity price at time $t \geq t_0$.

Instantaneous commodity depreciation w is a real composite function of time, i.e. $w(t) = w(p(t))$, where $w(p)$ is the continuous decreasing real function of instantaneous commodity price p and instantaneous commodity price p is a continuous decreasing real function of time t. If we monitor the development of instantaneous commodity depreciation at time interval $\langle t_0, t_e \rangle$, then for the first derivation of functions $w(p)$ and $p(t)$ it holds that $\frac{dw}{dp}(p) < 0$ for $p \in \langle p(t_e), p(t_0) \rangle$ and $\frac{dp}{dt}(t) < 0$ for $t \in \langle t_0, t_e \rangle$. It directly follows from these relationships that for the interval $\langle t_0, t_e \rangle$, $\frac{dw}{dt}(t) = \frac{dw}{dp}(p(t)).\ \frac{dp}{dt}(t) > 0$. This means that instantaneous commodity depreciation w is a continuous increasing real function of time t, which corresponds to trends for common commodities over time. Then, instantaneous commodity relative depreciation *RD* is also a continuous real function at interval $\langle t_0, t_e \rangle$ and $\frac{dRD}{dt}(t) > 0$ for every time $t \in (t_0, t_e)$.

Let us assume that the magnitude of instantaneous commodity relative depreciation *RD* over time *t* increases with acceleration and the acceleration of instantaneous commodity relative depreciation increases in direct proportion to the instantaneous speed of change of instantaneous commodity relative depreciation at time *t*. The motion equation of instantaneous commodity relative depreciation is thus (Zeithamer, 2011b)

$$\frac{d^2RD}{dt^2}(t) = B\frac{dRD}{dt}(t), \tag{4}$$

where B is the constant of proportionality, $B > 0$. In addition, let initial conditions be met where $RD(t_0) = RD_0 > 0$, $\frac{dRD}{dt}(t_0) = v_0 > 0$, so that the solution of differential equation (3) at interval $\langle t_0, t_e \rangle$ is then

$$RD(t) = RD_0 - \frac{v_0}{B} + \frac{v_0}{B}e^{B(t-t_0)}. \tag{5}$$

From here it directly follows that instantaneous commodity relative depreciation *RD* is a purely convex function at interval $\langle t_0, t_e \rangle$. This means that the increase in instantaneous commodity relative depreciation at interval $\langle t_0, t_e \rangle$ is progressive.

Let us assume that instantaneous commodity relative depreciation *RD* increases with acceleration at time *t* again and the acceleration of instantaneous commodity relative depreciation increases in direct proportion to the speed of change of relative depreciation at time *t* while the constant of proportionality is negative. The motion equation of instantaneous commodity relative depreciation is then (Zeithamer, 2011a; Zeithamer, 2011b)

$$\frac{d^2RD}{dt^2}(t) = -B\frac{dRD}{dt}(t), \tag{6}$$

where $(-B)$ is the constant of proportionality, $B > 0$. In addition, let initial conditions be met where $RD(t_0) = RD_0 > 0$, $\frac{dRD}{dt}(t_0) = v_0 > 0$, so that the solution of the differential equation (5) at interval $\langle t_0, t_e \rangle$ is then

$$RD(t) = RD_0 + \frac{v_0}{B} - \frac{v_0}{B}e^{-B(t-t_0)}. \tag{7}$$

From here it directly follows that instantaneous commodity relative depreciation RD is a purely concave function at interval $\langle t_0, t_e \rangle$. This means that the increase in instantaneous commodity relative depreciation at interval $\langle t_0, t_e \rangle$ is degressive. The progressive increase of instantaneous commodity relative depreciation is characteristic, for example, of certain types of food goods, while degressive increase of relative depreciation may be seen in certain commodities in the automotive industry. The same approach is used to derive a motion equation for the degressive and progressive development of the instantaneous relative price of a commodity over time.

3 Non-linear motion equation of commodity state with inflexion

In this section of our work we again presume the following conditions to be met: (1) the commodity is on one of the markets of the model of market structure with perfect competition at initial time t_0; (2) at time t_0 the commodity is found in its initial state, which is uniquely determined by the magnitude of instantaneous commodity depreciation $w(t_0) = w_0$.

Let the acceleration of $\frac{d^2 RD}{dt^2}$ of the instantaneous commodity relative depreciation be the sum of two components, i.e.

$$\frac{d^2 RD}{dt^2} = \left(\frac{d^2 RD}{dt^2} \right)_1 + \left(\frac{d^2 RD}{dt^2} \right)_2, \tag{8}$$

The first component of acceleration is a consequence of physical and chemical processes, which cause the first component of the instantaneous acceleration to increase in direct proportion to the magnitudes of rate of change of the instantaneous commodity relative depreciation, i.e.

$$\left(\frac{d^2 RD}{dt^2}(t) \right)_1 = B \frac{dRD}{dt}(t), \tag{9}$$

where B is the proportionality constant, $B > 0$ and $t \in \langle t_0, t_e \rangle$. The second component of acceleration results from socio-psychological processes (in physical and chemical approximation), which cause the second component of the instantaneous acceleration to be directly proportional to the product of the magnitude of rate of change of the instantaneous commodity relative depreciation $\frac{dRD}{dt}(t)$ and the magnitude of instantaneous commodity relative depreciation $RD(t)$, while the proportionality constant is negative, thus

$$\left(\frac{d^2RD}{dt^2}(t)\right)_2 = -A\frac{dRD}{dt}(t)RD(t), \qquad (10)$$

where $(-A)$ is the proportionality constant, $A > 0$, $t \in \langle t_0, t_e \rangle$.

By substituting relations (9) and (10) into equation (8), we obtain the following motion equation for the acceleration of instantaneous commodity relative depreciation (Zeithamer, 2012b)

$$\frac{d^2RD}{dt^2}(t) = B\frac{dRD}{dt}(t) - A\frac{dRD}{dt}(t)RD(t), \qquad (11)$$

where $A > 0$, $B > 0$, $t \in \langle t_0, t_e \rangle$.

One of the subsets of the set of solutions for motion equation (11) is given by

$$RD(t) = \frac{y_2 + y_1 e^{\sqrt{D}(t+C_2)}}{1 + e^{\sqrt{D}(t+C_2)}},$$

where for constants D, y_1, y_2, C_2 it follows that $D = B^2 + 2AC_1$, $y_1 = \frac{B+\sqrt{D}}{A}$, $y_2 = \frac{B-\sqrt{D}}{A}$, $0 < |y_2| < y_1, y_2 < 0$, $-\frac{B^2}{2A} < C_1 < 0$, $C_2 = \frac{1}{\sqrt{D}} ln\left(\left|\frac{y_2}{y_1}\right|\right) - t_p$. At time $|t_p|$ the value of instantaneous commodity relative depreciation is zero. The given subset of the solutions of motion equation (11) shows the progressive – degressive increase of instantaneous commodity relative depreciation with an inflexion point at time $t = -C_2$ and a limit at $lim_{t \to +\infty} RD(t) = y_1$.

Conclusion

Assuming that the market value of the commodity at time t is fully determined exclusively by the value of the instantaneous commodity price $p(t)$, methodological procedures taken from theoretical physics were used to construct motion equations for instantaneous commodity relative depreciation *RD*. Motion equations (4) and (6) for the progressive and degressive increase of instantaneous commodity relative depreciation are linear differential equations of the second order with constant coefficients assuming market structures with perfect competition. Motion equation (11) of instantaneous commodity relative depreciation for the progressive/degressive growth of depreciation is a non-linear differential equation of the second order with constant coefficients. Motion equation (11) was also derived for instantaneous commodity relative depreciation on a sequence of markets with perfect competition. In the solutions set for motion equation (11), there is the subset of solutions which model progressive/degressive growth of the magnitudes of instantaneous commodity relative depreciation with a single inflexion point.

Acknowledgment

The author is grateful to Mrs. Pavla Jará and the National Technical Library for their great effort and excellent work, which was indispensable in the completion of a large portion of this work. This paper is dedicated to Mrs. Věra Ruml Zeithamer and Mr. Josef Ruml Zeithamer and Mrs. Anna Ruml and Mr. František Ruml.

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