# A MULTI-CRITERIA EVALUATION OF ALTERNATIVES UNDER RISK

**Lenka Švecová – Jiří Fotr – Lucie Vrbová**

**Abstract**

Today, strategic decisions are rarely made under conditions of certainty, but rather under circumstances of risk and uncertainty. The decision making theory offers a well-developed methodological apparatus for decision making in inevitable and predictable situations. However, within a risk environment, the theory offers utilities such as the multi-attribute (multi-criteria) utility function. Regrettably this approach is too complicated for routine use by managers.

This paper presents different approaches to the evaluation of options under risk. Furthermore paper summarizes their advantages and limitations. In the second part paper proposes a new approach based on connecting methods and decision making tools in risk settings (in particular, probabilistic trees, risk decision matrices and rules) using conventional multi-criteria evaluation approaches deployed during conditions of certainty. This approach is understood as a normative framework for the multi-criteria evaluation of alternatives under risk. The application of this method to management improves the quality of decision making under conditions of risk, without a significant increase in demands on the decision maker.

**Key words:** multi-criteria evaluation, risk and uncertainty, utility function under risk, normative framework

**JEL Code:** M10, M21, D01

## Introduction

Multi-attribute evaluation under risk can be done in several ways (Keeney & Raiffa, 1993, p. 219, Goodwin & Wright, 2004, p. 95; Kepner & Tregoe, 2006, p. 94):

- assessing the best alternative in terms of its risk,
- inclusion of risk factors among the evaluation criteria,
- substitution of uncertain consequences of alternatives of their expected values due to specific criteria evaluation,

- using of utility function and construction of multi-attribute utility function under risk.

These approaches can be extended by a combination of some of the tools of decision making under risk (e.g., probability trees, decision matrix) with tools of multi-attribute decision making (MADM). Each of these approaches has its advantages and disadvantages, which can be assessed in terms of the following aspects: simplicity and clarity, difficulty for the evaluators, the degree of simplification and suitability for different types of problems.

## 1. Assessing the best alternative in term of its risk

Decision analysis within the Kepner-Tregoe ® methodology offers a way of integrating risk into the multi-attribute evaluation (Kepner & Tregoe, 2005, p. 94). This is a **detailed analysis of the first two most suitable alternatives** based on a multi-attribute evaluation in terms of their risk. In the cases that for the best alternative there are significant doubts about changing input assumptions and conditions, then after careful consideration of the pros and cons of both alternatives it is better to leave the best alternative and implement the less risky alternative, despite it being worse at this point.

This approach is relatively simple and less demanding on the evaluators, because it does not force him to directly quantify the effects of risk alternatives. We recommend that this approach is used in each of the final phases of the evaluation (although the assessment of risk alternatives is done previously), because it is a kind of the first step in preparation for an early warning system (Fuld, 2003, p. 20) in the implementation of the best alternative.

## 2. Inclusion of risk factors among the evaluation criteria

In this procedure, the individual risk factors are included as evaluation criteria. These risk factors are in the form of random variables whose future developments affect the consequences of strategic choices, both negatively and positively. The actual procedure of the multi-attribute evaluation of risk alternatives may be analogous in the case of quantitative criteria such as MADM. Partial evaluation of alternatives due to qualitative (as well as quantitative) criteria is determined by a direct expert evaluation. Total rating of alternatives is set as a weighted sum of the partial evaluations for the specific evaluation criteria.

The main advantage of this procedure is its simplicity and mainly a broad range of evaluation criteria, regardless of their nature. Because this process of MADM under risk basically consists of MADM such as certainty, resulting in limitations of their application:

- Dependency between the identified risk factors and consequences of alternatives due to certain criteria, which may significantly affect the occurrence of certain values of risk factors, is not respected. This deficiency can be weakened by respecting these dependencies in an expert determination of partial evaluation.
- Assessment using different procedures can lead to different results. Each method comes with certain assumptions as a base for the evaluation. Rank of alternatives is dependent on those assumptions.

Rather specific approach respecting negative and positive risks in the form of evaluation criteria method offers analytical hierarchical process (AHP) known also by its creator as Saaty's method (Saaty & Vargas, 2010, p. 13). This method divides hierarchically organized set of criteria into four groups, two of which relate to the impact of risk alternatives considered certain, and two groups involve risks and uncertainties. Evaluation of risk alternatives based on the criteria of each group then allows setting the total rating of alternatives in the form:

- share of the total rating of the positive aspects and negative aspects,
- difference between the positive and negative aspects, weighted by importance of individual criteria groups.

## 3. Substitution of uncertain consequences of alternatives of their expected values

This approach tries to respect the effects of risk alternatives with respect to certain criteria in the form of expected values of these effects. The basis is to replace the uncertain effects of alternatives due to the quantitative criteria of their expected (mean) values (Fotr & Švecová, 2006, p. 87). This step eliminates the lack of the previous procedure, as it is described above, when dependencies between identified risk factors and effects of risk alternatives are not respected. Further on, the process continues similar to MADM.

In case of quantitative criteria, expected values of the effects of alternatives may provide:

- Expert way, i.e. based on the expert's opinion. However, this puts high requirements on the experts and can lead to a significant simplification, when uncertain values of the effect of alternatives are replaced with mean values (which are usually the most likely), but their less probable values are not respected.
- In case of large number of risk factors, one can use scenario approaches (Cornelius et al., 2005, p. 92; van der Heijden, 2005, p. 11; Fotr & Švecová, 2006, p. 87), or Monte Carlo

simulation (Mun, 2006, p. 74; Fotr & Švecová & Špaček; 2009, p. 633). When using scenarios it is necessary to set conditional probabilities of all these factors(see tab. 1). During simulation, it is necessary to specify continuous probability distribution of the alternatives and their statistical characteristics (including mean values).

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**Tab. 1: Possible effects of alternatives during scenario application with multiple criteria**

| **Alternatives** | **Scenarios** | **Probability** | **Criterion 1** | | **Criterion 2** | | | **Criterion *m*** | |
|---|---|---|---|---|---|---|---|---|---|
| **Alternative 1** | scenario 1 | $p_1$ | $x_{11}^1$ | $E(x_1^1)$ | $x_{21}^1$ | $E(x_2^1)$ | ... | $x_{m1}^1$ | $E(x_m^1)$ |
| | scenario 2 | $p_2$ | $x_{12}^1$ | | $x_{22}^1$ | | | $x_{m2}^1$ | |
| | … | | | | | | | | |
| | scenario *s* | $p_s$ | $x_{1s}^1$ | | $x_{2s}^1$ | | | $x_{ms}^1$ | |
| **Alternative 2** | scenario 1 | $p_1$ | $x_{11}^2$ | $E(x_1^2)$ | $x_{21}^2$ | $E(x_2^2)$ | | $x_{m1}^2$ | $E(x_m^2)$ |
| | scenario 2 | $p_2$ | $x_{12}^2$ | | $x_{22}^2$ | | | $x_{m2}^2$ | |
| | … | | | | | | | | |
| | scenario *s* | $p_s$ | $x_{1s}^2$ | | $x_{2s}^2$ | | | $x_{ms}^2$ | |
| **…** | | | | | | | | | |
| **Alternative *n*** | scenario 1 | $p_1$ | $x_{11}^n$ | $E(x_1^n)$ | $x_{21}^n$ | $E(x_2^n)$ | | $x_{m1}^n$ | $E(x_m^n)$ |
| | scenario 2 | $p_2$ | $x_{12}^n$ | | $x_{22}^n$ | | | $x_{m2}^n$ | |
| | … | | | | | | | | |
| | scenario *s* | $p_s$ | $x_{1s}^n$ | | $x_{2s}^n$ | | | $x_{ms}^n$ | |

Source: authors

where $x_{ik}^j$ = effect of the alternative *j* of the criterion of evaluation *i* in case of scenario *k*,
$E(x_i^j)$ = expected value of effect of the alternative *j* of the criterion *i*
$p_k$ = probability of the scenario *k*
*s* = number of scenarios
*m* = number of criteria
*n* = number of alternatives

The expected value of effects of alternatives according to each criterion is defined by equation:

$$E(x_i^j) = \sum_{k=1}^{s} p_k \cdot x_{ik}^j, \text{ where } i = 1, 2 \ldots n \text{ and } j = 1, 2 \ldots m. \qquad (1)$$

The above approach leads to setting **the deterministic equivalent** of the task during multi-attribute decision making under risk, when we can use one of the methods of MADM for setting the total rating of the alternatives.

A disadvantage of this approach is that there may be a situation where the alternatives with the same expected effects are evaluated to be equal although these alternatives can have a significantly different degree of risk. Another limitation is also that this approach does not respect subject's attitude to risk. A solution is to not work with mean values, i.e. apply **multi-attribute utility function under risk** (hereinafter MAUF).

A disadvantage of application scenarios is increased labor intensity resulting from the necessity to determine the conditional probability distribution of individual risk factors. Another simplification of this approach is an expectation of the same set of scenarios for all criteria and alternatives. The situation where a set of scenarios for certain criteria, respectively alternatives, are different, is not considered. A possible way to eliminate this disadvantage is presented in chapter 5, and is actually an extension of this approach.

# 4. Application of the utility theory in MADM and MAUF

## 4.1 Utility theory as a basis for multi-attribute utility function under risk

The utility can be understood as a measure of satisfaction resulting from the consumption of goods and services. In terms of its measurability two main concepts are emerging, a cardinal and ordinal concept. Supporters of the cardinal utility theory assume direct measurability of utility, the ordinal theory supporters believe that man is able to rank alternatives by utility, but he is not able to directly determine its size. Decision theory often uses cardinal utility theory and assumes a direct measurability.

The utility theory (both cardinal and ordinal) has many critics, such criticism covers for instance: possibility of cardinal expression utility (Goodwin & Wright, 2004, p. 102), inaccuracies numerical expression of utility, applicability of the utility function is small (Slovic & Tversky, 1974, p. 368), irrational behavior of subject in determining utility expressed like the Allais paradox, assumption of the utility maximization. However, despite these criticisms, with the knowledge these limitations the utility function can be used in multi-attribute decision making under risk.

## 4.2 Multi-attribute utility function (MAUF)

MAUF results from a system of axioms of cardinal utility theory related to the behavior of subjects in risk situations.

**A construction of MAUF** is based on the quantification of the preferential decision maker's system and can be divided into the following steps (see Fotr, 1986, p. 283):

- verification of assumptions of the preferential and utility independence,
- construction of a partial utility function for each criterion,
- setting weights of the criteria, a use of a compensation method is recommended,

- construction of a multi-attribute utility function based on partial utility functions and criteria weights.

### 4.3 MADM based on MAUF

This evaluation is based on determining mean values of total utilities of each alternative and their ranking according to these values. If MAUF is additive, the mean value of the total benefits for the particular alternative is given by (2):

$$E[U(X)] = \sum_{i=1}^{m} v_i \cdot E[u_i(x_i)], \qquad (2)$$

where $E[U(X)]$ = mean value of the total utility of the alternative,

$E[u_i(x_i)]$ = mean value of the partial utility of the alternative due to the criterion $i$

Mean values of the utility for the discrete criteria this formula:

$$E[u_i(x_i)] = \sum_{k=1}^{s} p_k \cdot u_i(x_i). \qquad (3)$$

After calculating the mean values of total utilities for the various alternatives it is recommended to make sensitivity analysis. High sensitivity indicates inadequate construction of the MAUF.

The main advantage of the MAUF is its scientific verifiability; although for practical application it is very difficult and in practice is not used.

## 5. Linking instruments for decision making under risks with MCDM

As is clear from the preceding text, there are more opportunities to join the multi-attribute evaluation of alternatives under risk. Demanding application of the MAUF pushes this approach into the background. A possible solution is to link the methods and tools of decision making under risk (mainly probability trees, decision matrixes and decision making rules under the risk) with classical approaches of the MADM. The following text is a sort of a normative framework. It offers a solution to how to proceed in the MADM under risk. If we assume that the variants are already prepared and a set of criteria according to which the alternatives evaluated is defined, this normative framework is characterized by a sequence of steps:

- Identification of key risk factors and determining their significance
- Determination of the probability distribution of risk factors
- Scenarios development

- Construction of decision matrices
- Multi-attribute evaluation of alternatives

## 5.1 Identification of key risk factors and determining their significance

The first step is for each alternative **identify the risk factors** influencing its effects for the specific evaluation criteria. Identification of risk factors can be supported by a variety of methods and tools, which include a list of previously identified risk factors, and tools supporting the retrieval and display of these factors, such as interviews with experts, group discussions, respectively brainstorming sessions and cognitive (mental) maps.

As tools for **determining the significance** of risk factors one can use matrices (graphs) for risk evaluation and sensitivity analysis. Their application we can be arrange risk factors according to their relevance, when the firstfew factors when ordered mean the key risk factors.

## 5.2 Determination of the probability distribution of risk factors

Probability distributions of risk factors can be determined either based on subjective or objective probabilities. Subjective probabilities are an expression of opinion, belief, and conviction of an expert in the field to which the risk factor applies. They are based on his knowledge, intuition, past experiences, information equipment, etc. If there are numerical data from the past available for certain risk factors, their probability distribution can be determined using statistical methods. In this case we are talking about objective probabilities.

## 5.3 Scenario development

Based on the risk factors and their probability distribution it is possible to **develop quantitative scenarios.** They can be developed as a combination of key risk factors, probabilities that the scenarios happen are given by the product of the probabilities of risk factors forming the a given scenario. Suitable tools to display these scenarios are probabilistic trees. If risk factors cannot be quantified, it is possible to use qualitative scenarios, and it is recommended to develop two to five scenarios, according to Foster (Foster, 1993, p. 123) four scenarios are more suitable: most likely, unsurprising, dreamlike and pessimistic.

## 5.4 Construction of decision matrices

Develop scenarios can be used for more **decision making matrices**. These matrices usually represent risk alternatives in rows and scenarios in the columns. In the matrix cells there are the effects of alternatives for the selected evaluation criteria. Given that for some alternatives there can be defined partly different sets of scenarios, probabilities of their occurring will be different. In this case to display the effects of multiple alternatives it is necessary to use more decision matrices. In the matrices we set mean values of effects of individual alternatives according to the criteria. Then to determine the preferential order of alternatives, and the best alternative, we can apply some of the **rules for decision support under risk**.

The expected value of risk alternatives can also be determined directly using **Monte Carlo simulation**. The result is the probability distribution of the selected evaluation criteria expressed as mean and standard deviation. Simulation is an information and application-consuming and is used more when assessing highly capital-intensive alternatives.

For each evaluation criterion its own decision making matrix should therefore be drawn up, and the results (expected values of the chosen evaluation criteria for each alternative) will then enter into the MADM.

### 5.5 Multi-attribute decision making

For the MADM it is now necessary to set criteria weights. The theory offers a number of methods for determining weights, these include methods based on direct setting of weights (e.g. scoring method, raking method) or methods based on paired comparisons (e.g. Fuller triangle Saaty's method). For a large number of criteria it is recommended to use the criteria tree that allows evaluating the importance of the criteria stages. Specific is a compensation method for determining the weights of criteria, it is based on the assessment criteria of importance from the point of possible effects of alternatives according to these criteria. The advantage of this method is that the importance of the criteria takes into account the range of the effects of alternatives and is forcing the decision maker to consider the importance of the criteria in a wider perspective.

If we have already set mean values of utilities, it is possible to directly proceed to determine the preferential ranking of the alternatives based on the total expected utilities, calculated using equation (4):

$$E(U^j) = \sum_{i=1}^{m} w_i \cdot u_i^j \qquad (4)$$

where $E(U^j)$ = expected utility of the alternative $j$,

$w_i$ = weight (importance) of the criterion $m$,

$u_i^j$ = expected utility of the alternative $j$ for the criterion $i$.

If the mean values of the effects are in their original expression, it is necessary to **convert them to the same scale**. This can be done by direct transfer to the utilities (on the interval from 0 to 1 or on the scale from 0 to 10), or by simpler methods such as the method of linear partial utility functions, Saaty's method, etc. In particular, we consider suitable application of the method of linear partial utility functions.

## Conclusion

The economic crisis has certainly tested the preparedness of companies in terms of risk management, not only in terms of the operational risk management, but in particular in the area of strategic planning and assessment of strategic alternatives under conditions of risk caused by the changing environment. The actual broad issue of multi-attribute evaluation of risk in corporate practice is relatively neglected, especially because of the difficulty using methods and tools to support decision making under risk. Managerial decision theory offers possible approaches to risk multi-attribute evaluation, usability of these tools, however, has its limitations.

One way to integrate risk to the multi-attribute decision making is in the final stage of evaluation where the first two best alternatives go through a kind of a **risk audit**. This "audit" is suitable as a basis for the establishment of an early warning system, which should be part of risk management. It is also possible to **integrate risk factors among the evaluation criteria**.

Theoretical concept of **multi-attribute utility function under risk** is exact, but its applicability is subject to fulfilling number of assumptions. It has many opponents especially in the field of behavioral economics, who criticize the use of the basic premise of the utility function: the rationality of the decision maker.

A suitable way to reach multi-attribute evaluation under risk is to **link the methods and tools of risk decision making with decision making under certainty**. The proposed concept can be divided into several steps. These include: identification of key risk factors and determining their probabilities; further scenario development, especially using quantitative probabilistic trees. Developed scenarios can be then integrated into the decision making matrices and using the rule of the mean value for each variant and each evaluation criterion it is possible to determine the mean value of risk alternatives for given scenarios. The mean values provided this way can be used in the multi-attribute decision making under certainty.

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**Contact**

Lenka Švecová
University of Economics, Prague
W. Churchille Sq. 4, Prague 3, Czech Republic
lenka.svecova@vse.cz

Jiří Fotr
University of Economics, Prague
W. Churchille Sq. 4, Prague 3, Czech Republic
jiri.fotr@vse.cz

Lucie Vrbová
University of Economics, Prague
W. Churchille Sq. 4, Prague 3, Czech Republic
lucie.vrbova@vse.cz