STATISTICAL MODELING OF ANNUAL MONTHLY MAXIMUM RAINFALL IN UPPER NORTHERN REGION OF THAILAND

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Abstract

This paper focuses on the development of statistical modelling of the extreme rainfall in upper northern region of Thailand by using the generalized extreme value distribution. Annual maximum of monthly rainfall data for the years from 1971 to 2010 are modelled for eight locations in the upper north Thailand. We found evidences of stationary for all stations that are, one fitted the Weibull, the others fitted the Gumbel distributions. We derived the return levels estimate of the median extreme rainfall and constructed 95% confidence intervals for various return periods.

Key words: generalized extreme value distribution, stationary, return level

JEL Code: C1, C16, C52

Introduction

A number of the extreme precipitation events in the upper northern region are the major causes of severe floods in the Thailand's central region. Especially in the year 2011, Thailand has been facing the worst flood in 50 years. More than two-thirds of the Thailand's central region had affected. Since the north part is the birth place of the main river sources of the Chao Phraya River in Thailand's central region, which was one of the cause of widespread flooding. For this reason we then developed a statistical modelling of extreme rainfall in Thailand's upper northern region by using the generalized extreme value (GEV) distribution. The information obtained from our study is very important in terms of strategic planning and the prevention of drought related to public safety, among others. Many researchers had analyzed the extreme daily rainfall model for many regions and many countries by using the GEV distribution such as Durrans and Kirby (2004), Nadarajah (2005), Trefry *et al*. (2005), Feng *et al*. (2007), Nadarajah and Choi (2007), Overeem, *et al*. (2009) and Park, *et al*. (2011). This paper organized as follows. Methodology of this study was presented in Section 1. The detail of data used was show in Section 2, and the results and discussion are described in Section 3.

1 Methodology

Usually, when developing statistical models to deal with the extreme data to measure the stochastic behavior of the maximum and the minimum of independent and identically distributed (iid) random variables. Only the data above some threshold value (or below the low level) are used and are discarded for the rest. This method of modelling extreme requires the estimation of probability of the events that are more extreme than any that have already been observed (Coles, 2001). Suppose $X_1, X_2, \ldots, X_n$ are random sample of size n from GEV distribution with distribution function $F(x)$ is given by

$$F(x) = \exp\left\{-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi}\right\} \tag{1}$$

defined on x such that $\left(1+\xi\left(x-\mu\right)/\sigma\right)>0$ and with location parameter μ $\left(\infty<\mu<-\infty\right)$, scale parameter σ $\left(\sigma>0\right)$ and shape parameter ξ $\left(\infty<\xi<-\infty\right)$. If $\xi>0$ $\left(\xi<0\right)$ then the GEV distribution is reduced into the Frechet (Weibull) distribution and the Gumbel distribution is obtained in the limit as $\xi \to 0$.

This research focuses on the application of the GEV distribution to develop statistical models to monitor the rainfall data in eight locations in the upper north Thailand. The models of rainfall data are fitted the GEV model with the following variations of the location parameter, μ : both the GEV modeling of the stationary and the non-sationary with linear and quadratic trend in μ process when σ and ξ are constants. We estimate parameters μ, σ and ξ by the maximum likelihood estimation and determine the best fit model by the likelihood ratio test. Probability and quantile plots were used for checking a validity of the GEV model. In the GEV model, a return level of rainfall, X_T with a return period of T years is the level exceeded on average only once in every T years, $F(x_T)=1-1/T$. Thus, the X_T is given by

$$x_T = \mu - \frac{\sigma}{\xi}\left[1-\left(-\log\left(1-\frac{1}{T}\right)\right)^{-\xi}\right] \tag{2}$$

Especially case, the median estimation of the extreme rainfall (or the 2-year return level) is given by

$$Median = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}}\left[1-\left(-\log(2)\right)^{-\hat{\xi}}\right] \qquad (3)$$

The return level of rainfall X_T can be approximated by the chi-square distribution. Under the likelihood fuction $L(\mu,\sigma,\xi)$, $L_p(x_T) = \sup_{\sigma,\xi} L(x_T,\sigma,\xi)$ and $\chi^2_{1,1-\alpha}$ the $(1-\alpha)100\%$ quantile of the chi-squared distribution with one degree of freedom. The $(1-\alpha)100\%$ confidence interval for X_T can be obtained using the following expression

$$\left\{ X_T : 2\ln\left(\frac{L(\mu,\sigma,\xi)}{L_p(x_T)}\right) < \chi^2_{1,1-\alpha} \right\} \qquad (4)$$

2 Data

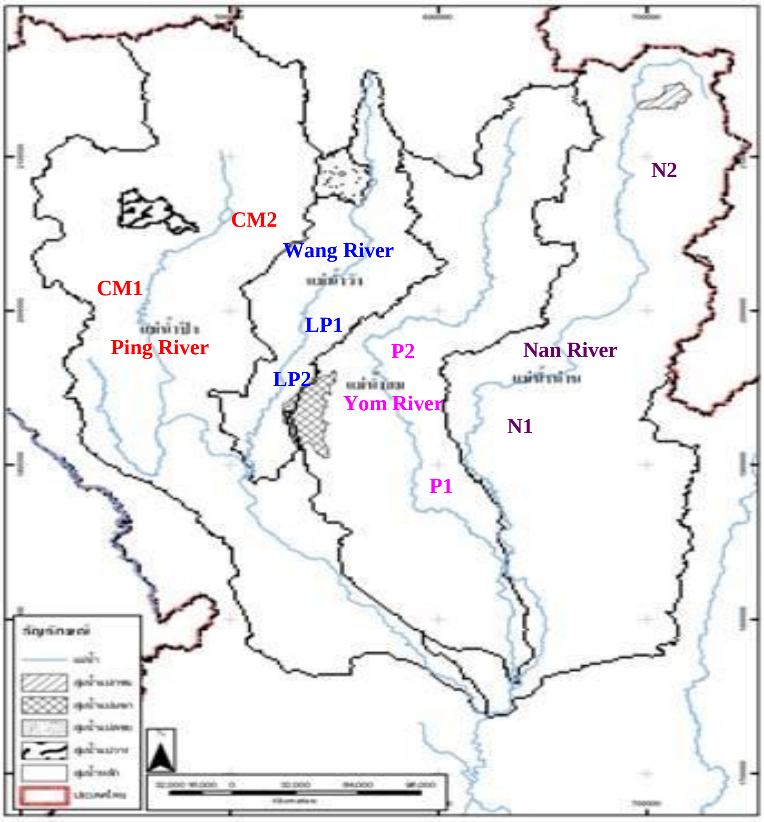
This paper we used the block-maxima method to define the extreme rainfall as the maximum of monthly rainfalls within each year. The data consist of annual maximum of monthly rainfall for the years from 1971 to 2010 in eight locations in upper north Thailand. These locations are selected from different geographical of the main river sources such as Ping River, Wang River, Yom River and Nan River of the Thailand's upper northern region pointed out in Fig. 1. The data were obtained from the hydrology and water management center for upper northern region of Thailand. We used the R Package 'ismev' (Stephenson, 2012) that is able to perform parameteric inferential analysis of the GEV distribution for each location in the phenomena listed below. Tab. 1 gives the details of data and summary of the monthly rainfall data in eight locations of the upper north Thailand.

Tab. 1: Descriptive statistics of the annual maximum of monthly rainfall data

Location	Years of data	Latitude	Longitude	Min.	Median (QD)	Max.
CM1:Mueang Chiang Mai	1971 – 2010	18° 47' 03"	99° 00' 30"	165.4	273 (60.2)	388.3
CM2: Mae Taeng	1984 – 2010	19° 12' 54	98° 58' 20"	193.0	309.9 (38.3)	508.6
LP1: Mueang Lampang	1971 – 2010	18° 17' 51"	99° 30' 56"	144.3	253.6 (64.7)	417.3
LP2: Mae Ta	1971 – 2010	18° 08' 36"	99° 37' 10"	114.0	266.4 (54.6)	464.4
P1: Mueang Phrae	1971 – 2010	18° 08' 15"	100° 07' 48"	174.7	273.5 (5736)	413.9
P2: Song	1974 – 2010	18 ° 35' 06"	100° 09' 17"	161.8	303.4 (60.6)	591.9
N1:Mueang Nan	1971 – 2010	18 ° 46' 25	100 ° 46' 59"	181.5	299.4 (60.4)	614.6
N2: Pua	1971 - 2010	8° 59' 13"	100° 56' 20"	192.2	341.2 (77.3)	585.9

Note: QD = semi inter-quartil range, unit: millimetre (mm)

Fig. 1: Geographical distribution of the eight stations in Thailand's upper northern region



CM1 = Mueang Chiang Mai

CM2 = Mae Taeng

LP1 = Mueang Lampang

LP2 = Mae Ta

P1 = Mueang Phrae

P2 = Song

N1 = Mueang Nan

N2 = Pua

Source: Adapted form the hydrology and water management center of Thailand's upper northern region

3 Results and Discussion

The models fitted to the rainfall data are stationary for all stations those were the Mueang Chiang Mai (CM1) station fitted the Weibull distribution and the others fitted Gumbel distributions. The summary of the best fitted models and the parameter estimation are exhibited in Tab. 2. We derived the return levels corresponding to 2, 10, 20, 50 and 100 years in each location of the upper north Thailand. The 95% confidence intervals for the return levels are also given in Tab. 3. Fig. 2 provides the plots for various return periods. The probability and quantile plots which were lie close to a unit diagonal supported to the fitted GEV model in each location in the appropiate phenomena listed above shown in the graphs of Fig. 3.

Tab. 2: The best fitted models and parameter estimates

Location	Best model	Parameter estimates		
		$\hat{\mu}$ (se)	$\hat{\sigma}$ (se)	$\hat{\xi}$ (se)
CM1:Mueang Chiang Mai	Weibull	260.55 (12.30)	70.71 (9.93)	-0.51 (0.12)
CM2: Mae Taeng	Gumbel	275.71 (12.63)	61.94 (8.87)	
LP1: Mueang Lampang	Gumbel	227.65 (10.66)	63.82 (7.94)	
LP2: Mae Ta	Gumbel	235.33 (11.23)	67.42 (7.90)	
P1: Mueang Phrae	Gumbel	248.64 (9.75)	58.40 (7.34)	
P2: Song	Gumbel	281.10 (11.54)	66.51 (8.24)	
N1:Mueang Nan	Gumbel	288.24 (11.36)	68.32 (8.53)	
N2: Pua	Gumbel	304.23 (14.03)	83.88 (10.28)	

Note: se = standard error

Tab. 3: The 95% return levels confidence intervals of the extreme rainfall

Location	Return levels (95% confidence interval)				
	Median	10-years	20-years	50-years	100-years
CM1:Mueang Chiang Mai	284 (261,307)	355 (339,372)	369 (353,385)	381 (363,398)	386 (367,406)
CM2: Mae Taeng	304 (276,331)	401 (361,441)	431 (383,480)	446 (403, 529)	488 (412,565)
LP1: Mueang Lampang	257 (230,284)	361 (324,399)	394 (341,446)	430 (349,511)	454 (346,562)
LP2: Mae Ta	267 (242,292)	369 (335,403)	399 (358,441)	433 (377,488)	454 (385,523)
P1: Mueang Phrae	274 (248,300)	374 (335,412)	407 (349,465)	445 (349,542)	472 (340,604)
P2: Song	308 (282,334)	424 (379,470)	465 (405,525)	516 (430,602)	552 (441,662)
N1:Mueang Nan	312 (286,339)	444 (388,500)	496 (415,577)	564 (435,694)	617 (439,794)
N2: Pua	345 (312,378)	476 (432,519)	514 (459,569)	556 (478,634)	583 (484,682)

Fig. 2: Return levels of the annual monthly maximum rainfall

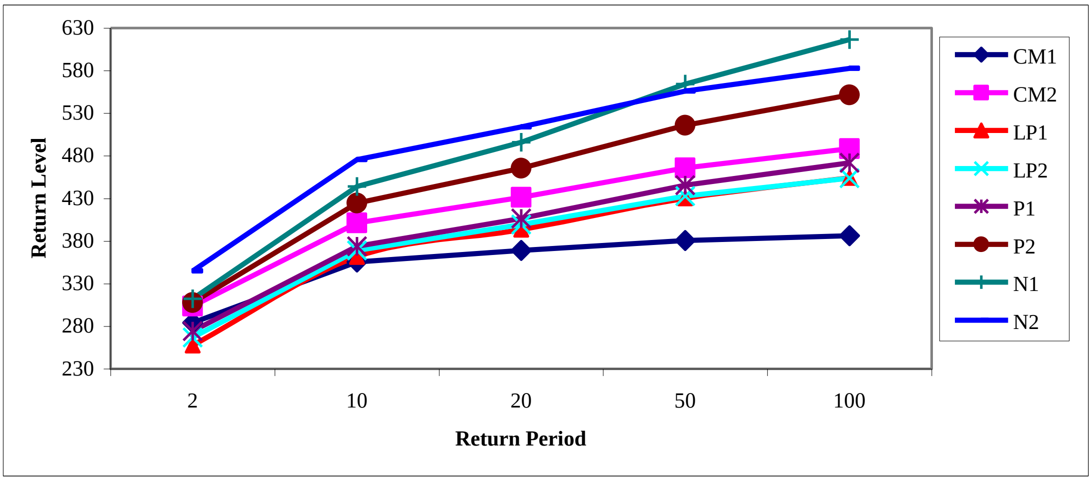
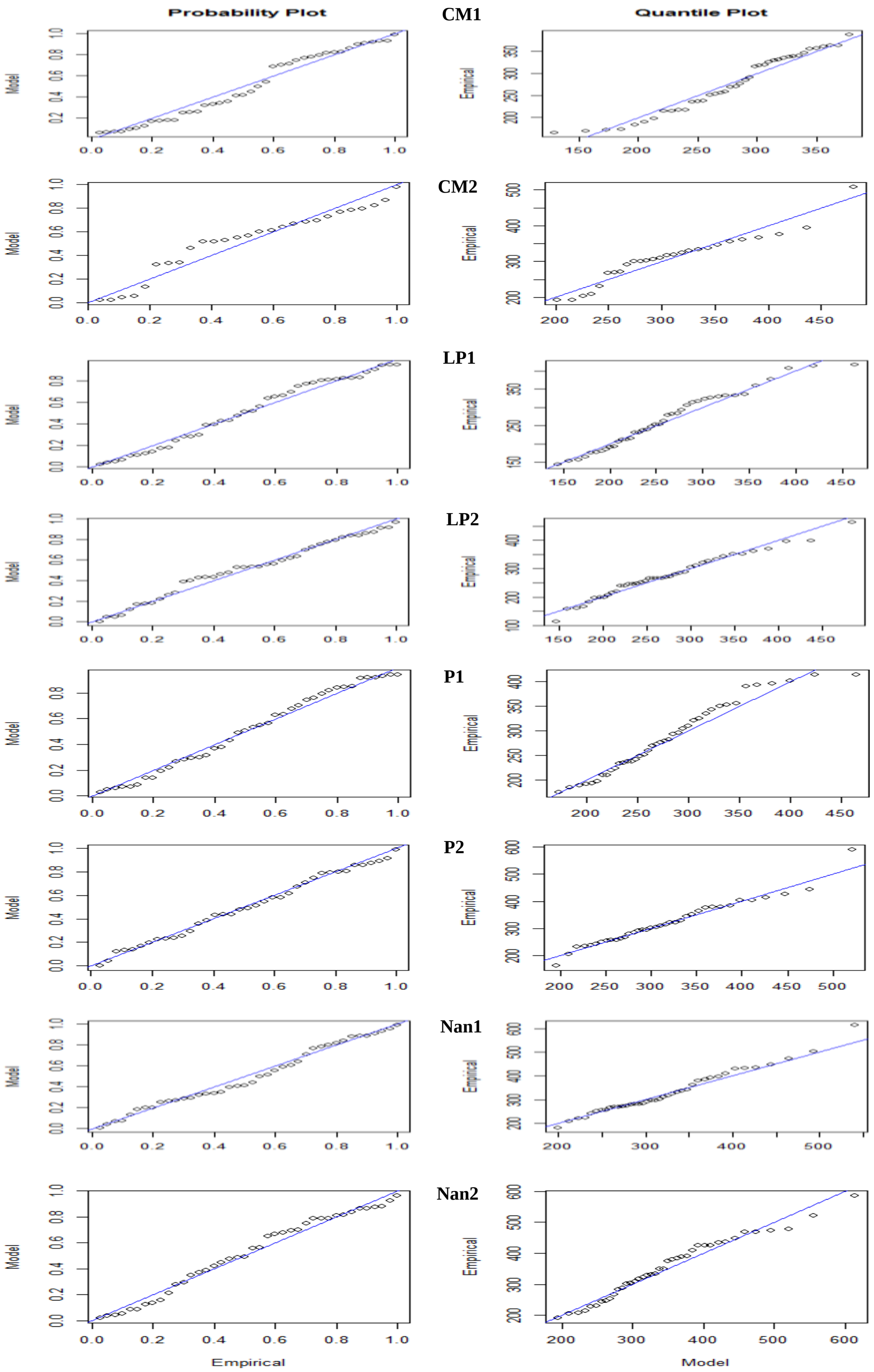


Fig. 3: Various diagnostic plots for annual monthly maximum rainfall model



Conclusion

The extreme rainfall in Thailand's upper Northern region is occurred during the middle of May through the middle of October in every year. We have devloped a statistical modelling of the annual maximum monthly rainfall data in the upper northern region of eight locations by selecting from different geographical origin of the main river sources such as Ping River, Wang River, Yom River and Nan River. By using the GEV distribution, the best model selection had an evidence of stationary for all stations. The Mueang Chiang Mai station fitted the Weibull distribution and the others fitted the Gumbel distributions. We provided the return levels and the 95% confidence intervals for return levels of the return periods of 2, 10, 20, 50 and 100 years for each station. The estimates return level of the Nan River and Yom River are higher than other stations. So it should be used to determine how to protect people by using the appropriate measures against the extreme rainfall that could lead to floods and other water-related natural disasters.

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