WORST-CASE SCENARIOS OPTIMIZATION AS A FINANCIAL STRESS TESTING TOOL FOR RISK MANAGEMENT

Mohamed El Ghourabi – Mourad Landolsi – Amira Dridi- Imed Gamoudi

Abstract

Financial stress testing (FST) is a key technique for quantifying financial vulnerabilities; it is an important risk management tool. FST should ask which scenarios lead to big loss with a given level of plausibility. However, traditional FSTs are criticized firstly for the plausibility that rose against stress testing and secondly, for being conducted outside the context of an econometric risk model. Hence, the probability of a sever scenario outcome is unknown and many scenarios yet plausible possibilities are ignored. The aim of this paper is to propose a new FST framework for analyzing stress scenarios for financial economic stability. Based on worst case scenario optimization, our approach is able first to identify the stressful periods with transparent plausibility and second to develop a methodology for conducting FST in the context of any financial-economic risk model. Applied to Tunisian economic system data, our proposed framework identifies more harmful scenarios that are equally plausible leading to stress periods not detected by classical methods.

Key words: Worst-Case Scenarios, Financial stress testing, Risk management

JEL Code: C14, C22,G21

Introduction

Stress is the product of a vulnerable structure and some exogenous shock. Financial fragility describes weaknesses in financial conditions and in the structure of the financial system. The size of the shock and the interaction of financial-system fragility determine the level of stress. A bank is exposed to the risk that the values of its assets and liabilities change in financial markets. All banks are potentially exposed to different types of economic risks, such as liquidity risk, credit risk and exchange-rate risk. Therefore, a bank failure basically can be associated with excessive risk-taking of bank managers. In fact, several empirical studies in the literature show that massive bank runs and withdrawals, lending booms and high increases in the foreign liabilities of the banking sector are among the major leading indicators of upcoming banking crises. That is why, researchers and risk managers are more interested in better understanding financial vulnerabilities of banks. One key measure of those fragilities is financial stress testing (FST). Committee on the global financial system defines FST as "a generic term describing various techniques used by financial firms to gauge their potential vulnerability to exceptional but plausible events". FST can be conducted by simulating historical stress episodes or by constructing hypothetical events built by stressing one or a group of risk factors. FST should ask which scenarios lead to big loss with a given level of plausibility. However, traditional FSTs are criticized firstly for the plausibility that rose against stress testing and secondly, for being conducted outside the context of an econometric risk model. Hence the probability of a sever scenario outcome is unknown and many scenarios yet with plausible possibilities are ignored. Many FSTs also fail to incorporate the characteristics that markets are known to exhibit in crisis periods, namely, increased probability of further large movements, increased co-movement between markets, greater implied volatility and reduced liquidity. The aim of this paper is to propose a new FST framework for analyzing stress scenarios for financial economic stability. For extreme scenarios, given that the distribution of risk factors is not elliptical, we introduce extreme value theory (EVT) as mentioned by Breuer and Csisz´ar (2010) that a given extreme scenario should be more plausible if the risk factor has fatter tails. However, for fat tails distributions where the two first moments are not known, the Mahalanobis distance as a plausibility measure is not applicable (Breuer and Csisz´ar, 2010). To overcome those pitfalls, we use an approach based on copula EVT to estimate upper bound of value at risk (VaR_{Max}) considered by several authors as a coherent risk measure used to estimate maximum loss (ML) value and hence to find the worst case scenario (WCS) namely the Copula EVT-WCS. Then, the main advantages of this approach are first the use of Copula EVT that focuses on extreme scenarios found in tail distributions. And second, for a fixed level of plausibility p, an explicit formula of WCS based on an estimation of (VaR_{Max}) is proposed.

The remainder of this paper is organized as follows. Section 1 presents an overview of FST. Section 2 presents credit risk models. Section 3 focuses on VaR based Copula EVT. Section 4 develops our proposed work relative to WCS Optimization. The empirical study is conducted in Section 5 and Section 6 provides the conclusion.

1 Financial Stress Testing

Some authors consider FST as a subgroup of risk modeling focusing on tail events that should be included in a risk model, others describe the selection of scenarios for the system-wide stress tests as an art rather than a science (e.g. Kupiec, 2001). Given difficulties involved in estimating an exact risk model, especially for FSTs, the selection of scenarios should be based on a measure of plausibility. FST can be conducted by simulating historical stress episodes or by constructing hypothetical events built by stressing one or a group of risk factors. An alternative methodology for computing stress losses is WCS: the greatest loss of the portfolio is calculated and risk managers examine which scenarios produced these extreme losses. For this purpose, we rely on an exact measure of stress that should be integrated in the FST process, this measure is called financial stress index (FSI) that will be used to overcome the drawbacks of FST. Dridi et al. (2012) proposed an FSI based on EVT and control chart to determine stress periods in the Tunisian banking system. For financial institutions, balance sheet structures, derived from aggregate financial statements, can indicate significant exposures to particular classes of assets and liabilities or income sources.

2 Credit risk models

Loan portfolio is the most significant source of risk; it is defined as the loss associated with unexpected changes in loans quality. The largest source of credit risk is loans that involve the risk of default of the counterpart. Measuring credit risk is an estimation of a number of different parameters namely probability of default (D), loss given default (LGD), which may involve estimating the value of collateral and the exposure at default (EAD). Some authors link credit risk to macroeconomic variables using econometric models. Pesola (2005) presents an econometric study of macroeconomic determinants for credit risk and other sources of banking fragility and distress in Finland. For Austria, Boss (2002) provides estimates of the relationship between macroeconomic variables and credit risk. For Norway, the Norges Bank has single equation models for household debt and house prices, and a model of corporate bankruptcies based on annual accounts for all Norwegian enterprises (Eklund et al., 2001). For Hong Kong, Gerlach et al. (2004) proposed an FST credit risk model based on a panel using bank-by-bank data. For the Czech Republic, Babouček and Jančar (2005) estimate a vector auto regression model with non-performing loans (NPL) and a set of macroeconomic variables. Similar models are also common in Financial Sector Assessment Program (FSAP) missions. For example, the technical note from the Spain FSAP includes an estimate of a regression explaining NPL on an aggregate level with financial sector indicators and a set of macroeconomic indicators.

Several shortcomings need to be considered when interpreting macroeconomic models of FST credit risk. In particular, the literature is dominated by linear statistical models. The linear approximation is reasonable when shocks are small, but for large shocks nonlinearity is likely to be important. Based on copula concept, we apply the method of worst case search over risk factor domains of certain plausibility to the analysis of portfolio credit risk.

3 Upper VaR based on copula concept

The aim of this section is to introduce the semi-parametric methodologies developed by Mesfioui and Quessy (2005) based on copula concept in order to propose in the following section, an explicit formula of WCS for credit portfolios of possibly dependent financial sectors' loans. The idea is conducted when the marginal returns distributions are in the domain of attraction of the generalized extreme value distribution and the dependence structure between financial assets remains unknown.

The concept of a copula has been first introduced by Sklar (1959) as a function which couples a joint distribution function with its univariate margins. The formal definition of this concept is a tool to model the dependence structure using the joint cumulative function of T observations. In this section, we give the basic concepts about the problem of VaR_{Max} for functions of dependent risks. The VaR at probability level $p \in [0,1]$ for a random variable X with distribution function F is defined as

$$VaR_p = F^{-1}(p). \qquad (1)$$

In order to evaluate the risk level of a portfolio of possibly dependent financial assets, we use copula. Let the multivariate distribution function of a random vector $X = (X_1, ..., X_d)$ be defined as $H(x_1,...,x_d) = P(X_1 < x_1,..., X_d < x_d)$ and denote by $F_1, ..., F_d$ the set of associated marginal distributions. The Sklar's theorem (Sklar, 1959) states that there exists a multidimensional copula C such that $H(x_1,...,x_d) = C(F_1(x_1),...,F_d(x_d))$ for all $(x_1,...,x_d) \in R^d$. For continuous marginal distributions C is unique. Let denote $F(FSI)$ the distribution function of the loans' stress, then

$$F_u(FSI) = \inf_{\sum w_i FSI_i = FSI} C_U(F_1(FSI_1),...,F_d(FSI_d)) \qquad (2)$$

The distribution function F_U, known respectively as the upper Fréchet bound, provide the best possible bounds of *F(FSI)* in terms of stochastic dominance. Following Mesfioui and Quessy (2005) result, VaR_{Max} for a given confidence level α, denoted is

$$VaR_{Max} = \inf_{\sum w_i FSI_i = p+d-1} \sum_{i=1}^{d} F_I^{-1}(\alpha). \qquad (3)$$

The above result is crucial since it allows to easily compute explicit upper VaR for possibly dependent risks as developed in the empirical part. One only needs to assume a distribution function for the marginal FSI_i.

4 Proposed Work: Worst-Case Scenarios Optimization

4.1 FSI-EVT Control Chart

We follow the approach proposed by Dridi et al. (2012) for FSI computation. We use a widely used technique namely variance-equal weight method which consists first to standardize the variables, then to aggregate those using identical weights. The FSI formula is the following:

$$FSI_t = \sum_{i=1}^{k} \frac{(x_{i,t} - \overline{x_i})}{S_i}, \qquad (4)$$

where k is the number of variables in the index, $\overline{x_i}$ is the mean of the variable X_i and S_i its standard deviation. Since our goal is a quantitative and continuous FSI relative to credit risk, we use loans for agriculture and fishing, industry, manufacturing, extractive industry and services. The choice of those variables is not restrictive and it is limited to the availability of the data. After computing the FSI, a model for their process is fitted. The latter is characterized by the presence of autocorrelation structure. Hence, linear and non linear time series model offer convenient ways of identification of the FSI model. Ghourabi and Limam (2007) show that the residual process, filtered from the original data, are independent and identically distributed with zero mean and constant variance. Based on the latter properties, we can use Vermaat's control chart for independent data (Vermaat, 2005) to monitor the FSI. Residual process is used in the FSI-EVT control chart to eliminate autocorrelated effect and detect disturbance affecting FSI process.

4.2 Proposed credit risk model

We need a model structure that can express the expected loss as a function of risk factors, then we will apply the method of worst case search over risk factor domains of certain plausibility to the analysis of portfolio credit risk, we are able at each point in time *t* to measure the expected loss of position *i* in sector *k* , for *k=1,…5.* we write the categories of risk factors separately as two vectors u_i and υ describing the innovation of macroeconomic variables at different future times.

The loan portfolio is partitioned into K sectors. The losses due to loan i from sector k are decomposed in any period t as:

$$L_{i,t} = D_{i,t}.LGD_k.EAD_i \qquad (5)$$

where LGD are homogenous within sectors, we consider $D_{i,t}$ as a default indicator. It is a binary variable equals 1 in case of default and 0 otherwise. EAD is the exposure of obligor I at default. We assume that the probability of default is homogeneous with each sector and write it as

$$\Pr(D_{i,t} = 1) = \pi_{k,t} \qquad (6)$$

We describe this variable with a probit model

$$\pi_{k,t} = \Phi(Z_{k,t}) \qquad (7)$$

With

$$z_{k,t} = \alpha_k + \sum_{j=1}^{q} \gamma_{t,j} \Delta x_{t-j} + \phi \Delta I_{t-1} + u_{k,t} \qquad (8)$$

I_t= *FSI*, $u_{k,t}$ is an idiosyncratic error term for sector k, *Φ(.)* is the cumulative distribution function of the standard normal distribution and

$$\Delta x_t = x_t - x_{t-1} \qquad (9)$$

Where $x_t = (\Delta Ln(GDP_t/GDP_{t-1})\;\; \text{Coverage Rate} \;\; \text{Inflation Rate} \;\; \text{Money Market Rate})^T$

4.3 Copula EVT-WCS analysis

Starting from a big loss which is VaR_{Max} and working backward to identify how such a loss could occur is commonly referred to among risk management professionals as reversing FST (Breuer et al., 2012). Given that we are working on extreme events and M L, we focus on tail distributions and hence we rely on copula EVT and VaR_{Max} introduced in section 3 in order to identify worst case scenario with a probability of realization p. That's why; we propose a reverse reasoning for WCS. We choose as ML the upper bound for *VaRp* namely VaR_{Max}, the loss achieved in the copula EVT-WCS. As outlined by Breuer et al. (2012), the ML is a coherent risk measure for any given feasible risk factor region.

In order to establish explicit bounds for VaR of a portfolio composed by possibly dependent assets, we combine the Dekkers et al.(1989) quantile estimator of large order with VaR bounds by Mesfioui and Quessy (2005), discussed above. Moreover, following the methodology of Vermaat et al.(2005), we provide location change invariant bound for quantile estimator of large order. The VaR_{Max} can interpreted as the WCS at a given level of plausibility, when there is no knowledge of the joint distribution of returns nor their dependence structure and only the marginal distributions are known.

<u>Proposition 1</u>

Define (Z_i) $1\leq i\leq d$ as a random variable with an absolutely continuous probability distribution function F_i. Suppose that $m_i/n(1-\alpha) \geq 1$. When no information is available about the dependence structure of $(Z_1, ..., Z_d)$, then the upper VaR bound under E-GARCH model is given by

$$\overline{VaR^*}(\alpha) = \sum_{i=1}^{d} B_i^{1/2}\left\{Z_{i(n-m_i)} + D_i\left(\frac{m_i}{nq_i(\alpha)}\right)\frac{1}{m_i}\sum_{j=1}^{m_i} Z_{i(n-j+1)} - Z_{i(n-m_i)}\right\} \quad (10)$$

where

$$q_i(\alpha) = \frac{L_i^{\frac{1}{G_{in}+1}}(1-\alpha)}{\sum_{i=1}^{d} L_i^{\frac{1}{G_{in}+1}}} \quad (11)$$

$$L_i = \lim_{k\to\infty} C_i = B_i\left(\frac{m_i}{n}\right)^{G_{in}}(\min(0,G_{in})-1)\frac{1}{m_i}\sum_{j=1}^{m_i} Z_{i(n-j+1)} - Z_{i(n-m_i)}\Big) \quad (12)$$

$$D_i\left(\frac{m_i}{nq_i(\alpha)}\right) = \frac{\left(\frac{m_i}{nq_i(\alpha)}\right)^{G_{in}} - 1}{G_{in}}(1-\min(0,G_{in})) \quad (13)$$

$$B_i = \sigma_i^{2\beta_i}\exp(\alpha_{i0} - \gamma_{i1}C_m(t)).\exp\left[\frac{\gamma_i + \alpha_i}{\sigma_i}\right] \quad (14)$$

5 Real case Study

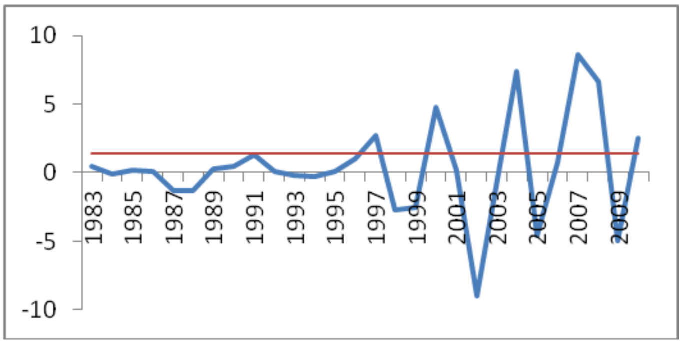
5.1 Data

Due to the annual frequency of some series, we compute a yearly index for the Tunisian banking sector from 1981 to 2010. We use mainly aggregate balance sheet data. For our case study, we choose five loan variables relative to five sectors: agriculture and fishing, industry, manufacturing, extractive industry and services. The choice of those variables is not restrictive and it is limited to the availability of the data as non performing loans which is not available for Tunisian banks for the studied period. We have 30 observations for each variable from 1981 to 2010.

5.2 FSI-EVT control chart

In the second step, we apply EVT control chart proposed by Dridi et al.(2012) as shown in Figure 1, we compute an overall FSI for five sectors namely agriculture and fishing, industry, manufacturing, extractive industry and services. Results show that 1997 is a stressed period which can be explained by the period of the Asian crisis, 2000 is also depicted as stress period mainly due to the increased loans for industry. Another stress period identified by our control chart is 2004. In fact, IMF (2004) considers that the period 2002-2004 has been much "stressed" because of serious weaknesses in the Tunisian banking sector due in part to the increase of non performing loans and difficult economic conditions in certain sectors such as in tourism. Moreover, we are able to identify 2010 as stressed period which makes sense because it corresponds to Tunisian Revolution.

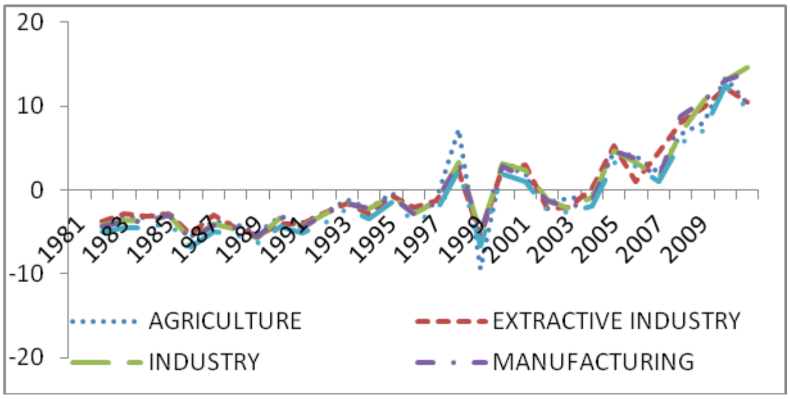
Fig. 1: FSI-EVT Control Chart



Source : authors' results

Moreover, we compute sectorial FSI to know stress levels for sectors mentioned previously. As shown in Figure 2, all sectors have almost the same shape. Indeed, as Figure 2 shows, the five sectors are far from being independent, and there is a specific dependence structure that needs a careful study to identify its nature. Therefore, we can use copula approach to deal with this dependence structure.

Fig. 2: Sectorial FSIs



Source : authors' results

5.3 Credit risk model

Tab. 3: Parameter estimates for probit models

Sectors K	Constant	ΔFSI $_{t-1}$	Money Market Rate	Inflation Rate	Coverage Rate	$LnGDP_t/GDP_{t-1}$
Agriculture	-1,2694**	0,1026**	0,7965**	0,5734**	-0,1249*	-5,0264
Industry	-1,5287**	0,2348**	0,9674**	0,5991**	-0,1287*	-6,2722*
Extractive Industry	-1,6108**	0,2533**	1,0166**	0,5981**	-0,1160*	-6,4036
Manufacturing	-1,3480**	0,1792**	0,8448**	0,5804**	-0,1265*	-4,6741
Service	-1,2083**	0,1267**	0,7406*	0,5558**	-0,1228*	-3,5511

(*) indicates significance at the 90% level and (**) indicates significance at the 95% level.

Source : authors' results

Table 3 presents relationship between sectorial FSIs and macroeconomic risk factors. GDP and Coverage rate vary in the opposite sense compared to FSI patterns that is when those two risk factors decreases, stress level will increase. As for Money market rate and inflation rate, they go in the same direction with FSI, which makes sense because stress level increase when inflation and money market rates increase. Intuitively, Industry and Extractive Industry turn out to be very sensitive to the macro-economic factors. A much weak relationship is registered for Agriculture and Service. For instance, the least effect is explained by the FSI of agriculture which should be clarified by the fact that Tunisian government usually supports this sector by rescheduling and facilities.

5.4 Copula EVT-WCS analysis

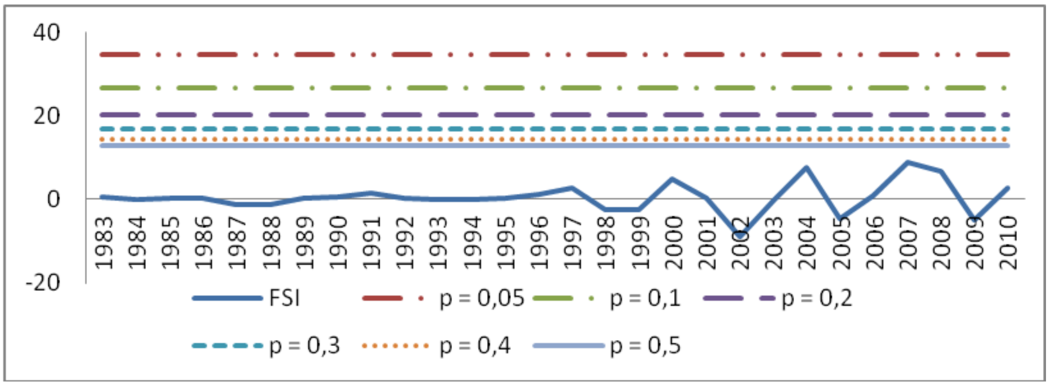
Tab. 4: WCS at level p for Tunisian credit risk

Level of Plausibility	*p = 0,05*	*p = 0,1*	*p = 0,2*	*p = 0,3*	*p = 0,4*	*p = 0,5*
WCS	34,4477196	26,619072	20,0377004	16,6852954	14,5048862	12,9185344

Source : authors' results

We apply our approach on Tunisian banking system; the value of WCS at level p of plausability is given in Table 4 and Figure 3. Following Figure 3, Tunisian banks do not achieve the WCS yet. As the value of the WCS increases, the level of plausibility decreases and approaches to zero. The very severe extreme scenario has a lower probability and usually tends to zero. Here, we talk about the trade-off between severity and plausibility outlined by Breuer et al. (2012), " *the more extreme the scenarios which are considered the less plausible they become: There is a trade-off between severity and plausibility of stress scenarios* ".

Fig3 : WCS at level p for Tunisian credit risk



Source : authors' results

Conclusion

How can one explain the fact that the less plausible extreme scenarios are realized and having very serious consequences? So, the financial institutions should seek for a tradeoff between the severity and the plausibility of stress scenarios. FST is only a starting point for analyzing the vulnerability of a financial system and can be particularly useful when they are conducted regularly, because this can provide information about changes in the risk profile of the system over time. An important shortcoming of FST is the danger to ignore harmful but plausible scenarios. This can create an illusion of safety. A way to overcome this disadvantage is to search systematically for worst case macro scenarios in some plausible domain. In this paper, we propose a reverse reasoning for WCS. We choose as ML, the upper bound for VaR namely VaR_{Max}, the loss achieved in the copula EVT-WCS in order to identify worst case scenario with a probability of realization α. Empirical results show that the more extreme the scenarios which are considered the less plausible they become.

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Contact

Mohamed El Ghourabi,
LARODEC-ISG-Tunis,
41, Rue de la Liberté, Le Bardo, 2000, Tunis, Tunisia.
mohamed.elghourabi@gmail.com

Mourad Landolsi,
ESSEC-Tunis,
4, Rue Abou Zakaria El Hafsi, Montfleury, 1089, Tunis, Tunisia
Landolsi.mourad@gmail.com

Amira Dridi, LARODEC-ISG-Tunis
41, Rue de la Liberté, Le Bardo, 2000, Tunis, Tunisia
amiradridi@laposte.net

Imed Gamoudi,
ESSEC-Tunis-Tunisia
4, Rue Abou Zakaria El Hafsi, Montfleury, 1089, Tunis, Tunisia.
imed.gamoudi@gmail.com