CALCULATION OF LTPD SINGLE SAMPLING PLANS FOR INSPECTION BY VARIABLES AND ITS SOFTWARE IMPLEMENTATION

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Abstract

In this paper we recall some properties of LTPD single sampling plans when the remainder of rejected lots is inspected. We consider two types of LTPD plans - for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected by attributes) and compare these plans with the corresponding Dodge-Romig LTPD plans by attributes. From the results of numerical investigations it follows that under the same protection of consumer the LTPD plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans. We discuss algorithm for calculation of these plans when the non-central t distribution is used for the operating characteristic. The calculation is considerably difficult and we use an original method, recently implemented in R software package. We briefly introduce our software and demonstrate its functionality for LTPD plans calculation and evaluation.

Key words: sampling inspection by variables, LTPD plans, software R

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Introduction

Acceptance sampling is one of the techniques used in quality control, either in vendor-buyer relationships or for management of within-company processes. The aim is to meet desired levels of protection against risk while keeping an eye on economic characteristics of the process. Inference is made based on inspection of a sample of items taken from a lot.

Depending on quality of the sample, the whole lot may be either accepted or rejected, or inspection of another sample may follow in case of double, multiple or sequential sampling plans. Acceptance sampling plans, specified by sample size and critical value (or acceptance number), determine the rules for this decision process.

There are many ways of classifying acceptance sampling. One such classification is according to whether an item is inspected by attributes, i.e. just classified as either good or defective (nonconforming) or by variables. Sampling plans for inspection by variables in many cases allow obtaining same level of protection as the corresponding sampling plans for inspection by attributes while using lower sample size. The basic notions of variables sampling plans are addressed in (Jennett and Welch, 1939).

Another important classification of sampling plans is according to type of quality levels which are considered. One possibility is that two quality levels (producer and consumer quality level) are specified, together with the corresponding probability of acceptance. Solution for finding such plans and its software implementation is discussed in (Kiermeier, 2008). Another problem is solved in (Dodge and Romig, 1998), where LTPD sampling plans for inspection by attributes when remainder of rejected lots is inspected, minimizing the mean number of items inspected per lot of process average quality are introduced. Acceptance sampling by variables when the remainder of rejected lots is inspected and LTPD sampling plans of Dodge-Romig type for inspection by variables is addressed in (Klůfa, 1994) and (Klůfa, 2010).

Since efficient implementation of procedures for calculation of these sampling plans had not been available, we implemented methods for calculation and evaluation of these plans in software package (Kaspříková, 2011).

1 LTPD plans for inspection by attributes

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes), (Dodge and Romig, 1998) consider sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\bar{p}; n; c) \tag{1}$$

under the condition

$$L(p_t; n; c) = 0.10 \tag{2}$$

(LTPD single sampling plans), where N is the number of items in the lot (the given parameter), $\bar{p}$ is the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, $P_t = 100 p_t$ is the lot tolerance per cent defective, denoted LTPD), n is the number of items in the sample *(n<N)*, c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c), *L(p, n, c)* is the operating characteristic (the probability of accepting a submitted lot with proportion defective p when using plan *n, c*).

Condition (2) protects the consumer against the acceptance of a bad lot – the probability of accepting a submitted lot of tolerance quality p_t (consumer's risk) shall be 0.10.

2 LTPD plans for inspection by variables and attributes

The problem to find LTPD plans for inspection by variables has been solved in (Klůfa, 1994) under the following assumptions:

Measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s.

The inspection procedure is as follows:

Draw a random sample of n items and compute, sample mean $\bar{x}$ and sample standard deviation s. Accept the lot if

$$\frac{U-\bar{x}}{s} \geq k, \text{ or } \frac{\bar{x}-L}{s} \geq k. \tag{3}$$

The task to be solved is determination of the sample size n and the critical value k. There are different solutions of this problem. In paper (Klůfa, 1994) similar conditions to the approach of (Dodge and Romig, 1998) were used for determination of n and k.

Now we shall formulate this problem. Let us consider *LTPD plans for inspection by variables and attributes* – all items from the sample are inspected by variables, but the remainder of rejected lots is inspected only by attributes. Let us denote

c_s^* - the cost of inspection of one item by attributes,

c_m^* - the cost of inspection of one item by variables.

Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is $n \cdot c_m^*$ with probability $L(p;n,k)$, and $\left[n \cdot c_m^* + (N-n) \cdot c_s^*\right]$ with probability $\left[1 - L(p;n,k)\right]$ The mean inspection cost per lot of process average quality is therefore

$$C_{ms} = n \cdot c_m^* + (N-n) \cdot c_s^* \cdot \left[1 - L(\bar{p};n,k)\right]. \tag{4}$$

Now we will look for the acceptance plan *(n,k)* minimizing the mean inspection cost per lot of process average quality C_{ms} under the condition

$$L(p_t;n;k) = 0.10 \tag{5}$$

The condition (5) is the same one as used for protection of the consumer in (Dodge and Romig, 1998). Let us introduce a function

$$I_{ms} = n \cdot c_m + (N-n) \cdot \left[1 - L(\bar{p};n,k)\right] \tag{6}$$

where

$$c_m = c_m^* / c_s^*. \tag{7}$$

Since

$$C_{ms} = I_{ms} \cdot c_s^*, \tag{8}$$

both function C_{ms} and I_{ms} have a minimum for the same acceptance plan *(n, k)*. Therefore, we shall look for the acceptance plan *(n, k)* minimizing (6) instead of (4) under the condition (5).

For these LTPD plans for inspection by variables and attributes *the new parameter* c_m was defined – see (7). This parameter must be estimated in each real situation. Usually is $c_m > 1$.

Putting formally $c_m = 1$ into (6) (I_{ms} in this case is denoted I_m) we obtain

$$I_m = N - (N - n) \cdot L(\bar{p}; n, k) \tag{9}$$

i.e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables. Consequently *the LTPD plans for inspection by variables* are a special case of *the LTPD plans by variables and attributes* for $c_m = 1$. From (9) is evident that for the determination of LTPD plans by variables it is not necessary to estimate c_m.

Summary: For the given parameters p_t, N, $\bar{p}$ and c_m we must determine the acceptance plan *(n,k)* for inspection by variables and attributes, minimizing I_{ms} in (6) under the condition (5).

In the first place we shall deal with the solution of the equation (5). The operating characteristic is (for both exact and approximate relation for operating characteristic see (Jennett and Welch, 1939) and (Johnson and Welch, 1940))

$$L(p; n, k) = \int_{k\sqrt{n}}^{\infty} g(t; n-1, u_{1-p}\sqrt{n})\, dt, \tag{10}$$

where $g(t; n-1, u_{1-p}\sqrt{n})$ is probability density function of non-central *t*-distribution with *(n-1)* degrees of freedom and noncentrality parameter $\lambda = u_{1-p}\sqrt{n}$.

Instead of (10), using the normal distribution as an approximation of the non-central t-distribution we have

$$L(p; n, k) = \Phi\left(\frac{u_{1-p} - k}{A}\right), \tag{11}$$

where

$$A = \sqrt{\frac{1}{n} + \frac{k^2}{2(n-1)}}. \tag{12}$$

The function Φ in (12) is a standard normal distribution function and u_{1-p} is a quantile of order 1 – *p*. The approximation (12) holds both for an upper specification limit *U* and for a lower specification limit *L*.

If we use (11) for operating characteristics, the equation $L(p_t; n, k) = 0.10$ has one and only one solution (Klůfa, 1994)

$$k = \frac{u_{1-p_t} - u_{0.10} \cdot h}{g}, \tag{13}$$

where

$$g = 1 - \frac{u_{0.10}^2}{2(n-1)}, \quad h = \sqrt{\frac{g}{n} + \frac{u_{1-p_t}^2}{2(n-1)}}. \tag{14}$$

This is an approximate solution of the equation (5). Exact solution of the equation (5) is

$$k = \frac{t_{0.9}(n-1, u_{1-p_t}\sqrt{n})}{\sqrt{n}}, \tag{15}$$

where $t_{0.9}(n-1, u_{1-p_t}\sqrt{n})$ is a quantile of order 0.9 of non-central t-distribution with *(n-1)* degrees of freedom and noncentrality parameter $\lambda = u_{1-p}\sqrt{n}$.

Inserting formula for *k* into I_{ms} function, we obtain a function of one variable *n*

$$I_{ms}(n) = n \cdot c_m + (N-n) \cdot \alpha(n), \tag{16}$$

where $\alpha(n)$ is the producer's risk (the probability of rejecting a lot of process average quality). Now we look for the sample size n minimizing (16). For theorem on relation between lot size and sample size see (Klůfa, 1994).

It was shown that under the same protection of the consumer the LTPD plans for inspection by variables and attributes are in many situations more economical than the corresponding Dodge-Romig LTPD attribute sampling plans. This conclusion is valid especially for the large lots and for the small values of the lot tolerance fraction defective – see (Klůfa, 1999).

3 Calculation and evaluation of LTPD plans for inspection by variables and attributes

LTPDvar (Kaspříková, 2011) is an add-on package to the R software (R Development Core Team 2008), in which methods for calculation of the LTPD plans by variables are implemented and which can be used for finding the appropriate plan as well as for evaluation of a plan. R software with LTPDvar package will be used for calculations in the following example.

Example. Let N = 450, p_t =0.01, $\bar{p}$ =0.0015, and c_m =1.7 (the cost of inspection of one item by variables is by 70% higher than the cost of inspection of one item by attributes). We will find the LTPD plan for inspection by variables and attributes and compare this plan with the corresponding Dodge-Romig LTPD plan for inspection by attributes.

Function planLTPD in LTPDvar package searches for the sample size n minimizing $I_{ms}(n)$ and gives plan with resulting n and corresponding k as output. In planLTPD function if input parameter method="napprox", approximate operating characteristic is used and the solution is obtained using procedure described in (Klůfa, 1994). If method="exact" (default), exact relation for operating characteristic is used and the optimization procedure searches for n in interval with centre at n resulting from planLTPD(..., method = "napprox"), allowing getting the solution rather quickly.

Approximate solution is obtained using the following command.

```
> planLTPD(N=450, pt=0.01, pbar=0.0015, cm=1.7, method="napprox")
An object of class "ACSPlan"
Slot "n":
[1] 67
```

Slot "k":

[1] 2.662032

Approximate solution is *n* = 67, *k* = 2.662032. For this plan, consumer's risk is only approximately 0.10.

For exact solution we make use of default value of method input parameter.

```
>  planLTPD(N=450, pt=0.01, pbar=0.0015, cm=1.7)
An object of class "ACSPlan"
Slot "n":
[1] 67

Slot "k":
[1] 2.670840
```

The LTPD plan for inspection by variables and attributes is *n* = 67, *k* = 2.67084.

For assessment of producer's risk we make use of operating characteristic function. Operating characteristic is in package LTPDvar available in OC function, which takes as its argument sample size, critical value and proportion defective.

```
> 1-OC(n=67, k=2.670840, p=0.0015)
[1] 0.1216439
```

The corresponding LTPD plan for inspection by attributes can be found in (Dodge and Domig, 1998). For given input parameters we get *n* = 180, *c* = 0. So the sample size of the Dodge-Romig plan for inspection by attributes is higher and moreover the producer's risk is higher, it can be calculated (see (Hald, 1981)) as

```
> 1-choose(0.0015*450,0)*choose((1-0.0015)*450,180)/ choose(450,180)
[1] 0.291528
```

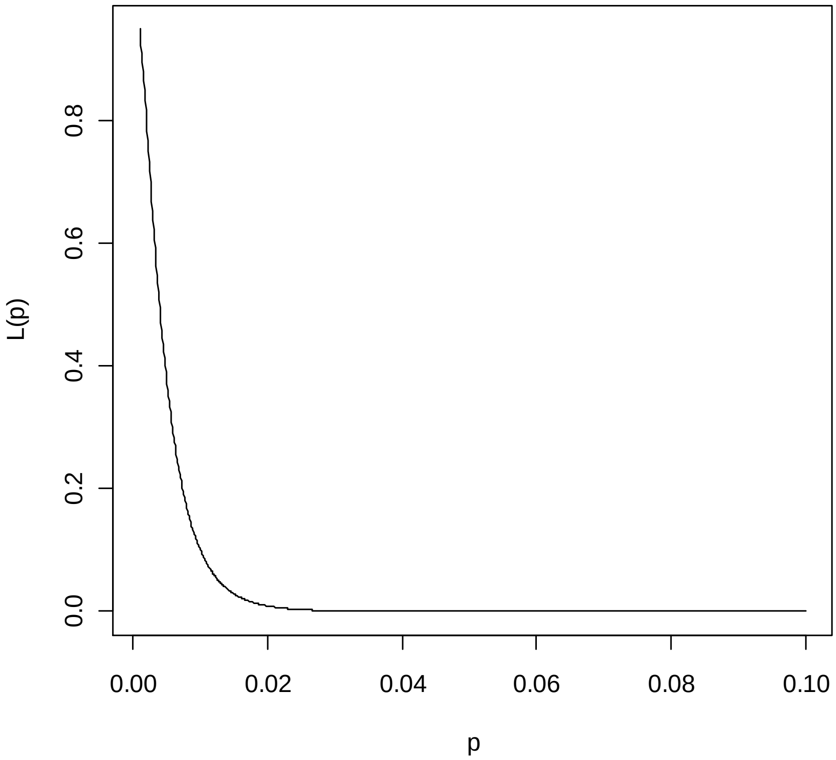
LTPDvar package makes use of S4 classes and methods, for details see e.g. (Gentleman, 2008). ACSPlan class has been defined for sampling plans. An output of planLTPD function is an object of class ACSPlan. Several methods exist for ACSPlan class, among others plot method has been created for this class, plotting proportion defective on the horizontal axis and probability of acceptance on the vertical axis.

Plot of operating characteristic curve corresponding to plan in our example can easily be produced with command

> plot(planLTPD(N=450, pt=0.01, pbar=0.0015, cm=1.7))

the resulting output graphic is shown in Fig. 1.

Fig. 1: Operating characteristic of plan n=67, k=2.670840



Source: authors using LTPDvar software

Conclusion

LTPD sampling plans for inspection by variables when remainder of rejected lots is inspected were discussed. It was shown that tools for finding such plans which minimize mean cost of inspection per lot of process average quality are now available. Besides calculation of plans, functionality of R software add-on package LTPDvar includes tools for evaluation of plans and for graphical display of operating characteristic of the sampling plan.

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